Władysław Kulpa

Some remarks on paracompactness of GO-spaces

In: Zdeněk Frolík (ed.): Abstracta. 7th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1979. pp. 40–41.

Persistent URL: http://dml.cz/dmlcz/701144

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by

W. Kulpa (Katowice).

Let $X = (X, <)$ be a linearly ordered set. A subset $C \subset X$ is said to be convex, whenever $a, b \in C$ and $a < b$ imply that $[a,b] \subset C$. A convex set $C \subset A$, $A \subset X$, is called a convexity-component of A, whenever $C' \cap C \neq \emptyset$ implies $C' \subset C$ for each convex set $C' \subset A$.

A linearly ordered topological space X is a triple $X = (X, <, \lambda(<))$ where $(X, <)$ is a linearly ordered set on which a topology is defined by the subbase of all sets $(\leftarrow, a), (b, \rightarrow)$ with $a, b \in X$.

A generalized ordered space X (abbreviated GO-space) is a triple $X = (X, <, \tau)$, where $\tau \supset \lambda(<)$ is a topology with a base consisting of convex subsets of X.

A continuous map $f: X \rightarrow M$ from a GO-space X is said to be convexity-paracompact (convexity-zerodimensional) iff each convexity component of $f^{-1}(m)$, $m \in M$, is paracompact (is a one-point set).

<u>Theorem 1.</u> If a GO-space X has a convexity-paracompact map into a metric space, then X is paracompact.

<u>Theorem 2.</u> If a GO-space X has a perfect map onto a Dieudonné complete space, then X is paracompact.

Theorem 1 strengthens a result of Faber [[illegible]] who has proved that each GO-space which has a convexity-zerodimensional map into a metric space must be paracompact. Theorem 2 is a strengthening of a result of Lutzer [4] who has proved that each Dieudonne complete GO-space is paracompact.

The above theorems one can obtain from the Pressing-Down Lemma and from the following results:

Factorization Theorem [3]. Let $f: X \to M$ be a continuous map from a GO-space $X = (X, <, \tau)$ into a metric space M. Then there exists a metric GO-space $Z = (Z, \prec, \mathcal{T})$ and continuous maps $g: X \xrightarrow{\text{onto}} Z$, $h: Z \to M$ such that $f = h \circ g$ and $g(x) \preceq g(y)$ whenever $x \leq y$, for each $x, y \in X$.

Lemma [illegible]. If a GO-space X has a continuous map $f: X \to M$ into a space M with a G_δ-diagonal such that for each $m \in M$, $f^{-1}(m)$ is paracompact, then X paracompact.

Lemma [illegible]. If $f: S \to M$ is a continuous map from a stationary set $S \subset \kappa$, $\kappa = \mathrm{cf}\,\kappa > \omega$, into a space M with a G_δ-diagonal, then there is an $\alpha < \kappa$ such that $f|S \cap [\alpha, \kappa)$ is constant.

The last Lemma generalizes a result of Lutzer [4] who has proved that if $f: S \to M$ is a continuous map of a stationary set $S \subset \kappa$, $\kappa = \mathrm{cf}\,\kappa > \omega$, into a metric space M, then $\mathrm{card}(f[S]) < \kappa$.

Theorem [1]. A GO-space X is not paracompact iff some closed subspace of X is homeomorphic to a stationary set $S \subset \kappa$, $\kappa = \mathrm{cf}\,\kappa > \omega$.

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Uniwersytet Śląski, Instytut Matematyki, ul. Bankowa 14
40-007 K a t o w i c e, Poland