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In: Zdeněk Frolík and Vladimír Souček and Jiří Vinárek (eds.): Proceedings of the 13th Winter School on Abstract Analysis, Section of Analysis. Circolo Matematico di Palermo, Palermo, 1985. Rendiconti del Circolo Matematico di Palermo, Serie II, Supplemento No. 10. pp. [111]–112.

Persistent URL: http://dml.cz/dmlcz/701867

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Let X be a set and let $\mathfrak{M}$ be an algebra of subsets of X. We denote by $ba(\mathfrak{M})$ the family of all real-valued quasi-measures, i.e., bounded additive set functions, on $\mathfrak{M}$. Let $\mathfrak{N}$ be another algebra of subsets of X and denote by $\mathfrak{F}$ the algebra generated by $\mathfrak{M} \cup \mathfrak{N}$. We are concerned with the following problem (suggested by [1], [3] and [4]):

Given $\mu \in ba(\mathfrak{M})$ and $\nu \in ba(\mathfrak{N})$ such that $\mu | \mathfrak{M} \cap \mathfrak{N} = \nu | \mathfrak{M} \cap \mathfrak{N}$, when does there exist a common extension $\varphi \in ba(\mathfrak{F})$ of μ and ν?

The answer is positive provided one of the following three

(1) Given two partitions $\{M_1, \ldots, M_m\} \subset \mathfrak{M}$ and $\{N_1, \ldots, N_n\} \subset \mathfrak{N}$ of X into non-empty sets, we have $M_i \cap N_j \neq \emptyset$ both for a fixed i and $j = 1, \ldots, n$ and for a fixed j and $i = 1, \ldots, m$.

(2) $\mathfrak{N}$ is finite.

(3) ν is complete and has finite range.

The answer is negative if $\mathfrak{M} \cap \mathfrak{N} = \{\emptyset, X\}$ and there exist $M_n \in \mathfrak{M}$ and $N_n \in \mathfrak{N}$ such that $\emptyset \neq M_1 \subset N_1 \subset M_2 \subset N_2 \subset \ldots \neq X$.

Condition (1) holds if $\mathfrak{M}$ and $\mathfrak{N}$ are independent in the sense of [3], p. 220, i.e., $M \cap N \neq \emptyset$ whenever $M \in \mathfrak{M}$ and $N \in \mathfrak{N}$ are non-empty. The sufficiency of (3) follows from that of (2) and [2], Proposition 1.

The proofs and other details will appear elswhere.

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