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A GAUGE THEORY FOR THE KADOMTSEV-PETVIASHVILI SYSTEM *

S. Kanemaki, W. Królikowski, and O. Suzuki

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1. A Lagrangian formalism of scalar fields
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Abstract

A Lagrangian formalism of scalar fields is considered and a new concept of "connection" is introduced. By this a gauge-theoretic understanding of the Sato theory on the K.-P. system is obtained. Our gauge group $\tilde{G}_-$ is the group consisting of pseudo-differential operators of non-positive orders with certain growth conditions. Then it can be concluded that the space R^* of elements of $\tilde{G}_-$ giving solutions of the K.-P. system defines the flat R^* -connection which we call the K.-P. connection. This connection can be regarded as a special gauge field.

Introduction

It is well known that various soliton equations can be obtained by using the theory of isospectral deformations of linear differential operators. A remarkable unification of soliton equations has been established by M. and Y. Sato [5] in terms of isospectral deformations of $D = d/dx$ in the category of pseudo-differential operators. This unified system of equations is called the Kadomtsev-Petviashvili system (= K.-P. system). They discovered the surprising fact: The space of solutions of the K.-P. system makes the Grassmann manifold of infinite dimension and moreover, any solution of

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the K.-P. system can be reduced to that of a system of certain linear equations. Several attempts of undetstandings on the Sato theory and its generalizations have been presented. Some of them are the method of Riemann-Hilbert transforms [10], the method of group-decompositions [4], [7] and the field-theoretic method [1]. Our attempt which we present here is a new one, which we call a gauge-theoretic understanding.

In this paper, we see that the K.-P. system can be understood in the view point of Uchiyama's gauge theory [9]. We note that our gauge group is an infinite dimensional Lie group. Hence our gauge theory for soliton equations is contrasted with that of Yang-Mills equations and nonlinear Heisenberg equation in dimensions of their gauge groups [3]. First, we consider the Lagrangian action:

$$\mathcal{L} = \int_{\mathbb{R}} \bar{\psi} D\psi \, dx \quad (D = d/dx)$$

for scalar fields ψ , $\bar{\psi}$, i.e., wave functions on the real line $\mathbb{R}$. We analyse the symmetry of $\mathcal{L}$ and obtain as the gauge group of the first kind a group consisting of invertible pseudo-differential operators with constant coefficients of the form:

$$\dots + c_n D^n + \dots + c_1 D + c_0 + c_{-1} D^{-1} + \dots + c_{-n} D^{-n} + \dots$$

Secondly, we apply the Uchiyama's gauge theory to our Lagrangian formalism. In this case, the gauge group of the second kind becomes a group consisting of invertible pseudo-differential operators with function coefficients of the form:

$$\dots + u_n(x) D^n + \dots + u_1(x) D + u_0(x) + u_{-1}(x) D^{-1} + \dots + u_{-n}(x) D^{-n} \\ + \dots$$

Then in order to obtain a new Lagrangian action which is invariant under this group, a connection, i.e., gauge field, necessarily arises in our consideration. It has a worth mentioning that pseudo-differential operators with negative orders, extended from usual differential operators, may be introduced as elements of the gauge group of the first or the second kind.

In Section 1, from a gauge group of pseudo-differential operators we introduce a new concept of "connection" on a fibre space over $\mathbb{R}$. Here we have to pay attention to the fact that our connection has been defined not only for a subgroup but also for a

special subset R of the gauge group, although R does not admit a structure of subgroup. We prove that the decomposition law of pseudo-differential operators into the parts of non-positive and negative orders gives rise to the flat connection (Theorem 1). This is our first step to a gauge-theoretic understanding on the K.-P. system. In Section 2, we shall treat the Lagrangian action of scalar fields $\psi, \bar{\psi}$ with infinitely many parameters $t = (t_1, t_2, \dots)$:

$$\mathcal{L}_t = \int_{\mathbb{R}} \bar{\psi} d\psi dx, \quad d = \sum_{n=1}^{\infty} (\partial/\partial t_n) dt_n.$$

For this Lagrangian action we consider the gauge groups $\mathcal{G}_0, \mathcal{G}$ of the first and the second kind, and then $\mathcal{G}$ -connections. Then we can conclude that the space R^* of elements of $\mathcal{G}$ giving solutions of the K.-P. system defines the flat R^* -connection which we call the K.-P. connection (Theorem 2).

