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# STRICT INDUCTIVE AND PROJECTIVE LIMITS, TWISTED SPACES AND QUOJECTIONS (*)

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The point of this survey is to bring together a collection of results showing the strong relationships existing among the various topics mentioned in the title (and also between the latter and the notions of continuous norm and total bounded set). We shall present the results in a chronological order, to better show the evolution of each topic.

In essence, the main question about locally convex spaces that are inductive or projective limits of a family of spaces (called *steps*) reduces to the following two problems, each being the converse of the other.

(P1) *If a certain property is shared by all the steps, is it also shared by the limit space?*

(P2) *Suppose that the limit space E has a certain property. Does this force E to have a particular structure, such as a "nice" decomposition into subspaces, and would the latter ones inherit the property originally assumed on E?*

Here a "nice" decomposition means one as a direct sum in the case of inductive limits or a product in the case of projective limits.

Problem (P1) has been by far the most studied, having been the

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object of an intensive investigation throughout the years, especially on what concerns separation, bounded sets, completeness and the nature of the locally convex topology of the limit space and also, for inductive limits, on the questions of closedness of step-wise closed subspaces, of the topologies that they inherit and of the extension of linear functionals that are step-wise continuous on subspaces. Because of natural reasons, the case to which most of the attention has been devoted is that of a limit of countably many steps and, just to give a few sample references, we mention the pioneering paper [7] of Dieudonné-Schwartz, the land-mark paper [21] of Grothendieck and then the papers [20], [9], [28] (from a bornological point of view), [33], [15] with its extensive bibliography) and [4], as well as the books [23], I and [22] (among others). Here, because one starts with the steps, it is possible to say a great deal about the limit space even in the case of general (countable) inductive limits or of general projective limits and so we shall not concern ourselves with Problem (P1).

Substantially different is, instead, the situation regarding Problem (P2). Because the starting assumption is now considerably weaker, one is forced to work with strict inductive and projective limits of countably many steps. Even so, the results have been very scarce until recent years and precisely until the appearance of the author's paper [29]. It is our purpose here to give a brief history of Problem (P2) up to the present day, but before doing this, we need to recall the definitions of a few notions that will be extensively used in the sequel, these being: continuous norm, total bounded set, strict inductive and projective limit, unconditional and absolute basis.

It is self-evident what the expression "A locally convex space has a continuous norm" means, while a bounded set is *total* if its linear span is dense in the space. Also, we abbreviate "Fréchet space" to $F$-space and refer to [22] for the definitions of $LB$-, $LF$- and $DF$-spaces. Next, an inductive (resp. projective) limit is *strict* if each step is a topological subspace (resp. a quotient) of the next. Strict inductive limits are classical, while strict projective limits have only acquired importance recently, essentially because of the results in [29]. Finally, a basis $(e_n)$ in a locally convex space $E$ is called *unconditional* if, for every $x = \sum_n \langle \phi_n, x\rangle e_n \in E$, we have

$$\sum_n \langle \phi_{\pi(n)}, x\rangle \, e_{\pi(n)} = x$$

for all permutations $\pi$ of $\mathbb{N}$ (= the set of positive integers), while $(e_n)$ is an *absolute basis* if, for every continuous seminorm $p$ on $E$ there is another one, $q$, such that

$$\sum_n |\langle \phi_n, x\rangle p(e_n)| \leq q(x) \qquad \text{for all } x \in E.$$

Clearly every absolute basis is unconditional. Also, in nuclear barrelled spaces (such as $F$- and $LF$-spaces) every basis is absolute, and hence unconditional, by the theorems of Banach-Steinhaus and Dynin-Mitiagin (see [31],10.2.1 or [22], 21,10,1). Furthermore, in nuclear $F$- or complete $DF$-spaces the sequence $(\phi_n)$ of coefficient functionals is an absolute basis for the strong dual. Finally, we refer to [20], [23], I, [22] and [4], §5 for the properties of strict (and general) countable inductive or projective limits and to [22] for what concerns absolute and unconditional bases and nuclear spaces, while we denote, as usual, by $\omega$ the $F$-space which is the product of countably many copies of the scalar field and by $s$ the $F$-space of rapidly decreasing sequences:

$$s = \{(\xi_n) \in \omega : p_k(\xi_n) = \sum_n n^k |\xi_n| < \infty \text{ for all } k \in \mathbb{N}\}.$$

