Jan Sobczyk
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QUANTUM DEFORMATION OF RELATIVISTIC SUPERSYMMETRY*

Jan Sobczyk

1) In this contribution I am going to review recent research on quantum deformations of Poincaré supergroup and superalgebra. It is based on a series of papers [1-5] and is motivated by both mathematics and physics. On the mathematical side, some new examples of noncommutative and noncocommutative Hopf superalgebras have been discovered. Moreover, it turns out that they have an interesting internal structure of graded bicrossproduct [5-6]. As far as physics is concerned the discussed deformations are closely related to quantum deformations of (not super!) Poincaré group and algebra which has become a subject of considerable interest in recent years [7] These deformations involve dimensional parameter κ (the classical case corresponds to the limit $\kappa \rightarrow \infty$) and some authors tried to impose limits on possible numerical values of κ coming from physical arguments [8]. Even if it is too early to say what is true physical significance of "quantum groups" it is certainly important to investigate different possibilities. Quantum groups appeared so far in physical literature in some two-dimensional models [9]. On the other hand there have been many efforts to consider quantum deformations of fundamental objects of relativistic physics: Poincaré group and Minkowski space [10]. In the case of Poincaré group and algebra some classification theorems are known. For example there exist a classification of possible quantum deformations of Poincaré group [11] and an analogous classification of classical r-matrices for Poincaré algebra [12]. On the physics side

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some authors tried to investigate consequences of deformations of Poincaré algebra in simple models [13]. Taking into account a role of supersymmetry in theoretical high energy physics it seems to be worthwhile to consider also quantum deformations of superPoincaré group and algebra.

In trying to construct quantum deformation of superPoincaré algebra one faces two technical difficulties. The first is that the superalgebra in question is not a semisimple one so that one cannot take immediate advantage of Drinfeld-Jimbo deformations [14]. The second is that on the superHopf algebra level we wish to keep a notion of reality. There are few strategies to overcome the first obstacle. One can start from some bigger semisimple Lie superalgebra for which the standard quantum deformation is known and then try to single out the deformation of superPoincaré algebra as a superHopf subalgebra. In [15] this program was almost realized. "Almost", as one necessarily arrives at extension of the structure we want obtain, namely at quantum deformation of superWeyl algebra. More effective procedure (however much more involved from the technical point of view) is to perform a suitable contraction. It is well known that this procedure leads to an interesting class of quantum deformation of space-time symmetries in different numbers of dimensions and with different signatures [16]. It is therefore suggestive to start from the quantum deformation of $OSp(1,4)$ which - classically - produces after contraction the superPoincaré algebra. In the paper [1] it was shown that in fact in this way it is possible to construct the required structure. In the rest of this paper we shall call it κ-deformed Poincaré superalgebra. As a typical feature in this type of computations, one has to "renormalize" the deformation parameter of $U_q(OSp(1,4))$ as $q = \exp\frac{1}{2\kappa R}$, where R is a contraction parameter.

2) For a sake of completeness we give first the definition of super (or Z_2-graded) Hopf algebra [17]

We assume that $\mathcal{H}$ is a Z_2-graded algebra, i.e. as a vector space $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$. We define the parity of an element $h \in \mathcal{H}$ as follows:

$$p(h) = 0 \text{ if } h \in \mathcal{H}_0; \qquad p(h) = 1 \text{ if } h \in \mathcal{H}_1. \tag{1}$$

We introduce the tensor product of two Z_2-graded algebras $\mathcal{H} \otimes \mathcal{H}'$ as Z_2-graded algebra with the following multiplication rule

$$(h \otimes h') \cdot (g \otimes g') = (-1)^{p(h')p(g)} hg \otimes h'g' \tag{2}$$

where $h, g \in \mathcal{H}$ and $h', g' \in \mathcal{H}'$.

The Hopf superalgebra $(\mathcal{H}, \Delta, \epsilon, S)$ is defined in the following way: $\mathcal{H}$ is a Z_2-graded algebra. The comultiplication map $\Delta\ : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$ is a homomorphism of $\mathcal{H}$ and is superassociative, i.e.

$$(\Delta \otimes 1) \otimes (\Delta) = (1 \otimes \Delta) \otimes (\Delta) \tag{3}$$

where the tensor product is defined in (2); besides $\Delta(1) = 1 \otimes 1$

The counit ϵ is linear map $\mathcal{H} \to C$, where

$$\epsilon(hh') = \epsilon(h)\epsilon(h'), \qquad (\epsilon \otimes 1) \otimes (\Delta(h) = (1 \otimes \epsilon)\Delta(h) = h \tag{4}$$

