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SOME TYPES OF POINTS IN N^*.

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We deal with some kinds of points in $N^* = \beta N \setminus N$, matrix points, R-points, O-points.

I. Matrix points. Recall, an independent matrix in N^* [1] is a family $\{A_{\alpha\beta} : \alpha \in 2^\omega, \beta \in 2^\omega\}$ of clopen subsets of N^* such that
(1) for all β $A_{\alpha\beta} \cap A_{\alpha'\beta} = \emptyset$ if $\alpha \neq \alpha'$, and
(2) if $\beta_0, \dots, \beta_n$ are distinct, then for all $\alpha_0, \dots, \alpha_n$ distinct or not, $\bigcap \{A_{\alpha_i\beta_i} : i \leq n\} \neq \emptyset$.

I.1. Definition. A point $x \in N^*$ is called a matrix point if there is an independent matrix as just defined, such that for any sequence $\sigma = \{U_i : i \in \omega\}$ of neighbourhoods of x there is $B(\sigma) \subset 2^\omega$ with $|B(\sigma)| < 2^\omega$ such that $x \in \bigcup \{A_{\alpha_i\beta_i} \cap U_i : i \in \omega\}$ where $\{\beta_i : i \in \omega\} \subset 2^\omega \setminus B(\sigma)$ are distinct and $\{\alpha_i : i \in \omega\} \subset 2^\omega$ [4]

A simple consequence of this definition is

I.2. Theorem. Let x be a matrix point in N^* for the matrix $\{A_{\alpha\beta} : \alpha \in 2^\omega, \beta \in 2^\omega\}$. Let $\{F_i : i \in \omega\}$ be a family of closed sets in N^*, not containing x. Suppose $B \subseteq 2^\omega$ and $|B| = 2^\omega$ and for any $\beta \in B$ there is an $\alpha \in 2^\omega$ with $A_{\alpha\beta} \cap (\bigcup \{F_i : i \in \omega\}) = \emptyset$. Then $x \notin \bigcup \{F_i : i \in \omega\}$

An immediate consequence of Theorem I.2 is the fact that no matrix point can be in the closure of the union of any countable family of closed sets in N^* each of which has Souslin number less than 2^ω.

I.3. Theorem. A matrix point of N^* is c-ok.

I.4. Theorem. There are 2^c matrix points in N^*.

II. Strictly R-points. A family $\lambda = \{A\}$ is precisely n-linked if an intersection of any n elements of λ is not empty and intersection of any $n+1$ elements of λ is empty [1].

For each $1 \leq n < \omega$ there is n-linked family λ in N^* such that $|\lambda| = 2^\omega$ and λ consists of clopen subsets of N^* [1].

Let $W = \bigcup \{U_n : n \in \omega\}$ be a union of clopen disjoint subsets of ,

N^* and $\pi_n = \{V_\alpha(n) : \alpha < 2^\omega\}$ be a precisely n-linked family of clopen subsets of U_n.

We call $\Pi = \{\pi_n : n \in \omega\}$ a foltering system. Define: $B' = \{O' : O'$ is clopen subset of $\overline{W}$ and for each $n \in \omega$ there is $V_\alpha(n) \in \pi_n$ such that $O' \cap U_n \supseteq V_\alpha(n)\}$. Define $\Phi(\Pi) = \cap\{O' : O' \in B'\}$.

II.1. Definition. A point $x \in \overline{W} \setminus W$ is called a strictly R-point if $x \in \Phi(\Pi)$ for some filtering system $\Pi = \{\pi_n : n \in \omega\}$ (see [2],[3])

II.2. Theorem. If x is strictly R-point, then $N^* \setminus \{x\}$ is not normal.

Note that matrix points are strictly R-points.

III. O-points. We prove that there are 2^c O-points in N^*, and so answer the E.van Douwen's question.

Recall that a set $A \subseteq N$ has a density 0 if $\lim\limits_{n\to\infty} \frac{|\{i \in A : i \leq n\}|}{n} = 0$ We will write $d(A) = 0$.

III.1. Definition. A point $x \in N^*$ is called a O-point if for each permutation $f : N \to N$ there is $A \subseteq N$ such that $A \in x$ and $d(f(A)) = 0$.

III.2. Theorem. There are 2^c O-points in N^*.

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