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In: Zdeněk Frolík (ed.): Abstracta. 9th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1981. pp. 109.

Persistent URL: http://dml.cz/dmlcz/701236

NINTH WINTER SCHOOL ON ABSTRACT ANALYSIS (1981)

# Conditional symmetries of stable measures on $R^n$

W.Linde and P.Mathé

A symmetric probability measure $\mu$ on the Borel sets of the n-dimensional Euclidean space $R^n$ is said to be p-stable , $0 < p \leq 2$, if for each $\alpha, \beta > 0$

$$\tau_\alpha(\mu) * \tau_\beta(\mu) = \tau_\gamma(\mu)$$

where $\tau_\alpha(\mu)(B) = \mu(\alpha^{-1}B)$ and $\gamma^p = \alpha^p + \beta^p$ .
This is equivalent to the fact that the characteristic function $\hat{\mu}$ of $\mu$ can be written as

$$\hat{\mu}(a) = \exp(-\int_{\partial U} |\langle x,a\rangle|^p \, d\sigma(x)) \quad , \; a \in R^n \quad ,$$

where $\sigma$ denotes a finite measure on the unit sphere $\partial U \subseteq R^n$ . While 2-stable ( Gaussian) measures are well investigated only few things are known about p-stable measures , $p < 2$ . For instance, if $\mu$ is Gaussian symmetric on $R^2$ then there is a real number c such that $\mu$ is conditional symmetric with respect to c , i.e.

$$(+) \quad \mu(\xi_1 \in B_1, c\xi_1 - \xi_2 \in B_2) = \mu(\xi_1 \in B_1, \xi_2 - c\xi_1 \in B_2)$$

for all Borel subsets $B_1, B_2 \subseteq R$ .Geometrically this means that $\mu$ is invariant under reflections around $\{\xi_2 = c\xi_1\}$ into the direction of $\{\xi_1 = 0\}$. In 1975 A.Tortrat stated that each p-stable symmetric measure on $R^2$ is conditional symmetric for some $c \in R$ . But this is false in general. The purpose of our work is to characterize those p-stable symmetric measures satifying (+) for some $c \in R$ . Finally, we give an example of a p-stable symmetric measure which isn't invariant under any reflection on $R^2$. Then the results are extended to the n-dimensional case.
A paper with the same title will appear in Ann.Inst.Henri Poincarè .