Jiří Souček

The complex probability theory as a basis of quantum theory

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Let us consider a probability theory (Ω, Σ, P), where Ω is an universal event, Σ an algebra of events and P the probability measure on Σ. The quantity $P(E)$, $E \in \Sigma$ is not directly observable. The observable quantity is a relative frequency $p(E) = N_{successful}/N_{all}$ and the Law of large numbers states that $p(E) = P(E)$. Thus $P(E)$ must be a positive real number for each observable event E. But in the quantum theory <u>there are principially unobservable events</u> (e.g. a trajectory of an electron). Probability $P(E)$ of such events need not be positive and may be (for example) a complex number. We shall show that such a complex theory of probability ($\mathbb{C}$-TP) forms a basis of quantum theory. Proof of so called Bell's inequalities (which disqualify the hidden parameters theories) are based on the positivity of the distribution of hidden parameters. In $\mathbb{C}$-TP these proofs do not go through and thus the hidden parameters theory is principially possible within $\mathbb{C}$-TP.

We suggest the following concept of the trajectory of an electron (in non-relativistic theory): it is the couple $\gamma = (\gamma_+, \gamma_-)$, $\gamma_+, \gamma_- : \mathbb{R} \to \mathbb{R}^4 = \{(x_0, \vec{x})\}$ of two trajectories γ_+, γ_- such that $\frac{d}{d\tau} x_0(\gamma_\pm(\tau)) \gtrless 0$. $\mathbb{C}$-TP is the system (Ω, Σ, Φ), where $\Omega = \{\gamma = (\gamma_+, \gamma_-)\}$, Σ is an apropriate algebra of subsets of Ω (we omit all mathematical details here) and Φ is a complex measure on Σ. Let us define (for $\gamma = (\gamma_+, \gamma_-)$, $E \in \Sigma$)

$$\gamma^+ = (\check{\gamma}_-, \check{\gamma}_+), \quad \check{\gamma}_\pm(\tau) = \gamma_\pm(-\tau), \quad E^+ = \{\gamma^+ | \gamma \in E\},$$

$$E \in \Sigma_{herm} \Longleftrightarrow E^+ = E,$$

$$\Omega_+ = \{\gamma_+ | (\gamma_+, \gamma_-) \in \Omega\}, \quad \Omega_- = \{\gamma_- | (\gamma_+, \gamma_-) \in \Omega\},$$

$$E \in \Sigma_{pure} \Longleftrightarrow E = A \times \check{A}, \text{ where } A \times \Omega_- \in \Sigma,$$

$$\check{A} = \{\check{\gamma}_+ | \gamma_+ \in A\},$$

$E \in \Sigma_{mix} \Longleftrightarrow E$ = disjoint union of pure events.

We suppose that $\Phi(E^+) = (\Phi(E))^*$ and that $\Phi(E) \geqq 0$ for $E \in \Sigma_{pure}$. In the application to quantum mechanics we suppose moreover that $\Phi(A \times B) = \Phi(A \times \Omega_-) \cdot \Phi(\Omega_+ \times B)$ (this means the statistical independence of γ_+ and γ_-) and that symbolically $\Phi(A \times \Omega_-) = \sum_{\gamma_+ \in A} \exp(iS[\gamma_+])$, $S[\gamma_+]$ is the action for γ_+ (i.e. $\Phi(\gamma_+ \times \Omega_-)$ is the Feynman's amplitude for the path γ_+).

Let $E = A \times \check{A} \in \Sigma_{pure}$. Then

$$\Phi(E) = \Phi(A \times \Omega_-) \cdot \Phi(\Omega_+ \times \check{A}) = \Phi(A \times \Omega_-) \cdot (\Phi(A \times \Omega_-))^* =$$

$$= \Big| \sum_{\gamma_+ \in A} \exp(iS[\gamma_+]) \Big|^2 .$$

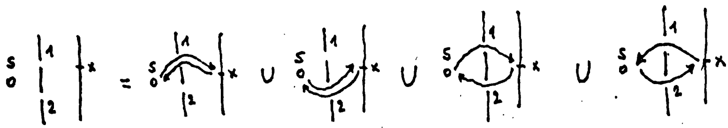
Thus the complex probability of a pure event is (after appropriate normalization) equal to the usual probability of an event A.

The following interpretation postulate, which relates events in the theory to the 'events' observed in real experiments, is supposed. Let the experiment is prepared in such a way that the presence of the electron is measured at space-time points $x_1, \dots, x_n$, $y_1, \dots, y_n$. Let us consider the event in which the presence of an electron was confirmed at $x_1, \dots, x_n$ and excluded at $y_1, \dots, y_n$. This situation is described in our theory by an event

$E_{x_1, \dots, x_n; y_1, \dots, y_n} = \{(\gamma_+, \gamma_-) | x_1, \dots, x_n \in \gamma_+ \cap \gamma_-, \; y_1, \dots, y_m \notin \notin \gamma_+ \cup \gamma_-\}$, where γ_+ means also the graph $\gamma_+(\mathbb{R})$.

The two-slit experiment.

Let an electron is emitted from the source s , then passes through two slits 1 and 2 and finishes at the point x at the screen (where is measured). We are interested in the probability P(x) of an arrival of electron at a point x . Such an event (say E) may be written as an disjoint union $E = E_{11} \cup E_{22} \cup E_{12} \cup E_{21}$, where $E_{kl} = \{ \gamma_+$ passes through the slit k , γ_- passes through slit l$\}$, k, l = 1,2 . In the diagram (for events!):



The probability $\Phi(E)$ is simply the usual one

$$\Phi(E) = \sum_{k,l=1}^{2} \Phi(E_{kl}) = |\langle x|1|S\rangle + \langle x|2|S\rangle|^2 .$$

If one of slits is alternatively closed, then the observed event is $E' = E_{11} \cup E_{22}$! and thus

$$\Phi(E') = \Phi(E_{11}) + \Phi(E_{22}) = |\langle x|1|S\rangle|^2 + |\langle x|2|S\rangle|^2 .$$

The same answer is true in the case, when electron is observed (e.g.) at the slit 1, because of the interpretation postulate. This means that in the experiments with and without measuring an electron at slits we are observing truly different events !! Only in this way, we believe, the rational understanding of the two-slit experiment (and of the nature of quantum superposition generally) may be obtained.

The physical interpretation of γ_+ and γ_- may be given. Let us suppose that there are two sorts of electrons e_+ and e_- – forward and backward ones with respect to the time direction of their evolution – and let e_+ and e_- move independently one from another. The complex probability of elementary event will be defined by

φ (e$_+$ moves along γ_+, e$_-$ moves arbitrarily) =

= Feynman amplitude for γ_+

φ (e$_+$ moves arbitrarily, e$_-$ moves along γ_-) =

= (Feynman amplitude for $_-$) .

Let the following event E_x corresponds to the observation of an electron at a space-time point x : E_x is an event, when there are simultaneously e_+ moving along γ_+ and e_- moving along γ_- and when both γ_+ and γ_- pass through x . This interpretation allows us to consider the Feynman amplitude directly as the complex probability (neglecting the fact that this is a complex number). The resulting amplitude $\sum_{\gamma \in E} \varphi(\gamma)$ will be always positive for an observable event E .

On this ground we can consider the motion of a quantum particle (it is a point-like object!) as an analog of Brownian motion when the usual theory of probability is substituted by the complex theory of probability (see the analogy between the heat and Schroedinger equations, their propagators etc.). We think that in this approache the deterministic interpretation (and the deeper understanding) of quantum theory will be reached.