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LONG RANGE ROTATORS

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ABSTRACT : In these notes we present a number of results which concern the absence of ordered phases at low dimensions for systems having a continuous rotation symmetry and long range interactions. The cases of classical spin systems and spin glasses in two dimensions and of lattice gauge systems in three dimensions will be considered.

1. INTRODUCTION

The mathematics involved in classical equilibrium statistical mechanics concern well known structures of probability theory. Thus, random fields with discrete argument correspond to spin systems in statistical mechanics.

Let $\mathcal{L}=\mathbb{Z}^d$ be a d-dimensional integer lattice. To each site $x\in\mathcal{L}$ we associate a N-dimensional vector $\vec{s}_x=\{s_x^1,\dots,s_x^N\}$ of unit length called spin. We assume that the spins have as a priori distribution the uniform measures $d\Omega_N$ on the sphere S^{N-1} and interact through the hamiltonian

$$H=-\sum_{x,y}\sum_{i=1}^{N} J^i(x-y)\,s_x^i s_y^i-\vec{h}\sum_x \vec{s}_x \tag{1}$$

where $J^1,\dots,J^N$ are real valued interaction potentials and $\vec{h}\in R^N$ represents the external magnetic field.

The configurations of the system are functions on $\mathcal{L}$ with values on S^{N-1}. On the set $K_{\mathcal{L}}$ of configurations we introduce the product topology and the corresponding Borel σ-algebra. Let $\xi\in K_{\mathcal{L}\setminus\Lambda}$ be a given configuration on $\mathcal{L}\setminus\Lambda$ where Λ is finite. The Gibbs measure, at the inverse temperature $\beta=1/T$ of the finite system restricted to the box Λ with boundary condition ξ outside Λ, is

$$d\mu_\Lambda=Z_\Lambda^{-1}\exp\beta\Big\{\sum_{\{x,y\}\cap\Lambda\neq\emptyset}\sum_{i=1}^{N}J^i(x-y)\,s_x^i s_y^i+\vec{h}\sum_{x\in\Lambda}\vec{s}_x\Big\}\prod_{x\in\Lambda}d\Omega_N(\vec{s}_x) \tag{2}$$

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where Z_Λ (the partition function) is the normalization constant chosen so that $\int d\mu_\Lambda = 1$, and the configuration $\{\vec{s}_x\}$ coincides with ξ on $\mathcal{L}\backslash\Lambda$.

A probability measure μ on $K_{\mathcal{L}}$ is a Gibbs measure or an equilibrium state if for all finite Λ its conditional probability given ξ outside Λ is given by (2).

For N = 1 we have the Ising model, for N = 2 the plane rotator and for N = 3 the classical Heisenberg model. Ferromagnetism corresponds to positive interactions. These systems are isotropic if $J^1 = \ldots = J^N$. In this case the hamiltonian at zero magnetic field

$$H = -\sum_{x,y} J(x-y)\,\vec{s}_x \vec{s}_y \tag{3}$$

is invariant under simultaneous rotation of all spins.

2. SPIN SYSTEMS

In (3) O(N) is the symmetry group. For $N \geqslant 2$, the Mermin and Wagner [1] result is well known and implies that there is no long range order in such systems for dimension d = 2 (except if the interaction is very long range). Then the spin-spin correlation function $\langle \vec{s}_x \vec{s}_y \rangle$ of any Gibbs state decays to zero at large distances

$$\lim_{|x-y| \to \infty} \langle \vec{s}_x \vec{s}_y \rangle = 0 \tag{4}$$

which is equivalent to saying that the one point correlation functions are invariant under O(N), and therefore the magnetization $\langle s_x^1 \rangle = 0$.

This is reinforced by the Dobrushin and Schlosman result [2] according to which, under very general assumptions, there is no breaking of a continuous symmetry in 2-dimensions, an interesting physical question for which optimal results are now available [3,4]. In this case all Gibbs states are invariant relative to the symmetry group of the hamiltonian.

If the converse case occurs-as for the Ising ferromagnet if $d \geqslant 2$ or for $N \geqslant 2$ if $d \geqslant 3$ [5] where (4) does not hold at low temperature- one says that the symmetry is spontaneously broken.

