Edward Grzegorek

On universal null and universally measurable sets, II

In: Zdeněk Frolík (ed.): Abstracta. 8th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1980. pp. 94--95.

Persistent URL: http://dml.cz/dmlcz/701185

On universal null and universally measurable sets, II

by

E. Grzegorek

Let I denote the unit interval, B the σ-field of Borel subsets of I, M the σ-field of universally measurable subset of I and let N denote the σ-ideal of universal null subset of I. By L we denote the σ-field of Lebesgue measurable subsets of I and by L_0 the σ-ideal of Lebesgue measure zero subsets of I.

Theorem. (i) There exists a continuous function $f : I \to I$ such that $f(N) \not\subset L$.

(ii) If $f : I \to I$ is a function of bounded variation on I then $f(M) \subset L$ (and hence $f(N) \subset L_0$).

(iii) There exists an infinitely differentiable function $f : I \to I$ such that $f(N) \not\subset M$.

(iv) Let $f : I \to I$ be a Borel measurable function. Then we have:

$$f(N) \subset M \text{ iff } f \text{ is bimeasurable,}$$

$$f(N) \subset N \text{ iff } f(N) \subset M \text{ iff } f(M) \subset M .$$

It follows from Theorem (i) the following

Corollary. There exists a universal null subset of $I \times I$ such that one of its projections onto I is not in L.

Theorems (i), (iii), (iv) and Corollary were earlier known under CH (or MA) and/or in a weaker form (Sierpiński 1938, Darst 1970, 1971, 1973, Laver 1976, Mauldin 1978, Grzegorek and Ryll-Nardzewski 1978).

Our Corollary immediately eliminates CH in the assumption of the solution by Darst [1] of a problem of Choquet about capacity and universal null sets.

Details and exact references can be found in [2].

References

[1] R.B. Darst: On the connection between Hausdorff measures and generalized capacity, Fund.Math.80 (1973), 1-3

[2] E. Grzegorek: On some results of Darst and Sierpiński concerning universal null and universally measurable sets, in preparation to Bull.Acad.Polon.Sci.Ser.Sci.Math.Astronom.Phys.