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Existence of non measurable sets


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EXISTENCE OF NON MEASURABLE SETS

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K. Kuratowski asked the following question:

(Q) Given a partition \( \{ A_\xi : \xi \in \mathcal{S} \} \) of the unit interval into pairwise disjoint sets of Lebesgue measure zero, does there exist a set \( S_0 \subseteq \mathcal{S} \) such that the union \( \bigcup_{\xi \in S_0} A_\xi \) is not Lebesgue measurable.

Assuming the continuum hypothesis, K. Kuratowski answered affirmatively this question.

Using models of set theory, namely generic extensions and ultrapower construction, I can prove the positive answer of (Q). The definability of the Lebesgue measure by a simple formula is essentially used in the proof.

Let us remark that existence of a real-valued measurable cardinal implies existence of an extension of the Lebesgue measure for which the answer is negative.

Similar result holds true for partitions into meager sets and the Baire property.

As far as I am informed, no elementary proofs of presented results are known.