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In: Zdeněk Frolík (ed.): Abstracta. 9th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1981. pp. 17–18.

Persistent URL: http://dml.cz/dmlcz/701217

NINTH WINTER SCHOOL ON ABSTRACT ANALYSIS (1981)

UNBOUNDED DESCENDING INFINITE CHAIN IN RUDIN-FROLÍK ORDER

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<u>Theorem.</u> There exists an ultrafilter p on the set $\mathbb{N}$ such that the set of types smaller than p in the Rudin-Frolík order [1,2] is isomorphic to the inverse order of the set of natural numbers.

Assuming the continuum hypothesis, this theorem has been proved by A. Louveau and R. C. Solomon [3]. Our proof does not need any set-theoretical assumption. Using the reduction of [3], one needs to construct an ultrafilter on $\mathbb{N}$ that well behaves in relation to a family F of discrete subsets of $\beta\mathbb{N}-\mathbb{N}$ of cardinality $2^{\aleph_0}$. The ultrafilter is constructed by the transfinite induction. On each step, exactly one discrete set of the family F is considered. Therefore, we must not finish the construction before the continuum'th step.

If each ultrafilter $p \in X$, X being discrete, $X \subseteq \beta\mathbb{N}-\mathbb{N}$, has the character $2^{\aleph_0}$, then also each ultrafilter $q \in \overline{X}$ has character $2^{\aleph_0}$. This fact is used for keeping the transfinite induction not to construct the ultrafilter too early.

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[2] M. E. Rudin: Partial orders on the types in $\beta\mathbb{N}$, Trans. Amer. Math. Soc., 155(1971), 353-362.

[3] R. C. Solomon: A type of $\beta\mathbb{N}$ with $\aleph_0$ relative types, Fundamenta Math., 79(1973), 209-212.