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Always of the first category sets

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ALWAYS OF THE FIRST CATEGORY SETS

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Results of this note were presented during 7th (compare [6]) and 12th Winter Schools of Abstract Analysis in Czechoslovakia. We prove in ZFC (using a theorem of D.Maharam and A.H. Stone [11]) that there is an always of the first category subset A of the real line R such that A + A is not of the first category in R. The lack of such example was pointed out in [2]. To prove this we investigate (often more carefully than necessary) a certain sub-σ-ideal of the σ-ideal of always of the first category subsets of R. Some remarks concerning universal null (= universal measure zero) subsets of R are also included.

Let X be a separable metric space. If every dense in itself subset of X is of the first category relative to itself, then X is said to be always of the first category. We denote by $\mathcal{K}(X)$, or simply $\mathcal{K}$ if X=R, the σ-ideal of the first category sets in X and by $\mathcal{K}^*(X)$, or $\mathcal{K}^*$ if X=R, the σ-ideal of always of the first category subsets of X. If Y is a metric space such that $X \subseteq Y$, then X is always of the first category iff for all perfect sets $P \subseteq Y$ the set $P \cap X$ is of the first category relative to P. References concerning $\mathcal{K}^*$ can be found e.g. in [10] and in the surveys articles [2] and [14]. We denote by $\mathcal{B}(X)$ the σ-field of Borel subsets of X. A space X is called a universal null set if there is no continuous probability measure on $\mathcal{B}(X)$ (for many equivalent definitions and references see [2] and [14]). We denote by $\mathcal{N}$ the σ-ideal of universal null subsets of R and by $\mathcal{L}_0$ the σ-ideal of Lebesgue measure zero subsets of R. A separable complete metric space is called Polish space. We need the following known

<u>Theorem.</u> If X and Y are uncountable Polish spaces without isolated points, then there is a Borel isomorphism f from X onto Y such that $f(\mathcal{K}(X)) = \mathcal{K}(Y)$.

This paper is in final form and no version of it will be submitted for publication elsewhere.

The Theorem follows from Lemma 1 in [13]. It also follows from Sikorski's theorem [17] and the result of Żaskowsky (see Example I, §35 in [18]) that any two separable nonatomic complete Boolean algebras are isomorphic. Any Borel isomorphism as in the above Theorem will be called category preserving (c. p.) isomorphism.

A family $\mathcal{J}$ of subsets of the real line R is called σ-ideal on R if $A_0, A_1, A_2, \ldots \in \mathcal{J}$ implies $\bigcup\{A_n: n=0,1,2,\ldots\} \in \mathcal{J}$ and $\mathcal{P}(A_0) \subseteq \mathcal{J}$, $\mathcal{J} \subsetneq \mathcal{P}(R)$ and for every $x \in R$ we have $\{x\} \in \mathcal{J}$. If $\mathcal{J}$ is a σ-ideal on R then we define (see [6])

$$\overline{\mathcal{J}} = \{A \subseteq R: \text{ for every } B \subsetneq R \text{ such that there exists a 1-1 Borel measurable function } f: B \to A \text{ we have } B \in \mathcal{J}\}.$$

If in the definition of $\overline{\mathcal{J}}$ we additionally assume that the function f maps B onto A, then such obtained family we denote by $\widetilde{\mathcal{J}}$.

<u>Proposition 1.</u> $\overline{\mathcal{J}}$ is a σ-ideal on R. $\overline{\mathcal{J}} \subseteq \mathcal{J}$, $\overline{\overline{\mathcal{J}}} = \overline{\mathcal{J}}$ and $\overline{\mathcal{J}} = \widetilde{\mathcal{J}}$.

<u>Proof.</u> The only nontrivial part is $\overline{\mathcal{J}} = \widetilde{\mathcal{J}}$. It is clear that $\overline{\mathcal{J}} \subseteq \widetilde{\mathcal{J}}$. Let now $A \in \widetilde{\mathcal{J}}$. Suppose that $A \notin \overline{\mathcal{J}}$. Hence there is $B \subseteq R$ such that $B \notin \mathcal{J}$ and there is a one to one Borel measurable function f from B into A. Clearly either $B \cap (-\infty, 0) \notin \mathcal{J}$ or $B \cap [0, \infty) \notin \mathcal{J}$. Assume that e.g. $B \cap (-\infty, 0) \notin \mathcal{J}$. Define $B_1 = B \cap (-\infty, 0)$. Let $B_2 \subseteq [0, \infty)$ be such that there is a Borel isomorphism h from B_2 onto $A \setminus f(B_1)$. Let $B_3 = B_1 \cup B_2$ and let k be a function from B_3 into A defined by $k(x) = f(x)$ if $x \in B_1$ and $k(x) = h(x)$ if $x \in B_2$. We have that k is a 1-1 Borel measurable function from B_3 onto A such that $B_3 \notin \mathcal{J}$. Hence a contradiction with $A \in \widetilde{\mathcal{J}}$.

