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Time-space duality and Salam-Weinberg model

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Time-space duality and Salam-Weinberg model.

J. Souček, V. Souček

Using the time-space duality (see [1]) the basic phenomenological in-put to S-W model (before spontaneous symmetry breaking) can be discuss and clarified. With aid of the space-time duality as the leading principle we can show the following facts:

1) There is a striking connection between the two - up to now unrelated - phenomenological facts:
   a) the weak interaction is left-handed
   b) elmg charges of electron, resp. neutrino, are -1, resp. 0.
2) It is possible to set up a simple axiomatic treatment of S-W model for leptons (without Higgs fields) - 2 plausible axioms suffice as an input.
3) As a consequence of this approach the relation $\theta_W = 30^\circ$ can be derived.

The time-space duality is formal symmetry (rather than duality) between complex and quaternionic quantum mechanics (see [1] or the other paper in this volume). Basic equations and correspondence between them is the following one:

$\boxed{\mathbb{C}\text{-QM}}$ $\qquad\qquad$ $\boxed{\mathbb{Q}\text{-QM}}$

$$i\partial_0\psi = (i\sigma_2 \otimes \sigma_k\partial_k + m\sigma_3 \otimes 1\!\!1)\psi \longleftrightarrow \vec{i}\vec{\partial}\chi = (P\partial_0 + m\sigma_3)\chi$$

where ψ is a Dirac spinor,

$$\sigma_2 \otimes \sigma_1 = \begin{bmatrix} 0 & -i\sigma_1 \\ i\sigma_1 & 0 \end{bmatrix}, \text{ etc.}$$

$$\partial_0 = \frac{\partial}{\partial x_0}\ ;\ \partial_k = \frac{\partial}{\partial x_k}\ ,\ k=1,2,3$$

where $\chi = \begin{matrix}\chi_1\\ \chi_2\end{matrix}$ is a doublet of quaternionic wave functions

$$\chi_j = \chi_j^0 + \vec{i}\,\vec{\chi}_j\ ,\ j=1,2$$

$\vec{i} = (i_1, i_2, i_3)$ ... quaternion units

$P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_2$

$$\partial_0 \longleftrightarrow \vec{\partial}$$
$$i \longleftrightarrow \vec{i}$$
$$x_0 \longleftrightarrow \vec{x}$$

For $m \neq 0$ the $\mathbb{C}$- (resp. $\mathbb{Q}$-) quantum mechanics describe bradyons (resp. tachyons), so there is no hope to test the time-space duality. But for massless particles the time-space duality is the symmetry between known objects and can be used to derive some practical consequences.

The time-space duality allows us to treat the massless part of S-W model with aid of two simple axioms.

To have a possibility to act with both $\mathbb{C}$- and $\mathbb{Q}$-gauge transformations on the same objects, we will consider wave functions with values in $\mathbb{CQ}$ (complex quaternions). If we want to write a Dirac equation for them, we need at least 2×2 matrices with values in $\mathbb{CQ}$, so we are forced to consider doublets of such functions. Hence the first axiom will be:

<u>Axiom 1:</u> The basic (Dirac) equation of the theory is the equation

$$(1) \qquad (i\mathbb{1}\,\partial_0 + \sigma_3 \vec{i}\,\vec{\partial} + m\sigma_1)\psi = 0\,, \quad \text{where} \quad \psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \quad \psi_i \in \mathbb{CQ}$$

The second axiom describe the action of gauge transformations on wave functions:

<u>Axiom 2:</u> Both $\mathbb{C}$- and $\mathbb{Q}$-gauge transformations acts only on some components of wave function, corresponding projectors ρ^t and ρ^s are dual to each other with respect to the time-space duality. More precisely, we suppose that the action of gauge transformations is described by

$$\psi \mapsto \rho^s(e^{i\alpha}\psi)$$
$$\psi \mapsto \rho^t(\psi e^{\vec{i}\vec{\alpha}})$$

and that ρ^s is the projection, constructed in the dual manner to ρ^t.

We need only one phenomenological input now - the fact that weak interactions are left-handed. First we will use this fact to derive the form of the parity operator $A^{(t)}$, $\gamma_5^{(t)}$-matrix and the projection operator ρ^t. To do this we shall decompose the wave function into complex components with aid of the substitution $\vec{i} \mapsto -i\vec{\sigma}$

$$\Psi_j = \Psi_0^j + \vec{i}\,\vec{\Psi}^j \ , \quad j = 1,2 \mapsto \Psi_j = \begin{bmatrix} \varphi_j^{11} & \varphi_j^{12} \\ \varphi_j^{21} & \varphi_j^{22} \end{bmatrix} \equiv \varphi_j \ , \ j=1,2 \ .$$

