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Jaroslav Kurzweil

A tribute to Bernard Bolzano

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## A TRIBUTE TO BERNARD BOLZANO

Jaroslav Kurzweil

Praha, ČSSR

Today, two hundred years since he was born, it is clear beyond any doubt that Bolzano was a man of true ingeniousness. Nonetheless, during his life he had practically no contact with mathematicians of his time, he worked quite alone, his work was unknown and therefore he had neither support nor criticism from other mathematicians. He felt rather uneasy about it. In his autobiography one may read:

"The only thing I could perhaps be proud of in my adult age were my own opinions, which I believed to have reached step by step by myself; and who knows how proud these discoveries could have made me, had they been generally accepted as correct! However, until now my concepts have hardly met with any approval at all, except for a small circle of those who once had been my pupils and whose judgement in these matters I must therefore consider biased. Those several works I have published were either not considered worth of any evaluation at all, or they were simply laughed at. Under these circumstances my confidence in the correctness of my own opinions, except those which immediately concerned the religion, has never strengthened enough to remove a certain slight doubt whether they all are not erroneous. How could I thus refer to these discoveries and get conceited!"

Bolzano was active in Mathematics from his youth till his death. In spite of his outstanding achievements, Mathematics never was his single field of interest. About the way he became interested in Mathematics he wrote:

"However, during my first weeks (at the University) I did not consider Mathematics, the very subject that was to attract me most in the future, worth of my attention at all. It was because of the man who at that time lectured this science, Professor Vydra, whose rather rude manner differed very distinctly from the fine manners of, say, Seibt or Blaha, though Vydra was highly reputable in all other respects. Besides, I still had no idea what is done in the science.

However, when I just incidentally opened Kästner's textbook at a page, where several rows were filled by stars, it so aroused my curiosity that I read the passage. Immediately I decided to study Mathematics, for I then expected to find in the science what I had such a long time looked for in vain. Namely, Kästner proved there

a fact which was usually quite neglected, since everybody was supposed to know all about it: he tried to explain to the reader the reason for one of his assertions; and this was what I liked best. Thus my special liking in Mathematics actually concerned only its purely speculative part: I appreciated in Mathematics what was simultaneously Philosophy. It was extremely difficult for me to grasp the first propositions of this science and I often went from one colleague of mine to another asking them for explanation. One of those to whom I applied rather often is now the owner of an inn and a park, which is just being established near the Prague New Gate."

Bolzano was a person of immense integrity, who saw his mission in clearing up the methods of logic. He attempted to build a spiritual universe, which would include ethics, religion and science, all linked together with clear relations of logic. In this universe a crucial role was played by the principle of supreme universal welfare, or the supreme moral law, by which Bolzano meant that a person should choose of all possible ways of acting the one that - after all consequences have been considered - will contribute most to the welfare of mankind, regardless of which part of the mankind will enjoy the profit. It was on this principle that Bolzano based his ethics and theology, to it he appealed in critical moments of his life.

Bolzano did not seek for the guaranty of validity of the supreme moral law in the obviousness of the moral feeling. His supreme moral law is a pure conceptional truth: its validity is independent of whether it is thought (the more so of the feelings with which it is thought), and even of whether there at all exist thinking beings! Hence the validity of the supreme moral law is independent even of God and the religion; in this respect Bolzano surpasses his great predecessor Leibniz, whose eternal truths, even if independent of God's will, were anchored in God's substance. How much deeper is our respect for the supreme moral law, says Bolzano, if we realize that even the most holy being, God himself, has to obey it! It is the very perfect concurrence of his will with the moral laws that his freedom consists in.

He appealed to the supreme moral law with particular urgency when making the decision to study theology after four years of mathematical and philosophical studies. He invited two or three of his best friends and made all efforts to deduce with the maximal objectivity that for him, priesthood is the most suitable of all professions. Nevertheless, in his autobiography written for Mrs.

Anna Hoffmann, he confesses that he was influenced by his mother's secret wish, as well as by his concern for a cousin of his:

"Thus the both circumstances caused that my approach was not quite unbiased, but that I too much emphasized the benefit I should have the opportunity to bring as a priest, and suppressed all I should have to sacrifice. In fact, I also expected, and not without reason, that the tender love of my sister, if Heaven save her, would make it easier for me to give up love of another kind. To be frank, I must nonetheless admit that I could hardly have realized, even if I wished to judge it quite impartially, that the sacrifice I was to bring would be as big as I later found it to be. My dear friend, what I have just confessed is a secret that I was hiding from my parents for all their lifetime. However, since the time that those are dead, who could be grieved by the thought that I was not quite happy I do not hesitate to confess the truth, if it may serve a warning to the others or bring another benefit. On the other hand, do not conclude from my words that I have just confided to you that I have ever regretted my joining this status. No, my best knowledge that I did it without low intentions has not allowed me to lose my conviction that what happened was surely right.