<u>Remark 1.</u> Marczewski [19] proved that $\overline{\mathcal{L}_0} = \mathcal{N}$ (see also Section IV in [2] and references there). On the other hand assuming CH (or MA) there is $X \in \mathcal{K}^*$ such that there is a Borel isomorphism f from X into R with $f(X) \notin \mathcal{K}$ (see [10] or [2] or [14]), so $\overline{\mathcal{K}} \subsetneq \mathcal{K}^*$.

J.C. Morgan II has proved [15] that there exists a subset X of R every homeomorphic image of which is in $\mathcal{K}$ but $X \notin \mathcal{K}^*$. On the other hand we have the following

<u>Proposition 2.</u> Let $X \subseteq R$. If every Borel isomorphic image of X into R is in $\mathcal{K}$ then every such image is also in $\mathcal{K}^*$.

Proof. In order to prove Proposition 2 it is enough to prove that for every X satisfying the assumption of Proposition 2 we have $X \in \mathcal{K}^*$. Suppose $X \notin \mathcal{K}^*$. Let P be a perfect subset of R such that $P \cap X \notin \mathcal{K}(P)$. Let g_1 be a c.p. isomorphism from P onto $(0,\infty)$. Let g_2 be any Borel isomorphism from $X \setminus P$ into $(-\infty, 0]$. Let h be the Borel isomorphism from X into R such that $h(x) = g_1(x)$ if $x \in P \cap X$ and $h(x) = g_2(x)$ if $x \in X \setminus P$. We have $h(X) \supseteq h(X \cap P) = g_1(X \cap P) \notin \mathcal{K}(0,\infty)$. Hence $h(X) \notin \mathcal{K}$ and so we have a contradiction.

Notice that if $\mathcal{J}$ and $\mathcal{I}$ are σ-ideals on R such that $\mathcal{J} \supseteq \mathcal{I}$ and $\overline{\mathcal{J}} \subseteq \mathcal{I}$ then $\overline{\mathcal{J}} = \overline{\mathcal{I}}$. Indeed, $\mathcal{J} \supseteq \mathcal{I}$ implies $\overline{\mathcal{J}} \supseteq \overline{\mathcal{I}}$. $\overline{\mathcal{J}} \subseteq \mathcal{I}$ implies $\overline{\overline{\mathcal{J}}} \subseteq \overline{\mathcal{I}}$ and hence, by Proposition 1, $\overline{\mathcal{J}} \subseteq \overline{\mathcal{I}}$. So $\overline{\mathcal{J}} = \overline{\mathcal{I}}$. Hence by Proposition 2 we have

Proposition 3. $\overline{\mathcal{K}} = \overline{\mathcal{K}^*}$.

We have the following

Theorem 1. Let $m_1 = \min\{|Y| : Y \subseteq R$ and $Y \notin \mathcal{K}\}$. There is $X \subseteq R$ such that $|X| = m_1$ and $X \in \overline{\mathcal{K}^*}$.

Before giving a proof we would like to make some remarks. A similar theorem for universal null sets can be found in [4] (compare also [5]). In [4] we proved that there is a subset $X \subseteq R$ such that $|X| = m_1$ and all Borel isomorphic images of X into R are in $\mathcal{K}$ (and so by Proposition 2 of the present note, in $\mathcal{K}^*$). The proof of Theorem 1 is similar to the proof in [4] but a little longer. Recall that [4] was based on some ideas from K. Prikry [16]. Instead of Theorem 1 I announced in [6] the following

Theorem 1'. Let m_1 be as in Theorem 1 and let $m_2 = \min\{|Y| : Y \subseteq R$ and $Y \notin \mathcal{K}^*\}$. Then there are $X_1, X_2 \subseteq R$ such that $|X_1| = m_1$, $|X_2| = m_2$, $X_1 \in \overline{\mathcal{K}}$ and $X_2 \in \overline{\mathcal{K}^*}$.