For $\mathbb{C}$-valued wave function $\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$ corresponding equations are $(m = 0)$

$$(i\mathbb{1} \otimes \mathbb{1}\,\partial_0 + i\sigma_3 \otimes \vec{\sigma}\,\vec{\partial} + m\sigma_1 \otimes \mathbb{1})\varphi = 0$$

and so in the $\mathbb{C}$-picture we obtain

$$A^{(t)}_{(\varphi)} = \sigma_1 \otimes \mathbb{1} \ , \quad \gamma_{5(\varphi)}^{(t)} = -\sigma_3 \otimes \mathbb{1} \ , \quad P^t_{(\varphi)} = \frac{1+\sigma_3}{2} \otimes \mathbb{1} \ .$$

After the translation back to $\mathbb{CQ}$-picture, we have

$$A^{(t)} = \sigma_1 \ , \quad \gamma_5^{(t)} = -\sigma_3 \ , \quad \rho^t = \frac{1+\sigma_3}{2} \ .$$

Now we can derive the form of the parity operator $A^{(s)}$, a dual $\gamma_5^{(s)}$-matrix and finaly a projection operator ρ^s in a completely dual manner. By means of the substitution $\Psi = \frac{1}{2}(\mathbb{1} + i\sigma_1)\chi$ we rewrite the equation (1) into the $\mathbb{Q}$-form $(m=0)$

$$(\vec{i}\,\vec{\partial} + P\partial_0 + im\sigma_3)\chi = 0$$

and we can now decompose the wave function χ into real and imaginary parts, both satisfying the same equation. The procedure dual to the one in $\mathbb{C}$-picture gives us now

$$A^{(s)}_{(\varkappa)} = \sigma_3 , \quad \gamma_{5\,(\varkappa)}^{(1)}(\varkappa) = P\varkappa i_3$$

(the choice of i_3 is conventional). After coming back to $\mathbb{C}\mathbb{Q}$--picture we obtain

$$A^{(s)} = iP , \quad \gamma_5^{(s)}(\psi) = \sigma_3 \psi i i_3$$

and finally

$$\rho^s(\psi) = 1/2(\psi - \sigma_3 \psi i i_3) .$$

So now we have the following infinitesimal gauge transformations

$$\delta_0 \psi = i\,\rho^s(\psi) , \quad \vec{\delta}\psi = \rho^t(\psi)\,\vec{i}$$

and we see that they form a Lie algebra $\mathcal{L} \cong su(2) \times u(1)$ with commutation relations

$$[\delta_i, \delta_j] = -2\varepsilon_{ijk}\delta_k ,$$

$$[\delta_0, \delta_1] = -\delta_2 , \quad [\delta_0, \delta_2] = \delta_1 ,$$

$$[\delta_0 \cdot \delta_3] = 0 .$$

The abstract model is now built up and we can identify it with usual S-W model before symmetry breaking. To identify weak, elmg and hypercharge infinitesimal generators in our Lie algebra we shall use the following known facts:

1) weak current is left-handed } with respect to the parity
2) elmg current is vectorial } operator $A^{(t)}$

3) a hypercharge generator is a central one

4) $Q = \frac{1}{2}(T_3+Y)$

From this facts we have

$$\delta_Q = k_Q(\delta_0 - \delta_3) , \quad \delta_Y = k_Y(-2\delta_0 + \delta_3) ; \quad k_Q , k_Y \in \mathbb{R} .$$

The relation $\delta_Q = 1/2(\delta_3 + \delta_Y)$ gives us $k_Y = 1$, $k_Q = -1$.

The knowledge of charge and hypercharge generators tells us that the physical content of our wave function φ (i.e. in the usual $\mathbb{C}$-picture) is

$$\varphi = \begin{bmatrix} e_L & \nu_L \\ e_R & \nu_R \end{bmatrix},$$

and for φ written in more usual form $\Phi = \begin{bmatrix} e_L \\ e_R \\ \nu_L \\ \nu_R \end{bmatrix}$ we have

$$\delta_Q = \begin{bmatrix} -1\!\!1 & & & \\ & -1\!\!1 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}, \quad \delta_Y = \begin{bmatrix} -1\!\!1 & & & \\ & -2\cdot 1\!\!1 & & \\ & & -1\!\!1 & \\ & & & 0 \end{bmatrix}.$$

(The right-handed neutrino ν_R is not contained in the standard S-W model, but with respect to the fact that it has no interaction with gauge fields, the physical content of the models is the same.)