Our discussions show that the space of solutions of soliton equations determines a special gauge field. Hence; we may expect to extend our discussions to the Yang-Mills equation and nonlinear Heisenberg equation by a gauge-theoretic version of the Sato theory on the Minkowski space-time [3].

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1. A Lagrangian formalism of scalar fields

We consider complex-valued functions defined on the real line $\mathbb{R}$ and a collection of operators including the differential operator $D = d/dx$. Let ψ and $\bar{\psi}$ denote two functions. Here $\bar{\psi}$ may not be the complex conjugate of ψ . Let S be an operator which maps a function ψ to the function $S\psi$. An operator $\bar{\psi}S$ formed with $\bar{\psi}$ and S is defined by $(\bar{\psi}S)\psi = \bar{\psi}(S\psi)$ for any ψ .

First, we deal with a Lagrangian action for ψ and $\bar{\psi}$ given by

$$(1.1) \quad \int_{\mathbb{R}} \bar{\psi} D\psi dx.$$

We restrict ourselves to the case where there exist invertible operators W satisfying the following action law: For a function ψ and an operator $\bar{\psi} (= \bar{\psi} \cdot 1)$, identified with the function $\bar{\psi}$, an operator W acts on the pair as

$$(1.2) \quad \psi \rightarrow \psi' = W\psi, \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} W^{-1}.$$

Under this action the function $\bar{\psi}\psi$ is invariant. We are interested in a set of operators W which makes a group G_0 and preserves $\bar{\psi}D\psi$ invariant, equivalently satisfies $WD = DW$. Choices of such groups are not unique. One of possible groups can be obtained by

$$(1.3) \quad G_0 = \{W | W = \sum_{n=-\infty}^{n=+\infty} c_n D^n \text{ with constant coefficients}\}.$$

Then the group G_0 is a subgroup of G_0 . For an invertible operator W we put

$$(1.4) \quad \psi_W = W\psi, \quad \bar{\psi}^W = \bar{\psi} W^{-1}.$$

PROPOSITION (1.5). *The Lagrangian action*

$$(1.6) \quad \mathcal{L}_0 = \int_{\mathbb{R}} \bar{\psi}^W D \psi_W dx, \quad W \in G_0$$

is invariant under the action of the group G_0 .

Proof. We choose arbitrary elements W and W' of G_0 and set ϕ by $W = \phi W'$, namely, $\phi = WW'^{-1}$. Since

$$(1.7) \quad \psi_W = \phi \psi_{W'}, \quad \bar{\psi}^W = \bar{\psi}^{W'} \phi^{-1},$$

we obtain $\bar{\psi}^W D \psi_W = \bar{\psi}^{W'} \phi^{-1} D \phi \psi_{W'} = \bar{\psi}^{W'} D \psi_{W'}$.

The group G_0 is called the gauge group of the first kind. Next we proceed to a group

$$(1.8) \quad G = \{W | W = \sum_{n=-\infty}^{n=+\infty} u_n(x) D^n \text{ with function coefficients}\}.$$

We call an element of G a formal pseudo-differential operator [5]. G is called the gauge group of the second kind. In order to obtain exact mathematical meanings, we have to restrict our considerations to special groups. For example, we may choose a group G consisting of elements W with the following condition: Every $u_n(x)$ is analytic function and there exists an integer n_0 such that $\text{ord } u_n(x) \geq n - n_0$ for any sufficiently large n ([4], [7], [8]). We have to pay attention to the fact that the Lagrangian action $\mathcal{L}_0$ is not invariant under G , because the commutator $[D, W] = DW - WD$ does not vanish identically. Here we note that the following equalities hold:

$$(1.9) \quad [D, W] = \sum (D u_n(x)) D^n \quad \text{for } W = \sum u_n(x) D^n$$

and

$$(1.10) \quad W D W^{-1} = -[D, W] W^{-1} + D \quad \text{for } W \in G.$$

The Uchiyama gauge theory [9] says that in order to get a new Lagrangian action which is invariant under the group of the second kind, a connection, i.e., a gauge field, has to be introduced. We call the disjoint union $\bigcup_{x \in \mathbb{R}} \{(\cdot)^W(x) \mid W \in G\}$ the *fibre space generated by G over $\mathbb{R}$* , or the *fibre G-space* simply. Then we can make the following definition:

Definition (1.11). Let G be a group of the operators described in (1.8) and let R be a subset of G . A collection $\{\varphi(W) \mid W \in R\}$ of operators is called an *R-connection* (with a range R on the fibre G -space), if (1) there exists a pair (G_1, ρ) constituted with an injective set-map $\rho: G_1 \rightarrow G$ of a group G_1 to G such that $R = \rho(G_1)$ and (2) $L_\Omega(W) \equiv D - \Omega(W)$ satisfies

$$(1.12) \quad L_\Omega(W) = \phi L_\Omega(W') \phi^{-1} \quad \text{for } W, W' \in R, \text{ where } W = \phi W'.$$

In particular, we call it a *G-connection* if in addition ρ is a group-isomorphism.

The following are examples of G -connections:

EXAMPLES (1) $\Omega(W) = D$. (2) $\Omega(W) = [D, W] W^{-1}$, in this case

$$(1.13) \quad L(W) \equiv L_\Omega(W) = W D W^{-1}.$$

(3) Let G' be a subgroup of G and $\iota: G' \rightarrow G$ be the natural inclusion mapping. If $\Omega(W)$ ($W \in G$) is a G -connection, then $\Omega(W)$ ($W \in G'$) becomes a G' -connection.

Immediately from (1.12) we see that if $\Omega_1(W)$ and $\Omega_2(W)$ are R -connections, then the relation

$$(1.14) \quad \Omega_1(W) - \Omega_2(W) = \phi(\Omega_1(W') - \Omega_2(W')) \phi^{-1}$$

holds for $W, W' \in R$, where $W = \phi W'$. This fact and Example (2) show that the following $\hat{\Omega}(W)$ given by

$$(1.15) \quad \hat{\Omega}(W) = W^{-1}([D, W] W^{-1} - \Omega(W)) W \quad \text{for } W \in R$$

satisfy the condition $\hat{\Omega}(W) = \hat{\Omega}(W')$ for any pair of W and W' of R , namely $\hat{\Omega}(W)$ does not depend on a choice of $W \in R$. Therefore,

we may write as $\hat{\Omega} = \hat{\Omega}(W)$. We call $\hat{\Omega}$ the *connection form* determined by $\Omega(W)$. An R -connection is called to be *flat* if its connection form vanishes identically, namely $\Omega(W) = [D, W] W^{-1}$.

By an application of Uchiyama theory to the Lagrangian action (1.6), we obtain

PROPOSITION (1.16). *Let $\Omega(W)$ be a G -connection. The Lagrangian action*

$$(1.17) \quad \mathcal{L} = \int_{\mathbb{R}} \bar{\psi}^W (D - \Omega(W)) \psi_W dx, \quad W \in G$$

is invariant under the group G .

Proof. For arbitrary elements W and W' , where $W = \phi W'$ in G we have

$$\bar{\psi}^W L(W) \psi_W = \bar{\psi}^{W'} \phi^{-1} (\phi L(W') \phi^{-1}) \phi \psi_{W'} = \bar{\psi}^{W'} L(W') \psi_{W'},$$

which implies the invariance of $\mathcal{L}$ under G .

The following group is important for a study on the K.-P. system. We put

$$(1.18) \quad G_- = \{ \sum_{n=0}^{\infty} v_n(x) D^{-n} \in G \mid v_0(x) = 1 \},$$

further, the space of operators $\mathfrak{G} = \{ \sum_{n=-\infty}^{n=+\infty} u_n(x) D^n \}$ and its complementary subspaces $\mathfrak{G}_+ = \{ \sum_{n=0}^{n=+\infty} u_n(x) D^n \}$ and $\mathfrak{G}_- = \{ \sum_{n=1}^{n=+\infty} u_{-n}(x) D^{-n} \}$. Then any element S of $\mathfrak{G}$ has the decomposition: $S = (S)_+ + (S)_-$ for $(S)_+ \in \mathfrak{G}_+$ and $(S)_- \in \mathfrak{G}_-$. Then we can prove