The fact that Theorem 1' is the same as Theorem 1 follows from the fact that $m_1 = m_2$ (see Remark 3) or Prop. 3. Theorem 1 also follows from Theorem 2 in [6], which was proved there, with the help of the c.p. isomorphism. Theorem 1 itself I discovered after learning from D. Fremlin [3] that he proved the existence of a set $A \in \mathcal{K}^*(R \times R)$ such that its projection is not in $\mathcal{K}(R)$. On the other hand the main part of the mentioned result of Fremlin follows easily from Theorem 1' itself. Indeed. From Theorem 1' we have that there are $A, B \subseteq \omega^{\omega}$ (= irrational numbers) such that $|A| = |B|$, $A \notin \mathcal{K}^*$ and $B \in \mathcal{K}^*$. Let f be any bijection from A onto B and let G be the graph of f. Since $\omega^{\omega} \times \omega^{\omega}$ is homeomorphic to $\omega^{\omega} \subseteq R$ we have that G is in $\mathcal{K}^*(R \times R)$ but clearly the projection of G onto the first axis does not belong to $\mathcal{K}^*$.

Now we give a full proof of Theorem1.

Proof of Theorem 1. By Propositions 1 and 3 it is enough to prove that there is $X \subseteq R$ such that $|X| = m_1$ and $X \in \widetilde{\mathcal{K}}$. It follows from the assumption of Theorem 1 that there is $Y \subseteq R$ such that $|Y| = m_1$, $Y \notin \mathcal{K}$ and Y is dense on R (add the rational numbers to Y from the definition of m_1). Observe that each subset A of Y such that $|A| < m_1$ is in $\mathcal{K}(Y)$. Let $\{y_{\alpha} : \alpha < m_1\}$ be a one-to-one enumeration of Y. For every $\alpha < m_1$ let F_{α} be an $\mathcal{F}_{\sigma}$ subset of Y such that $F_{\alpha} \in \mathcal{K}(Y)$ and $F_{\alpha} \supseteq \{y_{\alpha'} : \alpha' \leq \alpha\}$. We now define $Z \subseteq m_1 \times Y$ as follows: $Z = \bigcup_{\alpha < m_1} (\{\alpha\} \times F_{\alpha})$. Let $O_0, O_1, O_2, \ldots$ be a countable base for the topology of Y.

Setting

$$E_i = \{\alpha < m_1 : O_i \subseteq F_{\alpha}\} \text{ for every } i < \omega ,$$

we get

$$Z = \bigcup_{i < \omega} E_i \times O_i \quad \text{(compare [16] or see general theorem [1])}.$$

Let $\mathcal{A}$ be a countably generated and separating points σ-field on m_1. Let $\mathcal{C}$ be a σ-field on m_1 generated by $\mathcal{A}$ and the family $\{E_i : i < \omega\}$. It is clear that Z belongs to the product σ-field $\mathcal{C} \otimes \mathcal{B}(Y)$. We claim that the σ-field $\mathcal{C}$ has the following property

(*) for every $B \subseteq R$ such that there is a one-to-one $(\mathcal{B}(B), \mathcal{C})$-measurable function from B onto m_1 we have $B \in \mathcal{K}$.

It is clear that in order to prove (*) it is enough to prove (*) for B such that B is dense in R. Let f be a one-to-one $(\mathcal{B}(B), \mathcal{C})$-measurable function from B onto m_1. We have that there is a subset S of $B \times Y$ such that $S \in \mathcal{B}(B \times Y)$, $\{y : (b,y) \in S\} \in \mathcal{K}(Y)$ for every $b \in B$, and $|B \setminus \{b : (b,y) \in S\}| < m_1$ for every $y \in Y$ (put $S = \{(f^{-1}(b), y) : (b,y) \in Z\}$). Applying Kuratowski-Ulam category version of Fubini's theorem [10] we have that $B \in \mathcal{K}(B)$ and hence $B \in \mathcal{K}$. Let X be a subset of R such that there is a one-to-one $(\mathcal{B}(X), \mathcal{C})$-measurable function g from X onto m_1 (e.g. let f be a characteristic function of a countable sequence of sets generating $\mathcal{C}$ and put $X = f(m_1)$ and $g = f^{-1}$). It is clear that (*) implies that $X \in \widetilde{\mathcal{K}}$.

