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Discontinuous invariant functionals and traces

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Let E be a Banach space with a symmetric basis $(e_i)_{i\in\mathbb{N}}$. If σ is a permutation of the set $\mathbb{N}$ and $x = \sum_{i\in\mathbb{N}} x_i e_i \in E$, we let $x\circ\sigma = \sum_{i\in\mathbb{N}} x_{\sigma(i)} e_i$.

A linear map $f : E \longrightarrow \mathbb{R}$ is said to be invariant if $f(x\circ\sigma) = f(x)$ for each $x \in E$ and each permutation σ. Let $I(E)$ be the space of all invariant linear functionals on E. It was known that $I(c_0) = \{0\}$.

Theorem 1. a/ $I(l_p) = 0$ for $1<p<\infty$;

b/ $\dim I(l_1) > 1$ (in fact $= 2^{2^{\aleph_0}}$);

c/ there exists $E \neq l_1$ such that $I(E) \neq \{0\}$.

This result has an analogue for unitary ideals on the Hilbert space, H. Let

$$S_E = \{T \in B(H,H) : (s_j(T)) \in E\},$$

$s_j(T)$ being the s-numbers. A linear functional $\phi: S_E \longrightarrow C$ is said to be invariant if $\phi(UTU^{-1}) = \phi(T)$ for each $T \in S_E$ and each unitary operator U. We let $T(E)$ denote the space of all invariant linear fuлctionals on S_E. An element $\phi \in T(E)$ is called a trace if $\phi(P) = 1$ where $P : H \text{——} H$ is a rank one projection.

Theorem 2. a/ $T(l_p) = \{0\}$ for $1<p<\infty$;

b/ $\dim T(l_1) > 1$,

c/ there exists $E \neq l_1$ such that $T(E) \neq \{0\}$; in fact there is an invariant trace on E.

Remark. Parts b/ and c/ answer questions asked by Professor A.Pietsch in the first talk of the conference (cf. [1]).

In the talk we proved two parts of Theorem 1. Part a/

follows easily from the decomposition of the vector e_1 E due to R.Ocneanu. The main ingredient in the proof of b/ is the following lemma.

Lemma 3. For each $k > 0$ there is $\varepsilon(k) > 0$ such that, if

$$x = \sum_{i \leq k} (x_i - x_i \circ \sigma_i),$$

where $x_i \in l_1$, $\|x_i\| \leq 1$, σ_i are permutations and $x = e_1 + \sum_{i>1} a_i e_i$, then $|a_i| \geq \varepsilon(k)$ for some $i > 1$.

The proofs will appear elsewhere.

Remark. More facts are known now than it is formulated above, e.g. we have found a characterization of those E such that $I(E) = \{0\}$ (resp. $T(E) = \{0\}$).

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References.

[1] A.Pietsch, Operator ideals with a trace, to appear.