THEOREM 1. $\omega(W)$ ($W \in G_-$) is the flat G_- -connection if and only if

$$(1.19) \quad \omega(W) = -(L(W))_- \quad \text{for } W \in G_-.$$

Proof. For $W, W' \in G_-$, where $W = \phi W'$, it holds that

$$\begin{aligned} (L(W))_- &= (\phi(L(W')) \phi^{-1})_- = (\phi(L(W'))_+ \phi^{-1})_- + (\phi(L(W'))_- \phi^{-1})_- \\ &= (\phi D \phi^{-1})_- + \phi(L(W'))_- \phi^{-1} \\ &= (-[D, \phi] \phi^{-1} + D)_- + \phi(L(W'))_- \phi^{-1} \quad (\text{by (1.10)}) \\ &= -D + \phi D \phi^{-1} + \phi(L(W'))_- \phi^{-1}, \end{aligned}$$

which implies $D - \omega(W) = \phi(D - \omega(W')) \phi^{-1}$. Hence $\omega(W)$ is a G_- -connection. Comparing the non-positive orders of the both sides of (1.10), we obtain $\omega(W) = -(L(W))_- = [D, W] W^{-1}$, i.e., $\omega(W)$ is flat. Conversely, if $\omega(W)$ ($W \in G_-$) is the flat G_- -connection, then $\omega(W)$

reduces to $\omega(W) = [D, W] W^{-1} = -(L(W))_-$ by (1.10).

2. A gauge theory for the K.-P. system

We consider a Lagrangian formalism for scalar fields, $\psi = \psi(x, t)$ and $\bar{\psi} = \bar{\psi}(x, t)$ defined on the real line $(x \in) \mathbb{R}$ with infinitely many parameters

$$t = (t_1, t_2, \dots),$$

and for some collections of operators including $D = d/dx$ and $D_n = \partial/\partial t_n$. The total differential operator with respect to the parameters is denoted by

$$(2.1) \quad d = \sum_{n=1}^{\infty} D_n dt_n.$$

The Lagrangian action which we treat here is given by

$$(2.2) \quad \mathcal{L}(t) = \int_{\mathbb{R}} \bar{\psi}(x, t) d\psi(x, t) dx$$

for functions ψ and $\bar{\psi}$. We proceed to our discussions analogous to the one done in the previous section. We are interested in invertible operators $W = W(x, t)$, consisting together with the action law for ψ and $\bar{\psi}$:

$$(2.3) \quad \psi \rightarrow \psi' = W \psi (= \psi_W), \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} W^{-1} (= \bar{\psi}^W).$$

Hence, the function $\bar{\psi} \psi$ is invariant under this action.

First, we consider a group

$$(2.4) \quad \hat{G}_0 = \{W | W = \sum_{n=-\infty}^{n=+\infty} c_n(x) D^n\}.$$

In this case, we observe that coefficients $c_n(x)$ are constant with respect to t . Immediately, from $WD = dW$ for $W \in \hat{G}_0$ we have

PROPOSITION (2.5). *The Lagrangian*

$$(2.6) \quad \mathcal{L}_0 = \int_{\mathbb{R}} \bar{\psi}^W d\psi_W dx, \quad W \in \hat{G}_0,$$

possesses the symmetry of the group $\hat{G}_0$.

Following the Uchiyama theory, next we deal with a group

$$(2.7) \quad \hat{G} = \{W | W = \sum_{n=-\infty}^{n=+\infty} u_n(x, t) D^n \text{ with the property } (*)\}$$

(*) $u_n(x, t)$ ($n = 0, \pm 1, \pm 2, \dots$) are analytic functions of x satisfying the following growth condition: There exists an integer n_0 such that $\text{ord } u_n(x, t) \geq n - n_0$ for any sufficiently large n

(see [4], [7], [8]).

The Lagrangian action (2.6) gives rise to a gauge group $\tilde{\mathcal{G}}_0$ of the first kind and a gauge group $\tilde{\mathcal{G}}$ of the second kind respectively.