We shall begin with Bessaga and Pełczyński's classical result [2] dating back to 1957. It deals with $F$-spaces without continuous norms and concludes with the following:

(1) *An $F$-space has no continuous norm if and only if it contains $\omega$ as a (necessarily complemented) subspace.*

Now, there are plenty of $F$-spaces without continuous norms such as, e.g., countable products of Banach or $F$-spaces and the classical spaces $C(\Omega)$ or $\mathcal{E}(\Omega)$ of continuous or smooth functions on an open set $\Omega \subset R^m$, so that the following question arises quite naturally:

(P3) *Must every $F$-space without continuous norm be (isomorphic to) the product of a sequence of $F$-spaces with continuous norms? Is this true at least in the case of nuclear $F$-spaces?*

As we shall see, the above question will be of great importance in the sequel. For the moment, we note that while it is fairly easy to show that $C(R) \cong C([-1,1])^{\mathbb{N}}$ ($\cong$ denoting topological isomorphism), the following nuclear case, due to Mitiagin [26] in 1961, presents considerable difficulties.

(2) $\mathscr{E}(R) = \mathscr{E}([-1,1])^{\mathbb{N}}$ *(and hence* $= s^{\mathbb{N}}$; cf. [31], 10.3.9).

The result is achieved through the use of bases and this is by far no accident, as will be seen later. Also note that both $C(R)$ and $\mathscr{E}(R)$ are strict projective limits.

Another attempt at solving Problem (P3) was made by Dubinsky [10] in 1967, who showed

(3) *For an F-space which is also a perfect sequence space the answer to (P3) is positive.*

The above is obtained via a technical classification lemma in the dual space which makes up a sort of table of possible cases. Here matters come to a standstill until 1980.

Meanwhile, further isomorphism theorems were proved (cf., e.g., [31], 10.3) until 1978 when Valdivia [34], taking up Mitiagin's circle of ideas, was able to construct bases in the classical nuclear spaces $\mathscr{E}(\Omega)$ and $\mathscr{D}(\Omega)$ (= the test functions in $\Omega$) to obtain representations of these spaces as products or direct sums. Moreover, he proved that all the spaces $\mathscr{E}(\Omega)$ are isomorphic, and the same for the spaces $\mathscr{D}(\Omega)$, thus obtaining

(4) $\mathscr{E}(\Omega) = s^{\mathbb{N}}$ *and* $\mathscr{D}(\Omega) = s^{(\mathbb{N})}$.

These results were subsequently generalized by Vogt [35] to many spaces of functions and distributions.

We now come to the complete (negative) solution of Problem (P3) obtained by the author in 1980 (cf. [29]). Precisely, we have

(5) (a) *There is a strict projective limit of a sequence of reflexive Banach spaces which is not a product of a sequence of Banach spaces (and even of a sequence of F-spaces with continuous norms).*

(b) *There is a strict projective limit of a sequence of nuclear F-spaces which is not a product of a sequence of F-spaces with continuous norms.*

The following remarks are in order.

(i) The spaces in (5) were called *twisted* by the author and this is nowadays the accepted terminology. More in general, a locally convex space is called *twisted* if it is not isomorphic to a product of locally convex spaces with continuous norms.

(ii) Strict projective limits of sequences of Banach spaces were later called *quojections* by Bellenot and Dubinsky (cf. [1]) and *strictly regular* by Zarnadze (cf. [38] and also [4] and [6], were some properties of such spaces are analysed). We prefere the term quojection, since it describes better the structure of the space.

