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SOME REMARKS ON RAMSEY MATROIDS

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The purpose of this note is to summarize some of the results which are going to appear in [2] and to complement these results by stating some open problems and related consequences:

1. The following is the main result of [1]:

Theorem 1.1: For every simple matroid $M = M(X)$ and for every positive integer k there exists a matroid $N = N(Y)$ such that for every partition $Y = Y_1 \cup \ldots \cup Y_k$ there exists a matroid $M' = M'(X')$, $M' \simeq M$, such that $X' \subseteq Y_i$ for some Y_i.

The statement of 1.1 may be abbreviated by saying that matroids have singleton-Ramsey property. For many classes of structures (mainly of combinatorial type, such as graphs etc.) the existence of a singleton-Ramsey property may be established by simple means. The proof of 1.1 given in [1] (and also in [2]) is not of such simplicity.

Particularly, the following is not known:

Problem 1.2: Denote by $F(M)$ the minimal size of a matroid N which has the property stated in 1.1 . Is it true that there exists a constant α such that $F(M) \leq |X|^{\alpha}$ for every matroid $M = M(X)$?

It is also not known which classes of matroids have singleton-Ramsey property.

2. The following is the main result of [2]:

Theorem 2.1: For every simple matroid $M = M(X)$ and for every positive integer k there exists a matroid $N = N(Y)$ such that for every partition $a_i \cup \ldots \cup a_k$ of the set of all lines of N with exactly 2 elements there exists a matroid $M' = M'(X')$, $M' \simeq M$, such that all 2-point lines of M' are in one of the classes of the partition.

The statement of 2.1 may be summarized by saying that matroids have edge-Ramsey property (an edge meaning a flat with 2 independent points) .

A construction related to 2.1 is even less effective and therefore it is, at present, needless to state an analogy of 1.2 .

However, the following seems to be an interesting problem:

Problem 2.2: Which classes of matroids have edge-Ramsey property?

Perhaps this problem will have mostly a negative answer. E.g. it may be seen that the class of all graphical matroids does not have edge-Ramsey property. The same is true for transversal matroids.

The following problem is related to the existence of edge-Ramsey matroids and it seems to require a new technique:

Problem 2.3: Given a matroid $M = M(X)$ does there exists a matroid $N = N(Y)$ with the following property:

For every partition $a_1 \cup \ldots \cup a_n$ (n is a positive integer) of the set of all lines of N with exactly 2 points there exists a matroid $M' = M'(X')$, $M' \simeq M$, such that the partition of all 2-point lines restricted to M' is canonical.

Here we say that an equivalence $\sim$ on the set of all 2-lines of $M = M(X)$ is <u>canonical</u> if there exists an ordering $\leq$ of X such that one of the following possibilities holds for all 2-lines $x\,y$, $x'\,y'$ with $x < y$, $x' < y'$:

1. $xy \sim x'y'$ iff $x = x'$, $y = y'$
2. $xy \sim x'y'$ iff $x = x'$
3. $xy \sim x'y'$ iff $y = y'$
4. $xy \sim x'y'$

A positive solution to this problem would provide both an analogy of Erdös-Rado canonization lemma for matroids and a strenghtening of the selective property of matroids proved in [1] .

3. The above theorems were established by means of amalgams of matroids along a special set systems. The method of the proof has some further consequences.

For example the following may be proved using the basic construction given in [1] , [2]:

Given a matroid $M = M(X)$ denote by $\mathrm{Aut}\,M$ the group of all automorphisms $f : M \to M$.

<u>Theorem 3.1:</u> Let $M = M(X)$ be a matroid, G a subgroup of $\mathrm{Aut}\,(X)$. Then there exist a matroid $N = N(Y)$ with the following properties:

1. M is a restriction of N ;
2. $\mathrm{Aut}\,N \simeq G$;
3. every automorphism $f \in G$ extends uniquelly to an automorphism of N . (I.e. for every $f \in G$ there exists unique $\overline{f} \in \mathrm{Aut}\,N$ such that $\overline{f}|X = f$.)

This generalizes some of the results of Piff and Welsh, see [4], chapter 17 .

<u>Sketch of a proof:</u> We may assume without loss of generality that every point M lies on a line with at least 4 points. Consider a set $X' = X \times \{0,1\}$ and

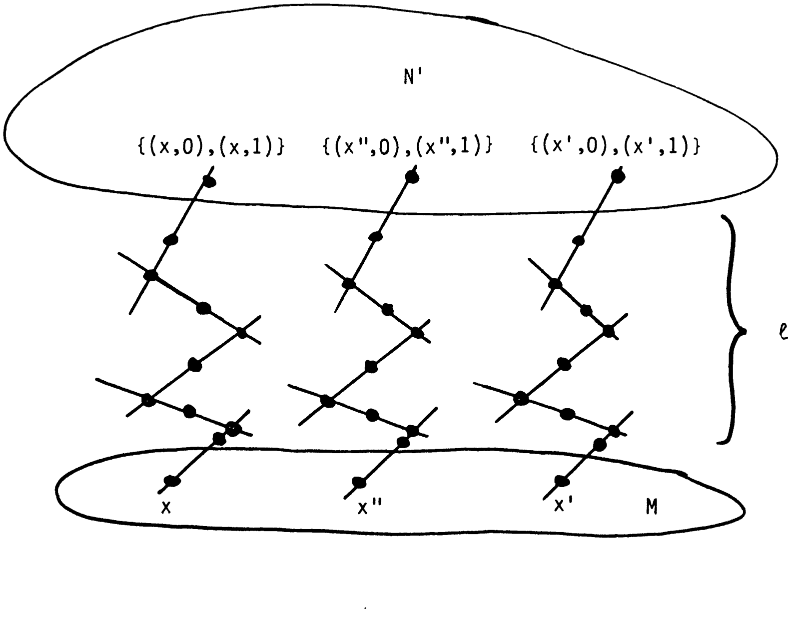
let G' be the group of all permutations $g' : X' \to X'$ defined by $g'(x,0) = (g(x),0)$, $g'(x,1) = (g(x),1)$, for a $g \in G$. Let (Y',E') be a graph which satisfies:

1. $\mathrm{Aut}\,(Y',E') \simeq G'$;
2. every $g' \in G'$ extends uniquelly to an automorphism of (Y',E') ;
3. (Y',E') is 3-connected and without triangles;
4. every edge of (Y',E') belongs to a cycle of length ≤ 7 ;
5. $\{\{(x,0),(x,1)\};\ x \in X\} \subseteq E'$.

The existence of such graphs follows from techniques given in [3] .

Let $N' = N(E')$ be the cycle matroid of the graph (Y',E') and let N be the amalgam of matroids N' and M and "chains of 3-lines" of length $\ell > \max\ \{7, r(N'), r(M)\}$ which is constructed in [1].
This is indicated on Fig. 1 .
As the amalgamation given in [1] is locally free, it is easy to see that N has all the desired properties:

Fig. 1



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