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On the Fréchet-Urysohn property in spaces of continuous functions

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A family $\mathcal{D}$ of sets is an ω*-cover* of a set X if, for every finite set $F \subseteq X$, there is a set $D \in \mathcal{D}$ such that $F \subseteq D$. A topological space X has the *Gerlits-Nagy property* (abbreviated *GNP*) if, for every open ω-cover $\mathcal{D}$ of X, there is a sequence $\langle D_n : n \in \omega \rangle \in {}^{\omega}\mathcal{D}$ such that $X \subseteq \bigcup_{m<\omega} \bigcap_{m \leq n < \omega} D_n$.

Gerlits and Nagy [1] introduced the GNP (which they called "property γ") and proved that the following are equivalent for every completely regular space X:

(1) X has the GNP ;

(2) $C(X)$ is a Fréchet-Urysohn space;

(3) $C(X)$ is countably bisequential;

(4) $C(X)$ is a w-space.

Here $C(X)$ is the space of all continuous real-valued functions defined on X, with the topology of pointwise convergence. The notion of a w-space is due to Gruenhage [2]; a simple characterization of w-spaces was found by Sharma [5].

Let R be the real line. It was shown in [1] that every subset of R, which has the GNP, also has Rothberger's [4] property C'' and is always of the first category; hence in Laver's [3] model, the only subsets of R having the *GNP* are the countable sets. On the other hand, it was shown in [1] that, assuming MA_{κ}, every subset of R of cardinality κ has the *GNP*; moreover, A.Hajnal has shown that the existence of an uncountable subset of R with the *GNP* is consistent with *ZFC+GCH*. The following theorem improves Hajnal's result:

<u>Theorem</u> Assuming *CH*, there is a set $X \subseteq R$ such that $|X| = 2^{\aleph_0}$ and X has the *GNP*.

As the *GNP* is clearly preserved by continuous mappings, we have the following corollary, which answers a question of Sierpiński [6, p.86]:

<u>Corollary.</u> Assuming *CH*, there is a set $X \subseteq R$ of cardinality $2^{\aleph_0}$ such that every continuous image of X has property C'' and is always of the first category.

<u>Problem.</u> Can *CH* be replaced by *MA* in the theorem above, or in the corollary?

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