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Conditional symmetries of stable measures on \mathbb{R}^n

W.Linde and P.Mathé

A symmetric probability measure μ on the Borel sets of the n -dimensional Euclidean space \mathbb{R}^n is said to be p -stable, $0 < p \leq 2$, if for each $\alpha, \beta > 0$

$$\tau_\alpha(\mu) * \tau_\beta(\mu) = \tau_{\alpha+\beta}(\mu)$$

where $\tau_\alpha(\mu)(B) = \mu(\alpha^{-1}B)$ and $\gamma^p = \alpha^p + \beta^p$.

This is equivalent to the fact that the characteristic function $\hat{\mu}$ of μ can be written as

$$\hat{\mu}(a) = \exp\left(-\int_{\partial U} |K(x, a)|^p d\mu(x)\right), \quad a \in \mathbb{R}^n,$$

where \mathcal{G} denotes a finite measure on the unit sphere $\partial U \subseteq \mathbb{R}^n$. While 2-stable (Gaussian) measures are well investigated only few things are known about p -stable measures, $p < 2$. For instance, if μ is Gaussian symmetric on \mathbb{R}^2 then there is a real number c such that μ is conditional symmetric with respect to c , i.e.

$$(+)\quad \mu(\mathcal{F}_1 \in B_1, c\mathcal{F}_1 - \mathcal{F}_2 \in B_2) = \mu(\mathcal{F}_1 \in B_1, \mathcal{F}_2 - c\mathcal{F}_1 \in B_2)$$

for all Borel subsets $B_1, B_2 \subseteq \mathbb{R}$. Geometrically this means that μ is invariant under reflections around $\{\mathcal{F}_2 = c\mathcal{F}_1\}$ into the direction of $\{\mathcal{F}_1 = 0\}$. In 1975 A.Torchat stated that each p -stable symmetric measure on \mathbb{R}^2 is conditional symmetric for some $c \in \mathbb{R}$. But this is false in general. The purpose of our work is to characterize those p -stable symmetric measures satisfying (+) for some $c \in \mathbb{R}$. Finally, we give an example of a p -stable symmetric measure which isn't invariant under any reflection on \mathbb{R}^2 . Then the results are extended to the n -dimensional case. A paper with the same title will appear in Ann.Inst.Henri Poincaré.