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UNBOUNDED DESCENDING INFINITE CHAIN IN
RUDIN-FROLÍK ORDER

L. Bukovský and E. Butkovičová

Theorem. There exists an ultrafilter p on the set \mathbb{N} such that the set of types smaller than p in the Rudin-Frolík order $[1,2]$ is isomorphic to the inverse order of the set of natural numbers.

Assuming the continuum hypothesis, this theorem has been proved by A. Louveau and R. C. Solomon [3]. Our proof does not need any set-theoretical assumption. Using the reduction of [3], one needs to construct an ultrafilter on \mathbb{N} that well behaves in relation to a family F of discrete subsets of $\beta\mathbb{N} - \mathbb{N}$ of cardinality 2^{\aleph_0} . The ultrafilter is constructed by the transfinite induction. On each step, exactly one discrete set of the family F is considered. Therefore, we must not finish the construction before the continuumth step.

If each ultrafilter $p \in X$, X being discrete, $X \subseteq \beta\mathbb{N} - \mathbb{N}$, has the character 2^{\aleph_0} , then also each ultrafilter $q \in \overline{X}$ has character 2^{\aleph_0} . This fact is used for keeping the transfinite induction not to construct the ultrafilter too early.

- [1] Z. Frolík: Sums of ultrafilters, Bull. Amer. Math. Soc., 73(1967), 87-91.

- [2] M. E. Rudin: Partial orders on the types in βN , Trans. Amer. Math. Soc., 155(1971), 353-362.
- [3] R. C. Solomon: A type of βN with \aleph_0 relative types, Fundamenta Math., 79(1973), 209-212.