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COMPARISON OF CRACK PROPAGATION CRITERIA IN LINEAR ELASTIC FRACTURE MECHANICS

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Abstract

In linear fracture mechanics, it is common to use the local Irwin criterion or the equivalent global Griffith criterion for decision whether the crack is propagating or not. In both cases, a quantity called the stress intensity factor can be used. In this paper, four methods are compared to calculate the stress intensity factor numerically; namely by using the stress values, the shape of a crack, nodal reactions and the global energetic method. The most accurate global energetic method is used to simulate the crack propagation in opening mode. In mixed mode, this method is compared with the frequently used maximum circumferential stress criterion.

1. Introduction

The description of crack propagation is one of the most important ingredients of linear elastic fracture mechanics (LEFM). The main questions are: At which loading level will the crack propagation begin and in which direction will the crack propagate?

The aim of this paper is to compare numerical implementations of most frequently used crack propagation criteria for opening mode and mixed mode in 2D.

2. Stress intensity factor concept

The stress intensity factor is a quantity used in LEFM to describe the asymptotic singular stress field near the crack tip. The stress in the vicinity of the crack tip is unbounded and grows in inverse proportion to the square root of distance from the tip. Under plane stress, the asymptotic stress field is described by

$$\sigma_x(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2-\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right), \quad (1)$$

$$\sigma_y(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}, \quad (2)$$

$$\tau_{xy}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} - \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right), \quad (3)$$

where K_I and K_{II} are the stress intensity factors for modes I and II which represent the loading and geometry conditions, r is the distance from the crack tip and θ is the polar angle; see e.g. [7].

3. Crack propagation in mode I (opening mode)

In mode I, the crack is opening without sliding. Therefore, we can assume that the crack will propagate in the original direction and we have to decide when the propagation starts.

3.1. Local Irwin criterion

This concept was introduced by Irwin [4] in 1957. The stress intensity factor is used to decide about the crack propagation. The propagation will start when the value of the stress intensity factor K_I reaches its critical value so-called fracture toughness denoted by K_c .

3.2. Global Griffith criterion

This criterion was introduced by Griffith [3] in 1920. The crack will grow if a sufficient amount of energy is released by its propagation. The criterion is based on the strain energy release rate, defined as

$$\mathcal{G}(u,a) = -\frac{1}{t}\frac{\partial W_e(u,a)}{\partial a}, \tag{4}$$

where $W_e(u,a)$ is the elastic strain energy considered as a function of the imposed displacement u and the crack length a. The beam thickness is denoted by t; see Figure 1.

Under plane stress and in mode I, both criteria are equivalent [2]. We can write

$$\mathcal{G}(u,a) = \frac{{K_I}^2}{E}. \tag{5}$$

The rules for crack propagation according to the local Irwin and global Griffith criteria are summarized in Table 1, where K_c resp. G_f are material properties called fracture toughness [$\mathrm{Nm}^{-3/2}$] and fracture energy [Nm^{-1}], resp.

Local criterion	**Global criterion**		**Crack behaviour**
$K_I < K_c$	$\mathcal{G} < G_f$	$\Rightarrow$	no crack propagation
$K_I = K_c$	$\mathcal{G} = G_f$	$\Rightarrow$	crack propagation
$K_I > K_c$	$\mathcal{G} > G_f$	$\Rightarrow$	inadmissible (in statics)

Table 1: Crack propagation rules according to the local and global criteria

3.3. Simulation in opening mode

Four methods have been used to calculate the stress intensity factor or the strain energy release rate in opening mode; namely by using (i) the stress values, (ii) the crack opening, (iii) nodal forces and (iv) the release of strain energy. The first three methods have a local character and deal with the values near the crack tip to calculate the stress intensity factor. The fourth method evaluates the change of the energy of the whole beam when the crack is extended. Three different types of triangular finite elements have been used for each method; namely (i) three-node element with linear approximation of displacement, (ii) six-node element with quadratic approximation and (iii) six-node quadratic element with singular shape functions on the edges starting from the crack tip; see Figure 2. Numerical simulations have been performed using the open-source finite element code OOFEM [6]. The relative error of all methods in a three-point bending test with geometry according to Figure 1 have been evaluated by comparing the computed value of the stress intensity factor with the ,,exact" values obtained by using approximate analytic formulas available in [2], [5] and [7]. The relative errors of all methods for all types of elements are shown in Table 2.

