Questions

ALEXEY OSTROVSKY

Neubiberg

Received 11. March 2005

1. Questions

In this section all the sets are supposed to be lying in the Cantor set **C**. We denote by **P** and **Q** the spaces of irrational and rational numbers.

(1) The author proved [2] that there is a compact quotient map $f : \mathbf{P} \to \mathbf{Q}^{\omega}$, but at the same time the image of **P** under every n-to-one quotient map is always G_δ.

Question 1. Can the image of a Borel set X under n-to-one quotient map have an arbitrarily high Borel class?

I do not know the answer even for compact quotient maps.

Question 1 would have to be solved in the first place for a simple cases of $X = \mathbf{P} \times \mathbf{Q}$.

Question 2. Is every Borel set X a countable union of pairwise disjoint closed in X sets X_i obtained (starting with a point) by multiplying and complementing[1]?

According to [3, Theorems 7, 12] it is sufficient to consider Δ^0_α-set X.

F. van Engelen in his dissertation gave a characteristic of homogeneous Borel sets of ambiguous class 3 with the same operations.

Bundeswehr University Munich, D-8557 Neubiberg, Germany

2000 *Mathematics Subject Classification.* Primary 54C10, 54H05, 54E40.

Key words and phrases. Quotient map, Borel sets.

[1] complementing by dense closed embedding. X_i are the sets M^I_α, A^I, M^{II}_α, A^{II}_α (and their products) defined in [2], p. 334.

References

[1] Engelen Van F., Homogeneous zero-dimensional absolute Borel sets *Ph.D. thesis* **CWI Tract 27** (1986)

[2] Ostrovsky, A., About quotient finite-to-one maps, *in: Continuous functions on topological spaces, Latv. Gos. Univ., Riga* (1986) 126 – 133 (in Russian)

[3] Ostrovsky, A., On a question of L. V. Keldysh concerning the structure of Borel sets, *Math. USSR Sbornik* **39, No 2** (1986) 317 – 337