After all, I have to recall that even when entering the study of theology I was not unconditionally decided to choose the status of a priest. Not even was I fully convinced of the truthfulness and the divine origin of the religion, the preacher of which I was to become. I wanted first to investigate into it and it was the study of theology where I expected to find the best opportunity to do so. In accordance with the result of my investigation I should have to decide for or against it. My decision in either case would be made easier by the fact that I had not entered any institution for students of theology but stayed and boarded in my father's house."

Theological studies corroborated Bolzano's earlier anticipation that the beginning of Christianity on Earth cannot be considered an immediate result of God's divine act. Bolzano felt intensively that the society of his time was pitiful in many respects and that changes were needed urgently; he sincerely wished to help to promote reforms. He came to the conclusion that the existence of Christianity and in particular of catholic religion is justified by its concurrence with the supreme moral law, as it results in the best moral profit for mankind. In 1805 he was ordained a priest and became professor of religious doctrine at Faculty of Philosophy of

Prague University, with the duty of teaching and holding sermons for academic youth every Sunday. Bolzano never wished to enter a monastery; the idea that he would give up the freedom of deciding about himself and even completely stop thinking, was unacceptable to him. In his notes we can read: "A member of a monastery is nothing more than a machine, the moving power being the prelate."

The chair of the religious doctrine had been established in order to promote the authority of both the church and the state. Bolzano made great efforts in order to revive and strengthen the religious feelings and beliefs of the students, but at the same time he vehemently criticized the establishment. Let us bring only two examples. In a sermon from 1811 Bolzano argues that global progress does not exclude a partial regress:

"In order to make sure about it, we unfortunately need not travel to distant countries; it is true even for our beloved country that instead of being every day wiser, better and happier, we have moved backwards during the last years; that the enlightenment recently awaked has again been replaced by darkness, that malpractice of all kinds has raised its shameless head more freely and with impunity, that the number of the suffering and poor in the country is increasing every day.

We declare as a general rule: in certain periods and regions, for a short time even on the whole Earth, the mankind obviously may decline in its perfection. However, each such decline necessarily prepares an even faster advancement. If we disregard particular moments but consider the whole history of the mankind, then it must be the wiser, the better and happier, the longer it has existed on this Earth."

In another sermon, Bolzano vehemently attacks the sovereigns who misuse the Christian religion to advocate bloodshed which they commit on the whole nations. They are "scoundrels who shamelessly play with the belief in immortality when justifying the murders they have committed on thousands and hundreds of thousands of people by the argument that according to our doctrine, death is no disaster for a man!"

Taking into account both his criticism of the establishment and his very unorthodox approach to the religious doctrine, we can well understand that a conflict was coming. Bolzano irritated many influential persons both in the state and the church administration. As a document of the hatred Bolzano evoked we may quote from a denunciation which was sent directly to Rome in 1819:

"At Prague University the religious doctrine is lectured upon by a certain priest, Bernard Bolzano by name, who is famous throughout Bohemia for his new and irregular way of teaching religion and for his sophistic scheming... . An example of his blasphemy is his rejection of reliable and irrefutable Catholic doctrines ... . The man, the coryphaeus of our false prophets, has from the very beginning had a number of pupils and followers who, being scattered all over the country, continue to spread among the mob the principles by which they were infected."

Bolzano was dismissed from the post of a university teacher in January 1820, being granted a small pension. The church sued him, but the suit was closed in 1825 without taking further measures against Bolzano. The main credit for the fact that the suit ended by a compromise goes to the Czech scholar Josef Dobrovský, who threatened to publish Bolzano's defence, submitting it to the whole educated Europe for judgment.

After the suit Bolzano lived in Prague and with the friendly family of Hoffmanns in Těchobuz. He strived for increasing the publicity of his ideas and though he was not allowed to publish in the Austrian monarchy nor abroad, he succeeded in publishing in Germany a number of more extensive as well as minor works from the fields of religion, philosophy, logic and mathematics. However, a number of his works was published only long after his death, as for example his Utopian essay "On the best state" in the year 1932. The results he achieved in mathematics are extremely important and their extent is admirable. Among other he wrote and published in 1837 an extensive work "Wissenschaftslehre" in four volumes, about 2400 pages. He analyzed in it the logical foundations of human knowledge and recognition. For various reasons his Wissenschaftslehre found no response in his time. Nonetheless, the value of this work is demonstrated by the fact that in the second half of the 20th century, several abridged versions have been published by various publishers.

The main results of Bolzano in mathematical analysis are well known. Some of them were published as soon as the beginning of the 19th century. However, his "Functionenlehre" (the theory of real functions) appeared only in 1933. It was the manuscript of this book where the famous Bolzano's function was discovered. A year later, Bolzano's "Zahlenlehre" was published. During the last months of his life, Bolzano completed his work "Paradoxien des Unendlichen", which was published in 1851. Here we find concepts very clo-

se to the later notions of the theory of sets. This book was known and referred to by Cantor, and some authors regard Bolzano as the founder of the theory of sets.

We commemorate Bernard Bolzano with high respect for his work, his unceasing activity and his profound belief in the value and power of exact thinking.