The antipode S is defined as linear antiisomorphism $\mathcal{H} \to \mathcal{H}$ with the property $(m(h \otimes h') = h \cdot h')$

$$m \circ (S \otimes 1) \circ \Delta(h) = m \circ (1 \otimes S) \circ \Delta(h) = \epsilon(h) \cdot 1 \tag{5}$$

If we introduce the graded flip operation:

$$\sigma(h \otimes h') = (-1)^{p(h)p(h')} h' \otimes h \cdot \tag{6}$$

and the graded opposite coproduct

$$\Delta'(h) = \sigma \circ \Delta(h) \tag{7}$$

we obtain also that

$$(S \otimes S)\Delta(h) = \Delta'(S(h)) \tag{8}$$

as well as

$$S(hh') = (-1)^{p(h)p(h')} S(h')S(h) \tag{9}$$

3) Let us now present a complete set of formulas defining the quantum deformation of superPoincaré algebra. The following notation is used: P_μ are momenta, L_j - boosts, M_j - $O(3)$ rotations and Q_A, $\bar{Q}_{\dot{A}}$ - two-component Weyl spinors.

$$[M_i, P_j] = i\epsilon_{ijk}P_k, \qquad [M_i, P_0] = 0 \tag{10}$$

$$[P_\mu, P_\nu] = 0, \qquad [L_i, M_j] = i\epsilon_{ijk}L_k, \qquad [L_i, P_0] = iP_i \tag{11}$$

$$[L_i, P_j] = i\kappa\delta_{ij} \sinh \frac{P_0}{\kappa}, \qquad [M_i, M_j] = i\epsilon_{ijk}M_k \tag{12}$$

$$[L_i, L_j] = -i\epsilon_{ijk}(M_k \cosh \frac{P_0}{\kappa} - \frac{1}{8\kappa}T_\kappa \sinh \frac{P_0}{2\kappa} + \frac{1}{16\kappa^2}P_k(T_0 - 4\vec{M}\vec{P})) \tag{13}$$

where

$$T_\mu = Q^A(\sigma_\mu)_{A\dot{B}}\bar{Q}^{\dot{B}} \qquad \sigma_\mu = (\vec{\sigma}, \mathbf{1}_2) \tag{14}$$

(T_μ satisfy the following commutation relations:

$$[T_i, T_j] = -4i\epsilon_{ijk}(P_k T_0 - 2\kappa \sinh\frac{P_0}{2\kappa} T_k), \qquad [T_0, T_i] = -4i\epsilon_{ijk}P_j T_k) \tag{15}$$

Further relations are:

$$[M_i, Q_A] = -\frac{1}{2}(\sigma_i)_A{}^B Q_B, \qquad [L_i, Q_A] = -\frac{i}{2}\cosh\frac{P_0}{2\kappa}(\sigma_i)_A{}^B Q_B \tag{16}$$

$$[P_\mu, Q_A] = [P_\mu, \bar{Q}_{\dot{A}}] = 0 \tag{17}$$

$$\{Q_A, \bar{Q}_{\dot{B}}\} = 4\kappa\delta_{A\dot{B}} \sinh\frac{P_0}{2\kappa} - 2P_i(\sigma_i)_{A\dot{B}} \tag{18}$$

$$\{Q_A, Q_B\} = \{\bar{Q}_{\dot{A}}, \bar{Q}_{\dot{B}}\} = 0 \tag{19}$$

$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(P_i) = P_i \otimes e^{\frac{P_0}{2\kappa}} + e^{-\frac{P_0}{2\kappa}} \otimes P_i \tag{20}$$

$$\Delta(M_i) = M_i \otimes 1 + 1 \otimes M_i \tag{21}$$

$$\Delta(L_i) = L_i \otimes e^{\frac{P_0}{2\kappa}} + e^{-\frac{P_0}{2\kappa}} L_i + \frac{1}{2\kappa}\epsilon_{ijk}(P_j \otimes M_k e^{\frac{P_0}{2\kappa}} + M_j e^{-\frac{P_0}{2\kappa}} \otimes P_k) +$$
$$+\frac{i}{8\kappa}(\sigma_i)_{\dot{A}B}(\bar{Q}_{\dot{A}} e^{-\frac{P_0}{4\kappa}} \otimes Q_B e^{\frac{P_0}{4\kappa}} + Q_B e^{-\frac{P_0}{4\kappa}} \otimes \bar{Q}_{\dot{A}} e^{\frac{P_0}{4\kappa}}) \tag{22}$$

