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EXTRAPOLATION OF FINITE ELEMENT SOLUTIONS ON NON-UNIFORM QUADRILATERAL MESHES

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It is known that, for an arbitrary mesh, extrapolation of the finite element solutions is impossible. In practice, fortunately, the finite element mesh is both flexible and regular. We found that the regularity but not the arbitrariness of the mesh can be used to make the extrapolation of finite element solutions to be possible. Actually, in Rannacher's survey, for the uniform triangular mesh or its variant, one step of extrapolation increased the accuracy of linear finite element solutions from $O(h^2)$ to $O(h^4)$. We shall show here that, for certain non-uniform quadrilateral mesh (including a graded mesh), one step of extrapolation increases the accuray of isoparametric bilinear finite element solutions from $O(h^2)$ to $O(h^3)$.

1. Rectangular Domain with Rectangular Mesh

Let D be a rectangular domain with the edges parallel to x and y axes. Consider in D the boundary value problem: find $u \in H_0^1$ such that

$$a(u, v) = (f, v) \quad \text{for } v \in H_0^1 \tag{1}$$

with the bilinear form

$$a(u,v) = \int_D u_x(a_{11} v_x + a_{12} v_y) + u_y(a_{21} v_x + a_{22} v_y),$$

where a_{ij} and f are given smooth functions.

Let T^h be a rectangular partition over D. For an arbitrary element $e \in T^h$ let h_e and k_e denote the lengths of e along x and y directions. Set

$$d_e = \max(h_e, k_e), \quad h = \max_e d_e .$$

Let u^h be the piecewise bilinear finite element solution of (1) over T^h. In the proving of extrapolation estimate (2) we need the usual restrictions for T^h:

H1. There exists a constant $b \geq 1$ such that

$$h_e \geq ch^b, \qquad k_e \geq ch^b, \qquad \text{for } e \in T^h;$$

H2. There exist the constants c_1 and c_2 such that, for any two adjacent elements e and e',

$$c_1 h_e \leq h_{e'} \leq c_2 h_e , \qquad c_1 k_e \leq k_{e'} \leq c_2 k_e .$$

Theorem 1. Assume that $u \in C^3(\bar{D})$ and T^h satisfies H1 and H2. Then, for z being the nodal points of T^h,

$$\left| \tfrac{1}{3}(4u^{h/2} - u^h)(z) - u(z) \right| \leq ch^3 \left| \ln h \right|. \tag{2}$$

See [1] [2] for the proof.

2. Polygonal Domain with Quadrilateral Mesh

Let D be a convex polygonal domain. We decompose D into several Fixed convex macro-quadrilaterals and then link up some equi-proportionate points of the opposite edges in each macro-quadrilateral and form a quadrilateral mesh T^h which may not be uniform. Let u^h be the isoparametric bilinear finite element solution of (1) over T^h. Then Theorem 1 still hold true. See [1] for details.

3. Reentrant Domain with Graded Mesh

For simplicty we consider here the simple model of (1), i.e. the Poisson equation, and the simple reentrant domain D, i.e. the union of rectangles. Let $\{V_j\}_{j=1}^m$ be the corner points of D, a_j the corresponding interior angles and

$$b_j = \pi / a_j .$$

It is known that the solution u is not smooth at the corners V_j. To compensate for this we use the following mesh T^h which is graded near each corner. For a given n, the mesh points of T^h are given by

$$\left(\frac{i}{n} \right)^{q_j} \qquad (i = 1,\cdots,n)$$

along x / and y directions near V_j. Then, by choosing the grading exponents

$$q_j > \frac{3}{b_j} ,$$

one step of extrapolation increases the accuracy of bilinear finite element solution u^h from $O(h^2)$ to $O(h^3 \, |\ln h|^{1/2})$:

$$\left| \frac{1}{3} (4u^{h/2} - u^h)(z)-u(z) \right| \leq ch^3 \left| \ln h \right|^{1/2} \qquad (3)$$

for all the mesh points z of T^h.

We guess, for an estimate of $O(h^r)$ with $r < 3$ in (3), we need only

$$q_j > \frac{r}{b_j}$$

And, for an interior estimate of (3), we need only

$$q_j > \frac{3}{2b_j} .$$

REFERENCES

[1] Lin Qun: Finite element error expansion for non-uniform quadrilateral meshes, Systems Science and Mathematical Sciences, 2:3 (1989), 157-164.

[2] Lin Qun, Xie Ruefeng: Extrapolation of bilinear finite element solutions with non-uniform partition, to appear.

[3] Rolf Rannacher: Extrapolation technique in the finite element method (survey), Proceeding of the Summer School in Numerical Analysis 1987, Helsinki University of Technology, Reports C7, Feb. 1988, 80-113.