For $N \geqslant 2$ in d = 2, upper bounds on the decay of $\langle \vec{s}_x \vec{s}_y \rangle$ were established in [6,7,8] if the interaction is finite range, and recently the case of long range interactions has also been discussed [9,10]. Our results on this problem will be now briefly described.

Proposition 1 : Let d = 2 and $N \geqslant 2$. We assume that the interaction verifies

$$|J(x)| \leqslant A\,|x|^{-\alpha} \tag{5}$$

where A is some constant. If $\alpha > 4$, then the spin-spin correlation functions associated to any Gibbs state are bounded according to

$$|\langle \vec{s}_x \vec{s}_y \rangle| \leqslant B\,|x-y|^{-\lambda(\beta)} \tag{6}$$

where $\lambda(\beta)$ is a strictly positive non decreasing function of β behaving as $\lambda(\beta) = K/\beta$ for large β (B and K are constants).

The upper bound (6) is of the same type as that previously established by McBryan and Spencer [7] in the case of finite range interaction and improves the results of refs.[9,10]. We notice also Fröhlich-Spencer's result [11], proving that for the plane rotator ferromagnet with nearest neighbour interactions, the correlation functions $\langle \vec{s}_x \vec{s}_y \rangle$ have precisely a power low decay at low temperatures.

Proposition 2 : Under condition (5) with $\alpha = 4$ an upper bound on $\langle \vec{s}_x \vec{s}_y \rangle$ holds, which decays at infinity as an inverse power of $\log|x-y|$.

For ferromagnetic systems with an interaction such that $J(x) = |x|^{-\alpha}$ for large $|x|$, with α such that $2 < \alpha < 4$, one knowns [11,12] that the O(N) is spontaneously broken and the limit (4) is strictly positive at low temperature (the condition $\alpha > 2$ is needed for the existence of a thermodynamic behaviour).

On the other hand under condition (5) with $\alpha \geqslant 4$ all Gibbs states preserve the O(N) symmetry. This result can be recovered from Propositions 1 and 2 by using correlation inequalities [14].

3. GAUGE LATTICE SYSTEMS

Gauge theories on a lattice were introduced by Wilson [15] as a discrete space-time approximation to a quantum theory of gauge fields. In the case of a U(1) invariance they can be described as follows :

To each ordered link (x,y) of neighbouring sites $|x-y| = 1$ an element $A_{xy} = A_{yx}^{-1} \in U(1)$ is assigned (where U(1) is the group of complex numbers of modulus 1). A configuration on $\mathcal{L}$ is specified by giving the values of A_{xy} at each link. Given a closed path on the lattice γ , formed by the links $(x_1,x_2), (x_2,x_3), \ldots, (x_n,x_1)$ we denote by $A(\gamma)$ the product $A_{x_1x_2}A_{x_2x_3}\cdots A_{x_nx_1}$, by $\mathrm{Tr}A(\gamma)$ the real part of $A(\gamma)$ and by $|\gamma| = n$ the perimeter of the path γ . We consider the hamiltonian (which should be called euclidean action in the context of a field theory)

$$H = -\sum_{\gamma} J(\gamma) A(\gamma) \tag{7}$$

where the $J(\gamma)$ are real valued interaction potentials and the sum extends over the set of clased paths of $\mathcal{L}$. We assume that the variables A_{xy} have as priori distribution the uniform measure on the circle. The corresponding Gibbs states of the system are then defined as in Section 1.

The system defined by (8) has a local symmetry property. Namely, if we introduce extra variables $K_x \in U(1)$ at each site $x \in \mathcal{L}$, and replace A_{xy} by

$$A'_{xy} = K_x A_{xy} K_x^{-1} \tag{8}$$

the hamiltonian (7) remains invariant. As a consequence of this locality it follows that the Gibbs states are always invariant under the symmetry operations (8).