$$\Delta(Q_A) = Q_A \otimes e^{\frac{P_0}{4\kappa}} + e^{-\frac{P_0}{4\kappa}} \otimes Q_A \tag{23}$$

$$\Delta(\bar{Q}_{\dot{A}}) = \bar{Q}_{\dot{A}} \otimes e^{\frac{P_0}{4\kappa}} + e^{-\frac{P_0}{4\kappa}} \otimes \bar{Q}_{\dot{A}} \tag{24}$$

$$S(L_i) = -L_i + \frac{i}{\kappa}P_i = -L_i + \frac{3i}{2\kappa}P_i - \frac{i}{8\kappa}(Q\sigma_i\bar{Q} + \bar{Q}\sigma_i Q) \tag{25}$$

$$S(M_i) = -M_i \qquad S(P_\mu) = -P_\mu \tag{26}$$

$$\epsilon(X) = 0 \text{ for } X \in \{P_\mu, L_j, M_j, Q_A, \bar{Q}_{\dot{A}}\} \tag{27}$$

4) It is well known that the superPoincaré algebra has a structure of graded semidirect product. It is interesting to observe that the quantum deformed analog of this structure is graded bicrossproduct. Let us introduce this notion with mathematical accuracy.

We assume that $\mathcal{H}_1$, $\mathcal{H}_2$ are two Hopf superalgebras (we put $h, g, f \in \mathcal{H}_1$, $a, b, c \in \mathcal{H}_2$). We assume further that $\mathcal{H}_2$ is a right $\mathcal{H}_1$ module, with the action α satisfying the grading property (we use the notation $\alpha(a \otimes h) \equiv a \triangleleft h$)

$$ab \triangleleft h = (-1)^{p(h_{(1)})p(b)}(a \triangleleft h_{(1)})(b \triangleleft h_{(2)}) \tag{28}$$

$$a \triangleleft (hg) = (a \triangleleft h) \triangleleft g \tag{29}$$

$\mathcal{H}_1$ is a left $\mathcal{H}_2$-comodule cosuperalgebra with the action β satisfying $\beta(1) = 1 \otimes 1$ and the following properties (we shall often use the notation $\beta(h) = h^{(1)} \otimes h^{(2)}$)

$$(1 \otimes \beta) \circ \beta = (\Delta \otimes 1) \circ \beta \tag{30}$$

$$(\epsilon \otimes 1) \circ \beta(h) = 1_{\mathcal{H}_1} \otimes h \tag{31}$$

$$(1 \otimes \Delta)\beta(h) = m_{12}\sigma_{23}(\beta \otimes \beta)\Delta \tag{32}$$

The linear Z_2-graded space (superspace) $\mathcal{H}_1 \otimes \mathcal{H}_2$ is endowed with the Hopf superalgebra structure and defines the Z_2-graded bicrossproduct Hopf superalgebra $\mathcal{H}_1 \bowtie \mathcal{H}_2$ with the following definitions of multiplication, comultiplication, counit and antipode

$$(h \otimes a)(g \otimes b) = (-1)^{p(a)p(g_{(1)})} h g_{(1)} \otimes (a \triangleleft g_{(2)}) b \tag{33}$$

$$\Delta(h \otimes a) = (-1)^{p({h_{(2)}}^{(2)})p(a_{(1)})} h_{(1)} \otimes {h_{(2)}}^{(1)} a_{(1)} \otimes {h_{(2)}}^{(2)} \otimes a_{(1)} \tag{34}$$

$$\epsilon(h \otimes a) = \epsilon(h) \cdot \epsilon(a) \tag{35}$$

$$S(h \otimes a) = (-1)^{p(h^{(2)})[p(h^{(1)}) + p(a)]} (1 \otimes S(h^{(1)} a))(S(h^{(2)}) \otimes 1) \tag{36}$$

if the following compatibility conditions are satisfied (compare with [18]):

$$\epsilon(a \triangleleft h) = \epsilon(a)\epsilon(h) \tag{37}$$

$$\Delta(a \triangleleft h)(-1)^{p(a_{(2)})[p(h_{(1)}) + p({h_{(2)}}^{(1)})]} (a_{(1)} \triangleleft h_{(1)}) {h_{(2)}}^{(1)} \otimes a_{(2)} \triangleleft {h_{(2)}}^{(2)} \tag{38}$$