(iii) While (5)(a) is surprising, (5)(b) is even more so, since every nuclear F - space is a subspace of a product of a sequence of Hilbert spaces.

(iv) The construction in 5(a) is extremely general and, indeed, any sequence of reflexive, non-Hilbert, Banach spaces can be used, thanks to Lindenstrauss and Tzafriri's solution to the complemented subspace problem [24]. Moreover, quojections are reflexive if and only if they are strict projective limits of sequences of reflexive Banach spaces, in which case they are also totally reflexive in the sense of Grothendieck (see [21], Proposition 10 and also [4], (5.6)).

Recalling what was said at the beginning about bases we can then bring out the most important feature of twisted spaces, this deriving from (5) via the use of Dubinsky's lemma:

(6) (a) *No twisted quojection can have an unconditional basis.*
(b) *No twisted, nuclear F-space can have a basis.*

Further remarks (contained explicitly or implicitly in [29]; but see also [18]):

(v) The steps in both (5)(a) and (5)(b) can be chosen to have unconditional bases (= bases for 5(b)) thanks to results in [25],p.91 and in [37] respectively.

(vi) (5) and (6) exhibit for the first time non-trivial (i.e., non-products), non-normable, (reflexive) F-spaces without unconditional bases,as well as a completely new class of nuclear F-spaces without bases which are entirely different from all those previously constructed (cf.,e.g., the classical counter-example in [27]).

(vii) The constructions leading to (5) are first made in the dual space. To be precise, we obtain the following results which are also of independent interest (and again based on [24],[25], p. 91 and [37]

(7) (a) *There is a strict inductive limit of a sequence of (reflexive) Banach spaces (with unconditional bases) which is not (isomorphic to) the direct sum of a sequence of subspaces each having a total bounded set; hence such a limit space has no unconditional basis.*

(b) *There is a strict inductive limit of a sequence of nuclear LB-spaces (with bases) which is not (isomorphic to) the direct sum of a sequence of subspaces each having a total bounded set; hence such a limit space has no basis.*

(viii) (7) is the (negative) answer to the following problem which, in a sense, is dual to (P3).

(P4) *Must every LB- or DF-space without a total bounded set be (isomorphic to) a direct sum of a sequence of subspaces each having a total bounded set?*

(ix) It is clear that, once (7) is proved in the reflexive or nuclear case, (5) follows by reflexivity passing to the duals and hence (6).

(x) We conclude this series of remarks with the following observations, which are made here for the first time. In [27] (cf. also [32]) Mitiagin and Zobin even showed that there are nuclear $F$-spaces without bases of arbitrarily large diametral dimension different from the maximal one (since, as is easily seen, the only $F$-space of maximal diametral dimension is $\omega$) (see [22] for the definition of diametral dimension and its properties). Here we observe that the same is true of the spaces in (5)(b) as shown by the following. Clearly there are nuclear Köthe sequence spaces with continuous norms and arbitrarily large (but not maximal) diametral dimension. We choose one such space and denote it by $G$ (necessarily $\neq \omega$). Then we observe that Theorem VI (2.1.6) of [11] holds: indeed, although it rests on Proposition II (3.1.3) of [11] and the proof of such proposition, as given there, is incorrect, a correct proof was subsequently supplied in [13], Theorem 1, thus ensuring the validity of Theorem VI (2.1.6). Hence $G$ has a quotient space $H$ which has no SUPI (definition in [11], VI (1.1.8)) and, of course, the diametral dimension of $H$ is no smaller than that of G. Now the strong dual $G'$ has a total bounded set containing the natural basis of $G'$ (i.e., the coordinate vectors) and a look at the proof of the above theorem shows that also the dual $H' \subset G'$ has a total bounded set which, in turn, implies that $H = H''$ has a continuous norm. However, $H$ cannot be complemented in $G$; indeed, if it were. Since $G$ has an absolute basis and hence a 1 - UPI, also $H$ would have a 1-UPI by [11], (1.2.3) and, consequently, a SUPI, leading to a contradiction. Taking now $X_n = G'$ and $Y_n = H'$ for all $n$ in [29], 2, we obtain (with (8)(b) again following from (8)(a) by duality)