Proposition 4. There is $X \in \overline{\mathcal{K}^*}$ such that $X \subseteq \omega^\omega$ and there is a continuous function f from ω^ω into ω^ω such that $f(X) \notin \mathcal{K}$.

Proof. By Theorem 1 we have that there are $A, B \subseteq \omega^\omega$ such that $|A| = |B|$, $A \in \overline{\mathcal{K}^*}$ and $B \notin \mathcal{K}$. Let g be a one-to-one function from A onto B and let G be the graph of g. Let h be a homeomorphism from ω^ω onto $\omega^\omega \times \omega^\omega$ and let π be the projection from $\omega^\omega \times \omega^\omega$ onto ω^ω such that $\pi(G) = B$. Define $X = h^{-1}(G)$ and $f = \pi h$.

Theorem2. There exists $A \in \overline{\mathcal{K}^*}$ such that $A + A \notin \mathcal{K}$.

Proof. We need the following particular case of a theorem of D. Maharam and A.H. Stone ([11] or [12]): If Z is a separable metric space, then every Borel measurable function f from Z into R can be expressed as the sum of two one-to-one Borel measurable functions. Let X and f be as in Proposition 4. By the theorem of D. Maharam and A.H. Stone there are one-to-one Borel measurable functions $f_i: \omega^\omega \to R$ $(i = 1, 2)$ such that $f = f_1 + f_2$. Since $f_i^{-1}: f_i(X) \to X$ $(i = 1, 2)$ are $\overset{1-1}{}$Borel measurable functions [10] we have that $f_i(X) \in \overline{\mathcal{K}^*}$ $(i = 1, 2)$. Define $A = f_1(X) \cup f_2(X)$. Clearly $A \in \overline{\mathcal{K}^*}$. Since $\mathcal{K} \not\ni f(X) \subseteq f_1(X) + f_2(X) \subseteq A + A$ we have $A + A \notin \mathcal{K}$.

Corollary. There exists $A \in \mathcal{K}^*$ such that $A + A \notin \mathcal{K}$.

The lack of such example was pointed out in [2]. It is well known [2] that assuming CH (or Martin's Axiom) there is $A \in \mathcal{K}^*$ such that $A + A = R$. Miller (compare [14]) proved that ZFC + (all $X \in \mathcal{K}^*$ have cardinality at most $\aleph_1$) is consistent if ZFC is consistent. Hence it is unprovable in ZFC that there is $A \in \mathcal{K}^*$ such that $A + A = R$.

In [7] we observed that there exist $N \in \mathcal{N}$ such that $N + N \notin \mathcal{N}$. It was left as an open problem if there exist $N \in \mathcal{N}$ such that $N + N \notin \mathcal{L}_0$. Theorem of D. Maharam and A.H. Stone is just what we need to see that the answer to that question is yes.

Theorem 3. There exists $N \in \mathcal{N}$ such that $N + N \notin \mathcal{L}_0$.

Proof. It is known (Theorem 2 (i) in [5]) that there is a continuous function $f: R \to R$ such that there is $X \in \mathcal{N}$ with $f(X) \notin \mathcal{L}_0$. Let $f = f_1 + f_2$, where f_1 and f_2 are one-to-one Borel measurable functions. Put $N = f_1(X) \cup f_2(X)$.

Remark 2.

a) In the definition of $\overline{\mathcal{J}}$, Propositions 1,2,3 and Theorem 1 the real line can be replaced by any uncountable Polish space without isolated points. Proofs are similar.

b) Let Z be a Polish space without isolated points and let f be a c.p. isomorphism from Z onto R. Then $f(\overline{\mathcal{K}^*}(Z)) = \overline{\mathcal{K}^*}(R)$.

c) Let $X \in \overline{\mathcal{K}^*}$ and let Y be a separable metric space without isolated points such that there exists a one-to-one Borel measurable function f from Y into X. Let Z be a Polish space without isolated points such that $Y \subseteq Z$. Then $Y \in \overline{\mathcal{K}^*}(Z)$.

Proof. Part a) is trivial. In order to prove part b) it is enough, by Proposition 3 and Remark 2 a), to prove that $f(\overline{\mathcal{K}(Z)}) = \overline{\mathcal{K}(R)}$, but the last equallity is easy to check. To prove c) let g be a c.p. isomorphism from Z onto R and let $X_1 = g(Y)$. Since $f(g^{-1}\upharpoonright X_1)$ is a one-to-one Borel function from X_1 into X we have $X_1 \in \overline{\mathcal{K}^*}$. Since $Y = g^{-1}(X_1)$, by b) we have $Y \in \overline{\mathcal{K}^*}(Z)$.