$$\beta(hg) = (-1)^{p(h^{(2)})[p(g_{(1)}) + p({g_{(2)}}^{(1)})]} (h^{(1)} \triangleleft g_{(1)}) {g_{(2)}}^{(1)} \otimes h^{(2)} {g_{(2)}}^{(2)} \tag{39}$$

$${h_{(1)}}^{(1)} (a \triangleleft h_{(2)}) \otimes {h_{(1)}}^{(2)} = (-1)^{p(a)p({h_{(2)}}^{(1)}) + p(h_{(1)})p({h_{(2)}}^{(2)})} (a \triangleleft h_{(1)}) {h_{(2)}}^{(1)} \otimes {h_{(2)}}^{(2)} \tag{40}$$

5) Now we show that the κ-deformed Poincaré superalgebra has a structure of graded bicrossproduct. This structure is not immediately seen in formulas (10-27). Below we shall demonstrate how it can be described in terms on generators P^+_μ, L^+_j, M^+_j, Q^+_A and $\bar{Q}^+_{\dot{A}}$ which differ from previously introduced generators (without a subscript $^+$) by a simple (but nonlinear) redefinition. The statement proven in [3] is that

$$U_\kappa(\mathcal{P}_{4;1}) = T^\kappa_{4;2} \bowtie O(1,3;2) \tag{41}$$

where $U_\kappa(\mathcal{P}_{4;1})$ denotes the κ-deformation of superPoincaré algebra, $O(1,3;2)$ is the undeformed superHopf subalgebra of superPoincaré algebra generated by Lorentz transformations and $\bar{Q}_{\dot{A}}$, and $T^{\kappa}_{4;2}$ is a Hopf superalgebra generated by four-momenta and Weyl spinors $\bar{Q}_A$ with defining relations:

$$[P\mu^+, P^+_\nu] = 0, \qquad [\bar{Q}^+_{\dot{A}}, P^+_\mu] = 0 \tag{42}$$

$$\Delta P^+_0 = P^+_0 \otimes \mathbf{1} + \mathbf{1} \otimes P^+_0, \qquad \Delta P^+_i = P^+_i \otimes \mathbf{1} + e^{-\frac{P^+_0}{\kappa}} \otimes P^+_i \tag{43}$$

$$\Delta(\bar{Q}^+_{\dot{A}}) = \mathbf{1} \otimes \bar{Q}^+_{\dot{A}} + \bar{Q}^+_{\dot{A}} \otimes e^{\frac{P^+_0}{2\kappa}} \tag{44}$$

$$S(\bar{Q}^+_{\dot{A}}) = -\bar{Q}^+_{\dot{A}} e^{-\frac{P^+_0}{2\kappa}} \tag{45}$$

The actions and coaction defining the bicrossproduct are mappings

$$\alpha : T^{\kappa}_{4;2} \otimes O(1,3;2) \rightarrow T^{\kappa}_{4;2} \tag{46}$$

$$\beta : O(1,3;2) \rightarrow T^{\kappa}_{4;2} \otimes O(1,3;2) \tag{47}$$

$$P^+_j \triangleleft L^{(+)}_i = -i\delta_{ij}\left[\frac{\kappa}{2}(1 - e^{-\frac{2P^+_0}{\kappa}}) + \frac{1}{2\kappa}\vec{P}^{(+)^2}\right] + \frac{i}{\kappa}P^+_i P^+_j \tag{48}$$

$$\bar{Q}^{(+)}_{\dot{A}} \triangleleft L^{(+)}_i = \frac{i}{2}e^{-\frac{P^+_0}{\kappa}}(\bar{Q}^{(+)}\sigma_i)_{\dot{A}} - \frac{1}{2\kappa}\epsilon_{ikl}P^+_k(\bar{Q}^{(+)}\sigma_i)_{\dot{A}} \tag{49}$$

$$\bar{Q}^{(+)}_{\dot{B}} \triangleleft Q^{(+)}_A = 4\kappa\delta_{A\dot{B}}\sinh\frac{P^+_0}{2\kappa} - 2e^{\frac{P^+_0}{2\kappa}}P^+_i(\sigma_i)_{A\dot{B}} \tag{50}$$

$$\beta\left(L^{(+)}_i\right) = e^{-\frac{P^+_0}{\kappa}} \otimes L^+_i + \frac{1}{\kappa}\epsilon_{ijk}P^+_j \otimes M^+_k + \frac{i}{4\kappa}(\sigma_i)_{A\dot{B}}e^{\frac{P^+_0}{\kappa}}\bar{Q}^+_{\dot{A}} \otimes Q^+_A \tag{51}$$

$$\beta\left(Q^+_A\right) = e^{-\frac{P^+_0}{2\kappa}} \otimes Q^+_A \tag{52}$$

All the details can be found in [5].

