Charles Stegall
More facts about conjugate Banach spaces with the Radon-Nikodym property


Persistent URL: [http://dml.cz/dmlcz/702016](http://dml.cz/dmlcz/702016)

**Terms of use:**

© Univerzita Karlova v Praze, 1994

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use.*

This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library*[http://project.dml.cz](http://project.dml.cz)*
More Facts about Conjugate Banach Spaces with the Radon-Nikodym Property

C. STEGALL

Linz*)

Received 15. March 1994

Corrected proofs of the following results of [S0] are given: if X is an Asplund space (respectively, X is a subspace of a gsg space) and K is a Corson compact then any operator from X to C(K) interpolates through a Banach space Y such that Y is both Asplund and hereditarily weakly compactly generated (respectively, Y is wcg). If K is a Corson compact that is the continuous image of a so called Radon-Nikodym compact then K is an Eberlein compact.

We use the same terminology and notation as in [S0]. We slightly rephrase the Lemma on page 48. The proof should be clear from the inequalities

\[
\|((f_1 \vee g_1) - h_1) - ((f_2 \vee g_2) - h_2)\|
\leq \|f_1 - f_2\| + \|g_1 - g_2\| + \|h_1 - h_2\|
\]

Lemma. Suppose that C_1, C_2 and C_3 are bounded and equimeasurable subsets of C(K). Then

\[
\{f \vee g) - h : f \in C_1, g \in C_2, h \in C_3\}
\]

is equimeasurable.

We give a correct proof of the Lemma on pages 49—50 of [S0]

Lemma. Let K be a compact Hausdorff space and C a subset of C(K) that is equimeasurable and separates the points of K (in some circles K is called a Radon-Nikodym compact). Let F be any subset of C(K) that is point countable. There exists a subset G of C(K) that is both equimeasurable and point countable and the algebra A generated by G contains F.

*) Institut für Mathematik, Johannes Kepler Universität, A-4040 Linz, Austria

We are grateful to the participants of the kk Analysis Seminar of October 1993 in Salzburg. Particularly, we thank M. Fabian for pointing out to us the errors in [S0] and P. Holicky for a number of comments.
Proof. We may assume that \( C \) is a convex and symmetrical subset of the unit ball with \( 1_K \in C \). It is easy to check that \( C \cdot C \) is also equimeasurable and, by induction, \( C^n \) is equimeasurable. It follows that

\[
E = \sum_{n} 2^{-n}C^n
\]

is equimeasurable. From the Stone-Weierstraß theorem we know that \( \cup_n n \cdot E \) is uniformly dense in \( C(K) \). For fixed positive integers \( m \) and \( n \) define

\[
F_{m,n} = F \cap \left( (n \cdot E) + B\left(0, \frac{1}{m}\right) \right).
\]

Observe, that for a fixed \( m \), \( \cup_n F_{m,n} = F \). For each \( f_{m,n,\alpha} \in F_{m,n} \) choose \( h_{m,n,\alpha} \in E \) so that \( \|nh_{m,n,\alpha} - f_{m,n,\alpha}\| < \frac{1}{m} \). Define

\[
u_{m,n,\alpha} = \left( nh_{m,n,\alpha} \vee \frac{1}{m} \right) - \frac{1}{m},
\]

which is non negative and observe that

\[
n^{-1}u_{m,n,\alpha} = \left( h_{m,n,\alpha} \vee \frac{1}{mn} \right) - \frac{1}{mn} \in \left( E \vee \frac{1}{mn} \right) - \frac{1}{mn}.
\]

Thus, \( G = \{n^{-1}u_{m,n,\alpha} : m, \ n, \ \alpha\} \) is equimeasurable. Fix \( k \in K \), \( m \) and \( n \); if \( n^{-1}u_{m,n,\alpha}(k) > 0 \) then

\[
nh_{m,n,\alpha}(k) > \frac{1}{m}
\]

which implies that \( |f_{m,n,\alpha}(k)| > 0 \). Thus, \( \{n^{-1}u_{m,n,\alpha} : \alpha\} \) is also point countable. Thus, we have that \( G \) is point countable because it is the countable union of point countable sets. Also, \( F \) is a subset of the closed algebra \( A \) generated by \( G \) and

\[
G \subseteq \cup_{m,n} \left( E \vee \frac{1}{mn} 1_K \right) - \frac{1}{mn} 1_K.
\]

Thus, the state space of \( A \) is both a Corson compact and a Radon-Nikodym compact.

The remarks beginning in the last paragraph of page 52 and continuing on page 53 are, at best, incomplete, and should be ignored.

Bibliography