$\mathcal{L}_0$ is not invariant under $\tilde{\mathcal{G}}$, since commutators

$$[D_m, W] = \Sigma (D_m u_n(x, t)) D^n \quad (m = 1, 2, \dots)$$

for $W = \Sigma u_n(x, t) D^n$, do not vanish identically, i.e., $[d, W] \neq 0$. Hence we have to make

Definition (2.8). Let $\tilde{R}$ be a subset of the group $\tilde{\mathcal{G}}$ described in (2.7). A set $\{\Omega(W) | W \in \tilde{R}\}$ of operators is called a *multiconnection* (or *total connection*) with a range $\tilde{R}$ (or, simply $\tilde{R}$ -*connection*) on the fibre $\tilde{\mathcal{G}}$ -space, if $\Omega(W)$ has the form $\Omega(W) = \Sigma_n \Omega_n(W) d t_n$ whose $\Omega_n(W)$ is a connection with a range $\tilde{R}$ with respect to D_n :

$$D_n - \Omega_n(W) = \phi (D_n - \Omega_n(W')) \phi^{-1}$$

for W and $W' \in \tilde{R}$, where $W = \phi W' (\phi \in \tilde{\mathcal{G}})$. $\Omega_n(W)$ is called the *partial connection* of $\Omega(W)$.

We note that a multiconnection $\Omega(W)$ with a range $\tilde{R}$ implies

$$d - \Omega(W) = \Sigma_n \phi (D_n - \Omega_n(W')) \phi^{-1} = \phi (d - \Omega(W')) \phi^{-1}$$

for $W, W' \in \tilde{R}$ with $W = \phi W'$.

By use of Uchiyama's Theorem, we obtain

PROPOSITION (2.9). Let $\Omega(W)$ be a $\tilde{\mathcal{G}}$ -connection. The Lagrangian

$$\mathcal{L} = \int_{\mathbb{R}} \bar{\psi}^W (d - \Omega(W)) \psi_W dx \quad \text{for } W \in \tilde{\mathcal{G}}$$

is invariant under the group $\tilde{\mathcal{G}}$.

We set

$$(2.10) \quad \tilde{\mathcal{G}}_+ = \{ \Sigma_{n=0}^{n=+\infty} u_n D^n \in \tilde{\mathcal{G}} | u_0 \neq 0 \}, \quad \tilde{\mathcal{G}}_- = \{ \Sigma_{n=0}^{n=+\infty} u_{-n} D^{-n} \in \tilde{\mathcal{G}} | u_0 \equiv 1 \}.$$

Corresponding to $\tilde{\mathcal{G}}, \tilde{\mathcal{G}}_+$ and $\tilde{\mathcal{G}}_-$, we consider the spaces of operators $\tilde{\mathcal{G}} = \{ \Sigma_{n=-\infty}^{n=+\infty} u_n D^n \}$, its complementary subspaces

$$(2.11) \quad \tilde{\mathcal{G}}_+ = \{ \Sigma_{n=0}^{n=+\infty} u_n(x, t) D^n \}, \quad \tilde{\mathcal{G}}_- = \{ \Sigma_{n=1}^{n=\infty} u_{-n}(x, t) D^{-n} \},$$

that is the direct sum $\tilde{\mathcal{G}} = \tilde{\mathcal{G}}_+ \oplus \tilde{\mathcal{G}}_-$. Hence, any element $X \in \tilde{\mathcal{G}}$ is written as $X = (X)_+ + (X)_-$ for $(X)_+ \in \tilde{\mathcal{G}}_+$ and $(X)_- \in \tilde{\mathcal{G}}_-$.