The discovered graded bicrossproduct structure is important for many reasons. In enables to define a natural action of the κ-deformed superPoincaré algebra on the chiral superspace. Then it is a very useful tool in demonstrating the duality between the quantum deformation of the algebra and the group [19]. It is also possible that bicrossproduct structure might be helpful in solving a problem whether there exist a universal R-matrix for $U_\kappa(\mathcal{P}_{4;1})$.

To complete the discussion of $U_\kappa(\mathcal{P}_{4;1})$ let us mention that it is possible to find its Casimir operators. The first Casimir is completely analogous as in the case of κ-deformed Poincaré algebra

$$C_2 = \left(2\kappa\sinh\frac{P_0}{2\kappa}\right)^2 - \vec{P}^2 \tag{53}$$

In order to find the second Casimir it is useful to find a new basis for $U_\kappa(\mathcal{P}_{4;1})$ in which the algebraic sector is undeformed and all the deformation is contained in a very complicated coproduct. This program has been performed in [4] where one can find all the technical details. With the knowledge of Casimir operators one can try to develop a theory of representations of $U_\kappa(\mathcal{P}_{4;1})$.

6) An important information which can be deduced from the formulae (20-24) is a form of classical r-matrix. It is only necessary to analyze the lowest order deformation terms of the coproduct. It turns out that in the classical limit as $\kappa \to \infty$ $U_\kappa(\mathcal{P}_{4;1})$ gives rise to the Lie superbialgebra that is a coboundary. The corresponding classical r-matrix reads:

$$r(U_\kappa(\mathcal{P}_{4;1})) = L_j \wedge P_j - \frac{i}{4} Q_A \wedge \bar{Q}_{\dot{A}} \tag{54}$$

Classical r-matrices make Lie groups Poisson-Lie groups as they enable to define on the group Poisson bracket which is compatible with the group action. If we forget for a moment about all the dispute about the proper way to define supergroup we can do the same for r-matrix in the supersymmetric case. Poisson superbrackets one obtains contain information about lowest order deformation of the quantum supergroup. An obvious simple idea to obtain a quantum group is to change Poisson brackets into commutators. This procedure is in general ambiguous due to ordering problems. There are however examples in which these ambiguities are absent so that it leads to correct results. It is a way in which κ-deformation of Poincaré group was obtain [20]. If one repeats this procedure in the superPoincaré case, one discovers that unfortunately ordering ambiguities do appear. It turns out however that in the discussed case it is possible to choose the ordering is such a way that one obtains a consistent superHopf algebra - κ-deformation of the Poincaré supergroup. Moreover one obtains another example of graded bicrossproduct [2]. We present now some technical details. The coordinates on the Poincaré supergroup are taken to be: $(A_\alpha^\beta, A_{\dot{\alpha}}^{\dot{\beta}})$ (we use the spinorial representation of the Lorentz generators), "chiral" translations $z_\mu = x_\mu + \frac{i}{2}\theta_\alpha(\sigma_\mu)^{\alpha\dot{\beta}}\bar{\theta}_{\dot{\beta}}$, supertranslations θ and $\bar{\theta}$. We can introduce now two κ-deformed Hopf superalgebras:

i) The algebra of functions $C(z_\mu, \theta_\alpha)$ on κ-deformed chiral superspace (z_μ, θ_α), with the following Hopf superalgebra relations:

$$[z_i, z_j] = 0\,, \qquad [z_0, z_i] = -\frac{i}{\kappa} z_i \tag{55}$$

$$[z_0, \theta_\alpha] = -\frac{i}{2\kappa}\theta_\alpha, \quad [z_i, \theta_\alpha] = 0, \qquad \{\theta_\alpha, \theta_\beta\} = 0. \tag{56}$$

and

$$\Delta(z_\mu) = z_\mu \otimes 1 + 1 \otimes z_\mu \tag{57}$$

$$\Delta(\theta_\alpha) = \theta_\alpha \otimes 1 + 1 \otimes \theta_\alpha \tag{58}$$

