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Towards the subquantum theory

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Towards the subquantum theory.

J. Souček, V. Souček

The aim of this note is to formulate a concept of a "quantum point particle" and to explain how the equations for probability amplitudes of them can be written and the eigenvalue equations for mass spectrum could be obtained. The notion of complex probability (see [1]) is used in this approach.

We shall consider 1+1-dimensional model of the theory. Let us first describe somewhat unusual character of quantum point particles, their meaning for standard quantum mechanics will be explain later. Classicaly, the probability distribution of a classical electron, the trajectory of which is described by a function $x(t)$, is the function

$$f(x,t) = \delta(x-x(t)) .$$

Suppose now that a "quantum point particle" has a well-defined trajectory and that the probability (complex probability! - see [1]) distribution of it is described by

$$(1) \qquad \Psi(x,t) = \delta(x-x(t))e^{i\varphi(t)}$$

where $x = x(t)$ is the trajectory of the particle.

To derive a form of the function $\varphi(t)$, let us consider first a free quantum point particle in rest; with respect to the homogeneity of time we have

$$\Psi = \delta(x)e^{-iEt} .$$

The physical meaning of E is of course the energy of the particle. Lorentz transformations give us for a free quantum point particle

$$(2) \qquad \Psi(x_0,x_1) = \delta(p\circ x)e^{-ip\cdot x} , \text{ where } \begin{array}{l} p\circ x = p_0x_1 - p_1x_0 \\ p\cdot x = p_0x_0 - p_1x_1 . \end{array}$$

Clearly $V = \frac{p_1}{p_0}$ is the velocity of the particle, p_0 its energy and p_1 its momentum. All states of a free quantum point particle is hence characterized by three parameters - x_1 $(x_0=0)$, p_0, p_1. So we have here one parameter more with respect to the classical mechanics and the question arises whether the momentum $p = (p_0,p_1)$ would satisfy a mass relation $p_0^2 - p_1^2 = m^2$, m fixed. In the kinematical description (2) we have one free parameter more (the function $\varphi(t)$). The dynamical description is in fact the infinitesimal form of kinematics, so there would be corresponding one free parameter, too. The new free parameter will be the energie p_0 of the quantum point particle, we shall have not any mass relation here and the correspondence will be

$$(x(t), \varphi(t)) \longleftrightarrow p_0 \, , \, p_1 \, .$$

The general probability distribution hence will depend on 2--momentum $p = (p_0,p_1)$ and will be a superposition of functions

$$\psi_{\overline{p_0},\overline{p_1}}(p_0,p_1,x_0,x_1) = \delta^{(2)}(p-\overline{p})\,\delta(p \circ x)e^{-ip\cdot x} \tag{3}$$

and of translations of them in space-time.

The propagator for our point particle can be written in the form

$$K_0(x_2,p_2|x_1,p_1) = \delta^{(2)}(p_2-p_1)\,\delta(p \circ x)e^{-ip\cdot x}\,\theta(p\cdot x)\,\mathcal{E}(p\cdot p) \, , \tag{4}$$

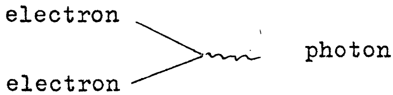
where $p = p_1$; $x = x_2-x_1$; $\theta(y) = 1/2(1+\mathcal{E}(y))$, $\mathcal{E}(y) = \operatorname{sgn} y$, $y \in \mathbb{R}$. The propagator K_0 is a Green function for the differential operator

$$\mathcal{D}_0 = i(p_0\,\partial_0 + p_1\,\partial_1) - p\cdot p \, , \quad \mathcal{D}_0 K_0 = i\,\delta \, .$$

All this firm the base of the description of free quantum point particles. The correct description of their interaction can be done in terms of Feynman diagrams.

Suppose that "point electrons" interact with "point photons".

An elementary interaction will be described by the graph



Generally speaking, a point-like electron is moving in a sea of point-like photons (the complex probability distribution of which is $\varphi(y, q)$) and the time evolution of the electron probability distribution will be influenced by interaction with the background sea of photons. The situation resembles something like quantum analogy of Brown motion. The best description of the situation is the Feynman approach of a propagator in a potential $\mathcal{V}$.

Suppose that an elementary interaction is described by a collision of a point electron with a potential $\mathcal{V}$. During the collision in the point x the momentum p is changed into a momentum p', the (complex) probability of the transition $p \to p'$ is given by $\mathcal{V}(x,p'-p)$. The potential $\mathcal{V}$ depends on photon probability distribution - $\mathcal{V} = \mathcal{V}[\varphi]$. The propagator $K(f|i)$ for interacting electrons satisfies the equation

$$K(f|i) = K_0(f\ i) + ai\int K_0(f|2)\mathcal{V}(2|1)K_0(1|i)d1d2 + \ldots$$

where $1 = (x_1,p_1)$, $2 = (x_2,p_2)$, $\mathcal{V}(2|1) = \delta(x_2-x_1)\mathcal{V}(x,p_2-p_1)$.
So we have the following equation for

$$K(f|i) = K_0(f|i) + ai\int K_0(f|2)\mathcal{V}(2|1)K(1|i)d1d2$$

$$\mathcal{D}_0^{(f)}K(f|i) = i\,\delta(f-i) + ai\int \mathcal{V}(f|1)K(1|i)d1 .$$

The (complex) probability distribution $\Psi(x,p)$ for point electrons will satisfy then the equation

$$\mathcal{D}_0\Psi(x,p) = ai\int \mathcal{V}(x,p-p')\Psi(x,p')dp' .$$

The same can be done for the point photon moving in the background point electron sea. The point photon probability distribution $\varphi(y, q)$ will then satisfy an analogical equation, some one like

$$\partial_0 \varphi(y, q) = \mathcal{W} * \varphi$$

but for a precise formulation of it the many-particle version of the probability distribution would be introduced. The dependence of the potentials $\mathcal{V}$, $\mathcal{W}$ on φ, ψ will be approximately

$$\mathcal{V}(x, q) = \varphi(x, q) , \quad \mathcal{W} \sim \psi^* \psi$$

so in the end we shall finish with the two coupled nonlinear equations for φ and ψ .

The interpretation of all this in usual terms is the following:

1) the physical vacuum described by the solution (ψ , φ) of our equations, which is Lorentz and translation invariant
2) the physical state describing one physical electron with momentum (P_0, P_1) is the solution of the equations

(5) $$\partial_\mu(\psi , \varphi) = P_\mu(\psi , \varphi) \quad \text{and} \quad Q(\psi , \varphi) = -e(\psi , \varphi)$$

where Q is the generator of phase transformation. The same definition can be written for physical photon also $(Q(\psi , \varphi) = 0)$.

There is one important possibility in this approach. The equations (5) are in fact eigenvalue equations for the mass (the equations are Lorentz invariant) and for suitable boundary conditions it could have a discrete spectrum.

The intuitive picture behind such equation for mass spectrum is that the point particle during the time continuously interact with point particles forming the vacuum; the desctructive interference distinguish then the discrete values of masses, the particles with them survive the interaction with vacuum up to asymptotic domain. Our model is in fact the model where virtual particles are considered as the real one and the vacuum is considered as the statistical ensemble of them (with respect to complex probability). The trajectory of a quantum point particle is principally unobservable, because there is no means to avoid influence of the vacuum on it, so in the end we are left with (complex) probability description of a point electron, which can be then related to the usual wave function of physical electron.