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## On universal null and universally measurable sets, II

by

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Let  $I$  denote the unit interval,  $\mathcal{B}$  the  $\sigma$ -field of Borel subsets of  $I$ ,  $\mathcal{M}$  the  $\sigma$ -field of universally measurable subset of  $I$  and let  $\mathcal{N}$  denote the  $\sigma$ -ideal of universal null subset of  $I$ . By  $\mathcal{L}$  we denote the  $\sigma$ -field of Lebesgue measurable subsets of  $I$  and by  $\mathcal{L}_0$  the  $\sigma$ -ideal of Lebesgue measure zero subsets of  $I$ .

**Theorem.** (i) There exists a continuous function  $f : I \rightarrow I$  such that  $f(\mathcal{N}) \not\subset \mathcal{L}$ .

(ii) If  $f : I \rightarrow I$  is a function of bounded variation on  $I$  then  $f(\mathcal{M}) \subset \mathcal{L}$  (and hence  $f(\mathcal{N}) \subset \mathcal{L}_0$ ).

(iii) There exists an infinitely differentiable function  $f : I \rightarrow I$  such that  $f(\mathcal{N}) \not\subset \mathcal{M}$ .

(iv) Let  $f : I \rightarrow I$  be a Borel measurable function.

Then we have:

$f(\mathcal{N}) \subset \mathcal{M}$  iff  $f$  is bimeasurable,

$f(\mathcal{N}) \subset \mathcal{N}$  iff  $f(\mathcal{N}) \subset \mathcal{M}$  iff  $f(\mathcal{M}) \subset \mathcal{M}$ .

It follows from Theorem (i) the following

**Corollary.** There exists a universal null subset of  $I \times I$  such that one of its projections onto  $I$  is not in  $\mathcal{L}$ .

Theorems (i), (iii), (iv) and Corollary were earlier known under CH (or MA) and/or in a weaker form (Sierpiński 1938, Darst 1970, 1971, 1973, Laver 1976, Mauldin 1978, Grzegorek and Ryll-Nardzewski 1978).

Our Corollary immediately eliminates CH in the assumption of the solution by Darst [1] of a problem of Choquet about capacity and universal null sets.

Details and exact references can be found in [2] .

#### References

- [1] R.B. Darst: On the connection between Hausdorff measures and generalized capacity, Fund.Math.80 (1973), 1-3
- [2] E. Grzegorek: On some results of Darst and Sierpiński concerning universal null and universally measurable sets, in preparation to Bull.Acad.Polon.Sci.Ser.Sci.Math. Astronom.Phys.