(8) (a) *There are strict inductive limits of sequences of nuclear LB-spaces which have arbitrarily large (but not maximal) diametral dimensions and which are not direct sums of sequences of subspaces with total bounded sets, and hence have no bases.*

(b) *There are twisted, nuclear F-spaces of arbitrarily large diametral dimension.*

We must point out, now, that the kind of considerations associated to results of the type of Proposition II (3.1.3) and Theorem VI (2.1.6) of [11] naturally lead to the following problem (cf. [12]):

(P5) *Characterize all those F-spaces that admit a nuclear Köthe quotient,*

i.e., a quotient which *is nuclear and has a basis and a continuous norm.*

Remarks:

(xi) The continuous norm is crucial since, already in 1936, Eidelheit [14] showed that any non-normable F-space has $\omega$ as a quotient.

(xii) Nuclearity is also crucial in the sense that the problem is likely to be much more difficult without it. Indeed, in the latter case the answer is unknown even in the Banach space case (and is not even known if every Banach space has a separable quotient [25]). Thus nuclearity rules out Banach spaces but the above points at the difficulty of the problem.

(xiii) The analogue of (P5) for subspaces was solved in 1961 (cf. [3]). For comments on problem (P5) we refer to [12], [13] and the author's brief survey [30]. Here we shall confine ourselves to discussing the most recent result, due to Bellenot and Dubinsky [1], which solves (P5) in the separable case.

(9) *A separable F-space $E$ has a nuclear Köthe quotient if and only if $E'$ is not the union of an increasing sequence of Banach spaces $F_n$ with each $F_n$ being a closed subspace of $F_{n+1}$.*

We note at this point that *the condition on $E'$ almost forces $E$ to be a quojection*, while it is immediate to see that quojections fail to have nuclear Köthe quotients even in the non-separable case. This raises the problem

(P6) *Are quojections the only F-spaces without nuclear Köthe quotients?*

It would be nice if this question could be settled. All we have up to now is the following partial (positive) answer, also due to Bellenot and Dubinsky.

(10) *Within the class of separable, reflexive F-spaces, quojections are exactly those spaces without nuclear Köthe quotients.*

Remarks:
(xiv) (10) follows from the fact that (9) forces $E'^{b}$ (= the space of bounded linear functionals on $E'$) to be a quojection. Unfortunately, this is not enough to conclude that $E$ itself must be a quojection. Indeed, in [1] an example is given of an F-space $E$ with continuous norm such that $E'^{b}$ has no continuous norm. This example was subsequently improved by S.Dierolf and the author in [5], where an F-space $E$ with continuous norm is contructed such that the strong bidual $E''$ has no continuous norm, thus answering a question of Vogt.
(xv) We conclude our comments on problems (P5) and (P6) by recalling that quojections fail to have nuclear Köthe quotients in a very strong way. In fact, we have

(11)(a) *A quotient of a quojection has a continuous norm (if and) only if it is a Banach space.*

(b) *Every quotient of a quojection is either a Banach space or again a quojection.*

(c) *A quotient of a quojection is a Montel space if and only if it is either finite-dimensional or isomorphic to $\omega$.*

(a) and (b) follow from [1], Proposition 3, while (c) is a consequence of [18], Corollary 5.5 (1).

We now go back to (7) or, rather, to the kind of results exemplified by (7). In its spirit, we mention the following lemma due to

Dineen (cf. [8], 5.4.3), which sets out a new method for deriving, via (5), statements of type (6) from statements of type (7).