Due to the local invariance the choice of an order parameter is not straightforward. Wilson [15] has however suggested that the average

$$W(\Gamma) = \langle A(\Gamma) \rangle \tag{9}$$

may be used to define ordering. One takes Γ as the contour of a rectangle with sides of length L and T parallel to the coordenate axis of the lattice. The decay properties of the quantity

$$f(L) = \lim_{T \to \infty} |W(\Gamma)|^{1/T} \tag{10}$$

are related to the problem of quark confinement.

In a pure gauge field theory only the terms corresponding to the plaquettes or elementary squares of the lattice contribute to (7). If the dimension $d = 3$ an upper bound on $f(L)$ which decays to zero when $L \to \infty$, has been proved by Glimm and Jaffe [16] in this case. On the other hand it is known that if $d \geq 4$ the order parameter $f(L)$ does not tend to zero if β is large enough [17]. The same is true for $d \geq 3$ in the Z_2 theory (in which the variables A_{xy} are restricted to take the values -1,+1).

The generalized form (7) of the euclidean action appears in a theory of gauge and Fermi fields on a lattice after integrating over the Fermi variables. In this case we have the following result.

<u>Proposition 3</u> : <u>Let $d = 3$ and assume that the interaction potentials verify</u>

$$|J(\gamma)| \leq A \exp -|\gamma| \log |\gamma| \tag{11}$$

<u>for some $A > 0$. Then</u>

$$|W(\Gamma)| \leq B \exp -\lambda(\beta) T \log L \tag{12}$$

where $\lambda(\beta)$ is a stricly positive non-decreasing function of β , behaving as $\lambda(\beta) = K/\beta$ for large β (B and K are constants).

This proposition extends the decay property proved in [16] to long range interactions.

4. SPIN-GLASS SYSTEMS

A spin glass is a dilute magnetic alloy where magnetic impurities are diluted in a non magnetic metal. It is believed that the physical behaviour of such systems comes from a spin-spin interaction of the impurities which is long-range and rapidly oscillating. It appears that this oscillating property, which is essential to produce a spin glass, can be modelled, according to the ideas of Edwards and Anderson [18], by a spin system in which the interaction potentials J(x,y) are random variables.

Let $\langle . \rangle (J)$ denote expectation with respect to a Gibbs state corresponding to a given configuration of the interaction potentials J(x,y). Let E denote expectation with respect to the random variables J. Since the mean magnetization $E\{\langle s^i_x \rangle (J)\}$ is assumed to be zero according with the assumed probability distribution for the variables J, one considers the following order parameter [17]

$$q = E\{\langle \vec{s}_x \rangle (J) \langle \vec{s}_x \rangle (J)\} \tag{13}$$

which should be strictly positive in a spin glass phase.

Few rigorous results are known for these systems. Let us mention a first study by Vuillermot [19], the proof of the existence of the thermodynamic limit by Khanin and Sinai [20], and the results on the absence of phase transitions in one dimensional Ising systems by Khanin [21] and by Cassandro, Olivieri and Tirozzi [22]. We notice that in refs. [20,21,22] the random character of the interaction plays a crucial role since long range interactions are considered for which the corresponding statements should be false in the deterministic case. This is also the case for the following result.

We consider a plane rotator spin glass in d = 2 dimensions. The hamiltonian is given by

$$H = -\sum_{x,y} \frac{J(x,y)}{|x-y|^\alpha} \vec{s}_x \vec{s}_y \tag{13}$$

We assume that J(x,y) for $\{x,y\} \subset \mathcal{L}$ are independently distributed random variables with mean zero. For the sake of simplicity we take $J(x,y) = \pm 1$ with probability $\frac{1}{2}$. Under these conditions we have :

Proposition 4 : Let $P(J)$ be any Gibbs state corresponding to the system defined by the hamiltonian (13) for a given configuration of the $J(x,y)$. We assume that the dimension $d = 2$ and that $\alpha > 3$. Then, for almost all J, $P(J)$ is invariant by rotation of the spins.

One remarks that this statement implies

$$\langle s_x^1 \rangle (J) = 0 \tag{14}$$

for almost all J and in particular that the mean magnetization and the order parameter (13) are zero. In the deterministic case (that is, if $J(x,y) = 1$) there is a breakdown of symmetry as has been mentioned in Section 2.

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