Here we recall the K.-P. system. The operator $L = W D W^{-1}$ for

$W \in G_-$ derived from the flat connection implies that $L^n = W D^n W^{-1}$ and its decomposition $L^n = (L^n)_+ + (L^n)_-$. In this case, $(L^n)_+$ is the n -th differential operator. The K.-P. system is a system of equations defined by

$$(2.12) \quad \partial L / \partial t_n = [(L^n)_+, L] \quad (n = 1, 2, \dots).$$

When $W \in \tilde{G}_-$ is an element described in the solution $L = W D W^{-1}$ of the K.-P. system, we shall say that W gives a solution of the K.-P. system. It is known ([1], [5], [6]) that an element W of G_- gives a solution of the K.-P. system if and only if W satisfies

$$(2.13) \quad \partial W / \partial t_n + (L^n(W))_- W = 0 \quad (n = 1, 2, \dots).$$

The following theorem is our main result:

THEOREM 2. Let R^* be the space of all elements of $\tilde{G}_-$ each of which gives a solution of the K.-P. system. Then the set $\{\Omega_{K.P.}(W) | W \in R^*\}$ defined by

$$(2.14) \quad \Omega_{K.P.}(W) = \sum_n \Omega_n(W) dt_n, \quad \Omega_n(W) = -(L^n(W))_-$$

becomes the flat R^* -connection (say, the K.-P. connection) on the fibre $\tilde{G}_-$ -space over $\mathbb{R}$.

Remark. (1) The K.-P. connection is a direct generalization of the connection given in Theorem 1, when we identify t_1 with x and set $t_n = 0$ ($n = 2, 3, \dots$). (2) The flatness of the K.-P. connection is well known as the Zakharov-Shabat equation.

For the proof of this theorem we need the following two lemmas:

LEMMA 1 (Mulase's decomposition theorem [4]). The group $\tilde{G}$ described in (2.7) can be decomposed into

$$\tilde{G} = \tilde{G}_- \cdot \tilde{G}_+,$$

in a sense that any element $g \in \tilde{G}$ determines the unique pair of elements $g_1 \in \tilde{G}_-$ and $g_2 \in \tilde{G}_+$ such that $g = g_1 \cdot g_2$.

LEMMA 2 ([4], [6]). There exists a one to one correspondence between the space R^* and the space Q of solutions U of the initial value problem:

$$(2.15) \quad \partial U / \partial t_n = [D^n, U], \quad U|_{t=0} = U_0 \in G_-,$$

where G_- is given in (1.18). The exact correspondence is described in the following manner: A solution U of (2.15) determines an

element W of G_- by the decomposition $U = W^{-1}V$ in Lemma 1. Then $L(W) = WDW^{-1}$ gives a solution of (2.12). Conversely, for a solution W of (2.12), we can find a unique element V of $\tilde{G}_+$ such that $V|_{t=0} = \text{identity}$ and $U = W^{-1}V$ gives a solution of (2.15).

The proof of Theorem 2. Let U_0 be any element of G_- . U_0 determines the unique solution $U (\in \tilde{G}_-)$ of (2.15) by Lemma 2. U can be decomposed uniquely as $U = W^{-1}V$ with $W \in \tilde{G}_-$ and $V \in \tilde{G}_+$ by Lemma 1. This gives rise to a mapping $\rho: G_- \rightarrow \tilde{G}_-$ which maps U_0 to W . This mapping ρ is injective ([4], [6]). Then we see that $R^* = \rho(G_-)$. Next we show that $\Omega_{K.P.}(W)$ becomes an R^* -connection. Let W and W' be elements of R^* and set $\phi (\phi \in \tilde{G}_-)$ by $W = \phi W'$. It follows from

$$(\partial W / \partial t_n) = (\partial \phi / \partial t_n) W' + \phi (\partial W' / \partial t_n)$$

and from (2.13) that

$$-(L^n(W))_- W = -(\partial \phi / \partial t_n) W' - \phi (L^n(W'))_- W'.$$

Hence

$$\omega_n(W) = (\partial \phi / \partial t_n) \phi^{-1} + \phi \omega_n(W') \phi^{-1}$$

holds, which implies that $\omega_n(W)$ ($W \in R^*$) is a partial R^* -connection. Therefore, $\Omega_{K.P.}(W)$ ($W \in R^*$) is an R^* -connection. The flatness of the connection follows from (2.13):

$$\begin{aligned} 0 &= \sum_n (\partial W / \partial t_n + (L^n(W))_- W) dt_n = \sum_n (\partial W / \partial t_n - \omega_n(W)) W dt_n \\ &= [d, W] - \Omega_{K.P.}(W) W. \end{aligned}$$

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