(12) *Let E be the strict inductive limit of a sequence of nuclear F-spaces (i.e., a nuclear, strict LF-space). Then E has a basis (if and) only if it is the direct sum of a sequence of nuclear F-spaces with bases.*

This paved the way for the study of strict inductive and projective limits from this viewpoint. Indeed, Dineen's method was taken up by Floret and the author, who first in [17] extended (12) to a strict inductive limit $E$ of a sequence of $LF$-spaces such that $E$ has an unconditional basis and then in [18] pushed Dineen's lemma to its natural limits of validity obtaining (see [18] for the definition of a closed-graph pair)

(13) *Let $(\mathcal{D},\mathcal{R})$ be a closed-graph pair and let $(E_k)$ be a strict inductive sequence of complete spaces $E_k \in \mathcal{R}$ such that $ind_k E_k \equiv$ $\equiv E \in \mathcal{D}$. If E has an unconditional basis, then there are complemented subspaces $G_k \subset E_k$ with unconditional bases such that $E = \oplus_k G_k$ topologically.*

Remarks:

(xvi) The paper [18] shows how, for extremely large classes of strict inductive and projective limits, the property of having a basis implies the structural property of being, respectively, a direct sum or a product, thereby showing that the methods used in [26],[34] and [35] are, indeed, quite natural.

(xvii) More in general, situations outside the nuclear case are investigated in [18], the results being obtained under the assumption of the existence of an unconditional basis.

(xviii) As in [29] and [17], the results are always obtained in the setting of strict inductive limits. For strict projective limits the results are then obtained by duality from the corresponding results in the dual spaces. For this one needs a perfect duality between strict inductive and strict projective limits, which in this case is achieved via suitable, though simple, extensions (also proved in [18]) of some classical theorems on strong duals of homomorphisms (see [23],II). But all this still requires the assumption of reflexivity for the strict projective limits concerned and, in particular, for the case of F-spaces.

(xix) However, (13) still includes, directly or indirectly, all the results (5) - (7) and (12), as well as (16) below.
(xx) We point out that what was said in Remark (xviii) yields a proof of (6) which avoids Dubinsky's technical lemma, whose use always seemed rather unnatural in this context.

Finally, in [19] Floret and the author were able to prove the ultimate generalization of (6), obtaining

(14) *An F-space with an unconditional basis either has a continuous norm or is isomorphic to the product of a sequence of F-spaces with continuous norm and unconditional basis.*

Remarks:
(xxi) Hidden in (14) is the fact that *every F-space without a continuous norm is the strict projective limit of a sequence of F-spaces with continuous norms.*
(xxii) (14) removes the assumption of reflexivity from the results in [18] concerning F-spaces (cf. Remark (xviii)). This is achieved because no results are first obtained in the dual space and then transferred back to the original space by duality (which requires reflexivity); instead, we only compute in the dual space to get results directly in the original F-space.
(xxiii) (14) gives a classification of F-spaces with respect to the property of having an unconditional basis yielding, in particular, that

(15) *No twisted F-space can have an unconditional basis.*

(xxiv) A result analogous to (14) also holds for *the strict projective limit of a sequence of reflexive DF-spaces*, i.e. for the strong dual of a strict inductive limit of reflexive F-spaces (but this had already been proved in [18]).

We conclude this brief survey by mentioning the following parallel, but related, result of Floret [16] answering (in the negative) a question asked by L.A. de Moraes:

(16) *There is a strict inductive limit E of a sequence of nuclear F-spaces $E_n$ such that each $E_n$ has a continuous norm but E admits no continuous norm.*

Remarks:

(xxv) By (12) such a space $E$ cannot have a basis.

(xxvi) None of the spaces $E_n$ in (16) has the Bounded Approximation Property (see [16] or [11], VI,1 for the definition), since no $E_n$ is countably normed. Thus, such spaces $E_n$ provide additional counter-examples to those already constructed by Dubinsky [11],VI,3 and Vogt [36]. In this context we note that all the examples given of nuclear $F$-spaces without bases but with continuous norm (such as Mitiagin and Zobin's and those constructed in their wake) have the Bounded Approximation Property [12]. However, nothing is known about nuclear $F$-spaces without bases and without continuous norms, so that we terminate by asking the following question:

(P7) *Do twisted, nuclear F-spaces have the Bounded Approximation Property?*

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