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Cartesian-closed Coreflective Subcategories of Uniform Spaces

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(abstract)

Let Unif be the category of Hausdorff uniform spaces. We study subfamilies $\mathcal{A}$ of Unif whose coreflective hull co($\mathcal{A}$) is cartesian-closed. For coreflective subcategories of Unif this means that there must exist function spaces Y^X over the hom-sets U(X,Y) making the exponential law $Y^{X \times Z} = (Y^X)^Z$ valid for all spaces X,Y,Z in the category.

We are particularly interested in the function spaces $\hat{U}(X,Y)$, equipped with the uniformity of uniform convergence, as possible exponential spaces.

For an infinite cardinal α, let B(α) be the class of all uniform spaces which have covering character at most α and which admit all cardinals less than α.

Theorem. Let $\mathcal{A} \subseteq$ Unif and let c be the coreflector to co($\mathcal{A}$). The following are equivalent:

(1) co($\mathcal{A}$) is cartesian-closed with exponentials $Y^A = c\hat{U}(A,Y)$ for all $A \in \mathcal{A}$ and $Y \in$ co($\mathcal{A}$).

(2) There exists a finitely productive subfamily $\mathcal{A}'$ of locally fine spaces such that $\mathcal{A} \subseteq \mathcal{A}' \subseteq$ co($\mathcal{A}$).

(3) There exists a finitely productive subfamily $\mathcal{A}'$ of B(α), for some $\alpha \geq \aleph_0$, such that $\mathcal{A} \subseteq \mathcal{A}' \subseteq$ co($\mathcal{A}$).

If these conditions are satisfied, then the closed unit interval I belongs to co($\mathcal{A}$) if and only if all spaces in $\mathcal{A}$ are precompact, and $I \notin$ co($\mathcal{A}$) if and only if all spaces in $\mathcal{A}$ admit $\aleph_0$.

Examples. co(B(α)) is cartesian-closed for each $\alpha \geq \aleph_0$.

Cartesian-closed coreflective subcategories of Unif need not be formed in this way, however. For example, if $\mathcal{A}$ is the class of all precompact proximally discrete uniform spaces, then co($\mathcal{A}$) is cartesian-closed since its product preserves sums and quotients. However, there is no finitely productive family $\mathcal{A}'$ in Unif such that co($\mathcal{A}$) = co($\mathcal{A}'$). Therefore, some of the exponential spaces in this category must differ from $c\hat{U}(X,Y)$.