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Integral representation of functions and imbedding theorems for domains with the flexible Horn property [Summary]


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Let $\lambda = (\lambda_1, \ldots, \lambda_n) \in (0, \infty)^n$. A domain $G \subseteq \mathbb{R}^n$ will be said to have the flexible $\lambda$-horn property (the flexible cone property if $\lambda_1 = \ldots = \lambda_n$) if, for some $\delta > 0$, $T > 0$ and for any $x \in G$, there exists a curve $\rho(t^\lambda) = \rho(t^\lambda, x) \overset{\text{def}}{=} (\rho_1(t^{\lambda_1}), \ldots, \rho_n(t^{\lambda_n}))$, $0 \leq t \leq T$, possessing the following properties:

a) $\rho_i(u)$ are absolutely continuous on $[0, T^\lambda_i]$; $|\rho_i'(u)| \leq 1$ for a.a. $u \in [0, T^\lambda_i]$;

b) $\rho(0, x) = 0$, $x + \bigcup_{0 < t \leq T} [\rho(t^{\lambda}, x) + t^{\lambda} \delta^{\lambda} (-1, 1)^n] \subseteq G$.

The concept of a domain with the flexible cone property is more general than that of a domain with the cone property, with the F. John property, and of an $(\varepsilon, \delta)$-domain.

We get an integral representation of functions in terms of their derivatives and differences. On this basis imbeddings of anisotropic Sobolev spaces $W^{p_1, \ldots, p_n}_q(G) \subset L_q(G)$ are established, as well as estimates for $L_q$-moduli of continuity of functions, leading to imbeddings of spaces defined via differences.

A necessary condition is obtained for the Fourier multipliers from $L_p(\mathbb{R}^n)$ into $L_p(\mathbb{R}^n)$, $1 < p < \infty$. This generalizes the Hörmander criterion, relaxing the requirement on the smoothness in $L_q$ from $1 + [\frac{n}{2}]$ to $\frac{n}{2}$.

References


We consider the class of functions $u : (1, \infty) \rightarrow \mathbb{R}$ which stabilize to polynomials $P(t; u) = \sum_{m=0}^{r-1} a_m t^m$ ($r \in \mathbb{N}$ is fixed) as $t \to +\infty$.

For functions from this class the inequality
\[
|u(s)(t)| \leq c \left( \sum_{\mu=1}^{k} |u(1)_{1}^{(\mu)}| + \sum_{\nu=1}^{\ell} |a_{j_{\nu}}| + \sum_{v=1}^{d} |\phi v_{v}| \right)_{\ell_{p}(1, +\infty)},
\]
where $\phi$ is a given function (a weight), $t^{\alpha - 1} \in L_{q}(1, +\infty)$, $\alpha > r - 1$, $1/p + 1/q = 1$, $k + \ell \geq r$; $\{i_{\mu}\}_{\mu=1}^{k}$ and $\{j_{\nu}\}_{\nu=1}^{\ell}$ are admissible sets of indices $i, j \in \mathbb{Z}, r-1$, connected with the Pólya problem [1], $a_{j_{\nu}}$ are the coefficients of the polynomial $P(t; u)$, the constant $c > 0$ is independent of the function $u$ [2,3].

In the case $p = 2$ we prove existence and uniqueness of a function minimizing the corresponding quadratic functional in the class considered, $u^{(1)}_{1}, \mu = 1, \ldots, k$, and $a_{j_{\nu}}, \nu = 1, \ldots, \ell$, being fixed.

The conditions are explained which are satisfied by the solution to this problem with arbitrary values of $i$ and $j$ at the ends of the interval $(1, +\infty)$.