The following remark seems to be in [3] but without proof.

Remark 3 (Fremlin[3]). Let Z be a Polish space without isolated points, let $m_1(Z) = \min\{|Y| : Y \subseteq Z$ and $Y \notin \mathcal{K}(Z)\}$ and let $m_2(Z) = \min\{|Y| : Y \subseteq Z$ and $Y \notin \mathcal{K}^*(Z)\}$. Then we have $m_1(Z) = m_2(Z) = m_1(R)$. If Y is a separable metric space without isolated points such that $|Y| < m_1$ then $Y \in \mathcal{K}^*(Y)$.

Proof. It is clear that $m_2(Z) \leqslant m_1(Z)$. Let $S \subseteq Z$ be such that $|S| = m_2(Z)$ and $S \notin \mathcal{K}^*(Z)$. By Remark 2a and Proposition 2 there is $S_1 \subseteq Z$ such that $S_1 \notin \mathcal{K}(Z)$ and $|S_1| = |S| = m_2(Z)$. Hence $m_1(Z) \leqslant m_2(Z)$. The fact that that $m_1(Z) = m_1(R)$ we have immediatelly from the existence of c.p. isomorphism between Z and R. Let Z be a Polish space without isolated points such that $Z \supseteq Y$ (e.g. let Z be the Cantor completion of Y). Since $m_1 = m_2$ we have that $Y \in \mathcal{K}^*(Z)$.

Remark 4. Assume CH. Then $\overline{\mathcal{K}^*} \times \overline{\mathcal{K}^*} \subseteq \overline{\mathcal{K}^*}(R \times R)$.

Indeed. Let $A, B \in \overline{\mathcal{K}^*}$. CH implies that $A \times B$ is a countable union of graphs of partial functions from R into R. Since the projections are one-to-one continuous functions from graphs into A or B respectively we have that the graph of each partial function is in $\mathcal{K}^*(R\times R)$ and so $A\times B\in\mathcal{K}^*(R\times R)$. Similar argument works if instead of CH we assume only MA plus that $2^{\aleph_0}$ is a succesor cardinal.

From now X and Y will denote uncountable Polish spaces. Let $\mathcal{M}_0(X\times Y)$ be the σ-field of subsets of $X\times Y$ generated by $\mathcal{B}(X\times Y)\cup\mathcal{K}^*(X\times Y)$ and let $\mathcal{M}_1(X\times Y)$ be the σ-field on $X\times Y$ generated by $\mathcal{B}(X\times Y)\cup\mathcal{N}(X\times Y)$. The following consequence of Mazurkiewicz-Sierpiński theorem was observed in [9], Remark 4.

(+) If $C\subseteq X$ and $C\times Y\in\mathcal{M}_0(X\times Y)\cup\mathcal{M}_1(X\times Y)$, then $C\in\mathcal{B}(X)$.

In [8] I generalised (+) to the following

(++) If C is an uncountable subset of X and D is an uncountable analytic subset of Y such that $C\times D\in\mathcal{M}_0(X\times Y)\cup\mathcal{M}_1(X\times Y)$, then $C\in\mathcal{B}(X)$.

Now I would like to show how (++) follows immediatelly from (+). Namely we have the following (+++).

(+++) If C is an uncountable subset of X, D is a subset of Y such that D contains a homeomorphic image of the Cantor set and $C\times D\in\mathcal{M}_0(X\times Y)\cup\mathcal{M}_1(X\times Y)$, then $C\in\mathcal{B}(X)$ and $D\in\mathcal{B}(Y)$.

Indeed. Let K be a subset of D such that K is homeomorphic with the Cantor set. We have $C\times K\in\mathcal{M}_0(X\times K)\cup\mathcal{M}_1(X\times K)$. Hence, by (+), $C\in\mathcal{B}(X)$. Now again by (+), $D\in\mathcal{B}(X)$.

<u>Corollary</u>. If A and B are non-Borel universally measurable subsets of X and Y respectively (for the definition see e.g. [2]) such that at least one of them is not a universal null set, then $A\times B$ is a universally measurable subset of $X\times Y$ such that $A\times B\notin\mathcal{M}_0(X\times Y)\cup\mathcal{M}_1(X\times Y)$.

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