ii) The classical Hopf superalgebra of functions $C(A_{\alpha\beta}, A_{\dot\gamma\dot\delta}, \bar\theta_{\dot\alpha})$ on the superextension of classical Lorentz group, with the following defining relations:

$$[A_{\alpha\beta}, A_{\gamma\delta}] = [A_{\alpha\beta}, A_{\dot\gamma\dot\delta}] = [A_{\dot\alpha\dot\beta}, A_{\dot\gamma\dot\delta}] = 0 \tag{59}$$

$$[A_{\alpha\beta}, \bar\theta_{\dot\gamma}] = [A_{\dot\alpha\dot\beta}, \bar\theta_{\dot\gamma}] = 0 \qquad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = 0 \tag{60}$$

$$\Delta A_{\alpha\beta} = A_{\alpha\gamma} \otimes A_{\gamma\beta}\,, \qquad \Delta A_{\dot\alpha\dot\beta} = A_{\dot\alpha\dot\gamma} \otimes A_{\dot\gamma\dot\beta} \tag{61}$$

$$\Delta\bar\theta_{\bar\alpha} = \bar\theta_{\bar\alpha} \otimes 1 + (A_{\dot\alpha\dot\beta})^{-1} \otimes \bar\theta_{\dot\beta}\,. \tag{62}$$

In the paper [5] the following statement has been proven: The $D = 4$ κ-deformed Poincaré supergroup can be described as the graded bicrossproduct Hopf superalgebra

$$C_\kappa(\mathcal{P}_{4;1}) = C(z_\mu, \theta_\alpha) \bowtie C(A_{\alpha\beta}, A_{\dot\alpha\dot\beta}, \bar\theta_{\dot\alpha}) \tag{63}$$

with the following definition of the action α

$$\bar\theta_{\dot\alpha} \triangleleft z_i = -\frac{i}{2\kappa}\left[1 - (A^+A)^{-1}\right]_{\dot\alpha}{}^{\dot\beta}\bar\theta_{\dot\beta} \tag{64}$$

$$\bar\theta_{\dot\alpha} \triangleleft z_0 = -\frac{i}{2\kappa}(A^+A)_{\dot\alpha}{}^{\dot\beta}\bar\theta_{\dot\beta} \tag{65}$$

$$\bar\theta_{\dot\alpha} \triangleleft \theta_\beta = -\frac{i}{2\kappa}\left[1 - (AA^+)^{-1}\right]_{\dot\alpha\beta} \tag{66}$$

$$A_{\alpha\beta} \triangleleft z_i = \frac{1}{2\kappa}\left[(A\sigma_k)_{\alpha\beta}\Lambda_{ik}(A,\bar A) - (\sigma_i A)_{\alpha\beta}\right] \tag{67}$$

$$\bar A_{\dot\alpha\dot\beta} \triangleleft z_i = \frac{1}{2\kappa}\left[(\sigma_i\bar A)_{\dot\alpha\dot\beta}\Lambda_{lk}(A,\bar A) - (\bar A\sigma_i)_{\dot\alpha\dot\beta}]\right] \tag{68}$$

$$A_{\alpha\beta} \triangleleft z_0 = \frac{1}{2\kappa}(A\sigma_i)_\alpha{}^\beta\Lambda_{i0}(A,\bar A) \tag{69}$$

$$\bar A_{\dot\alpha\dot\beta} \triangleleft z_0 = \frac{1}{2\kappa}(\sigma_i\bar A)_{\dot\alpha}{}^{\dot\beta}\Lambda_{i0}(A,\bar A) \tag{70}$$

$$A_{\alpha\beta} \triangleleft \theta_\gamma = A_{\dot\alpha\dot\beta} \triangleleft \theta_\gamma = 0 \tag{71}$$

and coaction β

$$\beta(z_\mu) = \Lambda_\mu{}^\nu(A,\bar A) \otimes z_\nu - i(A^{-1)}\sigma_\mu)_{\alpha\dot\beta}\bar\theta_{\dot\beta} \otimes \theta_\alpha \tag{72}$$

$$\beta(\theta_\alpha) = (A^{-1})_{\beta\alpha} \otimes \theta_\beta \tag{73}$$

Taking into account very similar properties of κ-deformations of Poincaré and superPoincaré group and algebras it is very likely that also in the super case both Hopf algebras are dual to each other. But a proof of this hypothesis is still missing.

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INSTITUTE OF THEORETICAL PHYSICS
WROCLAW UNIVERSITY
PL. MAXA BORNA 9, 50-204 WROCLAW, POLAND.