Let us now write the Lagrangian for our theory. The interaction part $\mathcal{L}_I$ has the form

$$\mathcal{L}_I = \Phi^+ \beta^\mu D_\mu \Phi ,$$

where $\beta^0 = 1\!\!1 \otimes 1\!\!1 \otimes 1\!\!1$; $\beta^k = -1\!\!1 \otimes \sigma_3 \otimes \sigma_k$, $k = 1,2,3$;

$$D_\mu = \partial_\mu + A_\mu , \quad A_\mu = \sum_{A=0}^{3} A_\mu^A \ \delta_A \in \mathcal{L}$$

To write a Lagrangian for pure gauge fields it is necessary to prescribe a metric on the algebra $\mathcal{L}$. Then

$$\mathcal{L}_{YM} = (F_{\mu\nu} | F_{\mu\nu}) \ ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu , A_\nu].$$

The metric would be invariant with respect to a global gauge transformation, so on the su(2)-subalgebra $\{\delta_1, \delta_2, \delta_3\}$ it must be a multiple of Cartan-Killing metric and on the u(1)-part $\{\delta_Y\}$ we can choose an arbitrary nontrivial metric. Let us set

$$(\delta_Y | \delta_Y) = \lambda^2 , \quad (\delta_k | \delta_\ell) = b^2 \delta_{k\ell} , \quad (\delta_Y | \delta_k) = 0 \ ;$$
$$\ell, k = 1,2,3$$

where λ and b are the interaction constants. (In our formalism the interaction constants are contained not in $\mathcal{L}_I$, but in

$\mathcal{L}_{YM}$. The gauge fields are not, of course, canonical fields, i.e. they need not satisfy the usual comutation relations. We can divide them by suitable constants and for new canonical fields $\mathcal{L}_{YM}$ will have the usual form and the interaction constants will apear in $\mathcal{L}_I$ as usual.)

The interaction part $\mathcal{L}_I$ contains a pure spinor term, which gives us a metric $(\Phi|\Phi)$ for spinor fields. It is the important fact now that our infinitesimal generators $\vec{\delta}$, δ_0 are equally normed in described spinor norm (i.e. the eigenvalues are the same), because the space-time duality tells us now that the dual generators $\vec{\delta}$ and δ_0 would have the same norm in $\mathcal{L}$. The corresponding relation is

$$(\delta_j \mid \delta_j) = (\delta_0 \mid \delta_0) = b \ ; \quad j = 1,2,3$$

and from $\delta_Y = -2\,\delta_0 + \delta_3$ it follows that

$$b^2 = (\delta_0 \mid \delta_0) = \frac{1}{4}\,(\delta_Y - \delta_3 \mid \delta_Y - \delta_3) = \frac{\lambda^2 + b^2}{4} \ ,$$

hence

(2) $$\lambda = \sqrt{3}\cdot b \ .$$

We obtained the Lagrangian

$$\mathcal{L} = \Phi^+\beta^{\mu}(\partial_{\mu} + A^Y_{\mu}\,\delta_Y + \vec{A}_{\mu}\,\vec{\delta})\Phi + \lambda^2(F^Y_{\mu\nu})^2 + b^2(\vec{F}_{\mu\nu})^2 =$$

$$= \Phi^+\beta^{\mu}(\partial_{\mu} + \frac{1}{\lambda}\,(\lambda A^Y)\,\delta_Y + \frac{1}{b}\,(b\vec{A}_{\mu})\,\vec{\delta})\Phi + (\lambda F^Y_{\mu\nu})^2 +$$
$$+ (b\vec{F}_{\mu\nu})^2 \ ,$$

where the fields λA^Y_{μ} , $b\vec{A}_{\mu}$ are already the canonical fields. We can compare $\mathcal{L}$ with the usual S-W Lagrangian

$$\mathcal{L} = \bar{L}\left[\gamma^{\mu}(\partial_{\mu} - i\frac{g'}{2}\,a_{\mu} - i\,\frac{g}{2}\,\vec{\tau}\,\vec{b}_{\mu}\right]L + \bar{R}\,\gamma^{\mu}(\partial_{\mu} - iga_{\mu})R$$

and we obtain immediately $g = \frac{2}{b}$, $g' = \frac{2}{\lambda}$.

The relation (2) gives us now

$$\theta_W = 30^{\circ} \ .$$

There is an interesting interpretation of this fact. As we saw before, the correctly normed isospin generators are $\vec{\tau}$ instead of the usual $\vec{\tau}/2$, so "the correct" interaction constant is $g_W = 1/2\ g$. The relation $\theta_W = 30^{\circ}$ reads then as $e = g_W$ and this can be interpreted as the equal strength of elmg and weak interaction (before the symmetry breaking). Hence the space time-duality is the remedy to usual problems with the degeneracy of the group $U(1) \times SU(2)$ and we are left with the only one interaction constant.

At the same time, the nice connection between the structure of the space-time (we can consider it as $ix_0 + \vec{i}\vec{x}$) and S-W group $U(1) \times SU(2)$ (a generator is $\alpha_0 i\, \sigma^s + \vec{\alpha}\vec{I}\, \sigma^t$) can be easily seen in this approach.

Literature

[1] J. Souček: Quaternion quantum mechanics as the description of tachyons and quarks, Czech. Jour. of Phys., vol. B 29, 1979, 3, 315-318.