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On a Michael's conjecture concerning the Lindelöf property in the Cartesian products

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It is known that if Y is a hereditarily Lindelöf space and X is a separable metric space or a Lindelöf complete in the sense of Čech space or a C -scattered space then $Y \times X^{\aleph_0}$ is Lindelöf. Let us recall that X is a C -scattered space if every closed subset F of X contains compact set with non-empty interior with respect to F . The first result, mentioned above, is due to S. Willard, second one due to Z. Frolik, third one due to K. Alster.

Michael conjectured that if $Y \times X$ is Lindelöf for every hereditarily Lindelöf space Y then $Y \times X^{\aleph_0}$ is Lindelöf for every hereditarily Lindelöf space Y .

The answer to the Michael's conjecture is a negative one provided that the condition (*) holds.

The condition (*) says that

- (*) there exists an uncountable coanalytic subset of the Cantor set which does not contain uncountable compact subsets.

Gödel and P.S. Novikov proved that (*) holds under the Gödel's axiom constructibility. L. Bukovský, D.A. Martin, R.M. Solovay and P. Vopěnka defined a model of set theory such that $\aleph_1 < 2^{\aleph_0}$ and every subset of the Cantor set of cardinality $\aleph_1$ is coanalytic. R.M. Solovay proved that if a measurable number exists then every coanalytic set contains the Cantor set.

Under the condition (κ) I have obtained the following two examples.

Example 1 (κ) . There exists X such that for every hereditarily Lindelöf space Y and every natural number n the product $Y \times X^n$ is Lindelöf but X^{N_0} is not.

Example 2 (κ) . There exist a separable metric space M and a space Z such that for every Lindelöf space Y and every natural number n the products $Y \times Z^n$ and Z^{N_0} are Lindelöf but $M \times Z^{N_0}$ is not.

Let me finish with the following three problems.

(1) Let $Y \times X$ be a Lindelöf space for every Lindelöf space Y . Is it true that X^{N_0} is Lindelöf.

(2) Let $Y \times X$ be a Lindelöf space for every hereditarily Lindelöf space Y . Is it true that X^2 is Lindelöf.

(3) Is it possible to obtain Example 1 and 2 without set theoretical assumptions.