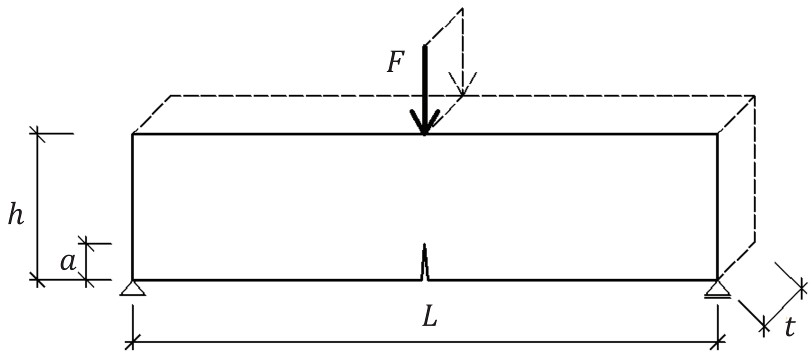

Figure 1: Geometry of the three-point bending test

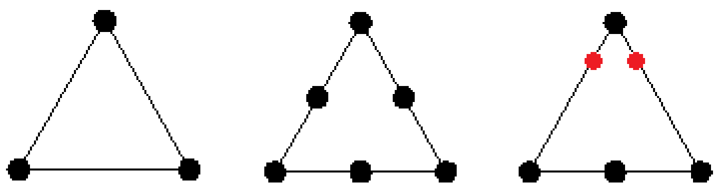

Figure 2: Used finite elements: linear (left), quadratic (middle), modified singular quadratic (right)

	Linear	Quadratic	Singular quadratic
Values of stress	> 30 %	10 - 30 %	10 - 30 %
Shape of a crack	< 10 %	2 %	1 %
Node reactions	5 - 10 %	< 5 %	< 5 %
Energetic method	5 %	2 %	0.5 %

Table 2: The relative errors of the computed stress intensity factor K_I for different methods and different finite elements

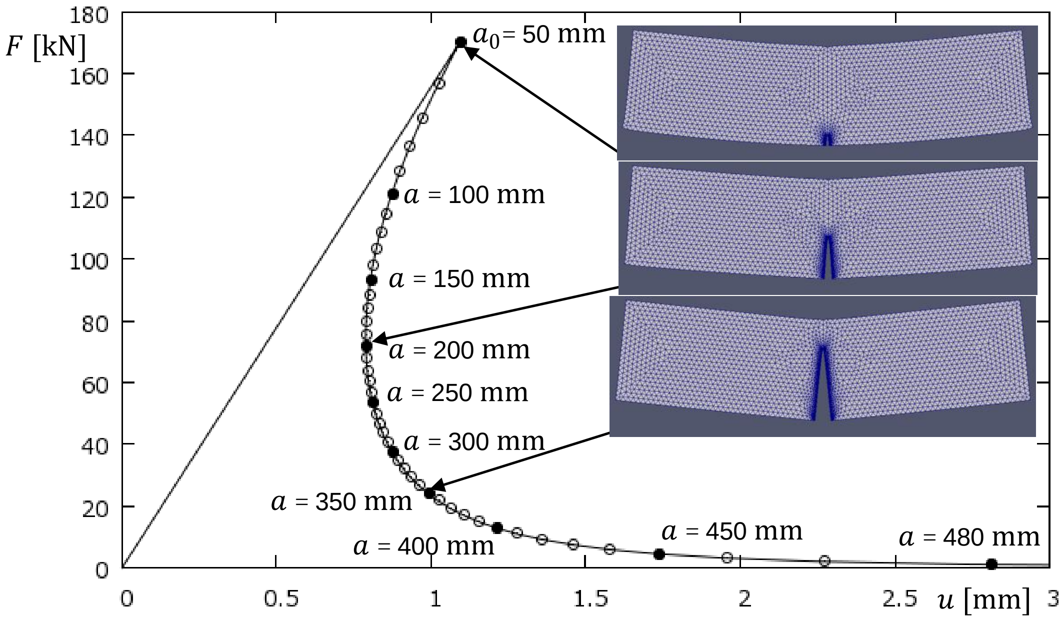

Figure 3: Force-displacement diagram of the three-point bending test

The global energy method using the modified quadratic elements is the most accurate but also the most time-consuming one. Figure 3 shows the load-displacement diagram obtained with this method for a beam of the following geometry: height $h = 0.5$ m, length $L = 2$ m, thickness $t = 0.2$ m, initial crack length $a_0 = 0.05$ m, elastic modulus $E = 20$ GPa, Poisson ratio $\nu = 0.2$ and fracture toughness $K_c = 4$ MNm$^{-3/2}$.

4. Crack propagation in mixed mode

The mixed mode represents a combination of tensile opening and in-plane shear. The direction of crack propagation is not known in advance and has to be determined by a suitable criterion.

4.1. Maximum circumferential stress criterion (MCSC)

This criterion determines the crack propagation direction based on the maximal circumferential stress σ_θ, which is defined as

$$\sigma_\theta(r,\theta) = \sigma_y \cos^2\theta + \sigma_x \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta. \tag{6}$$

Substituting from (1)–(3), we obtain

$$\sigma_\theta(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\cos^3\frac{\theta}{2} - 3\frac{K_{II}}{\sqrt{2\pi r}}\cos^2\frac{\theta}{2}\sin\frac{\theta}{2}. \tag{7}$$

The angle θ with maximal circumferential stress (MCSC1) is obtained by solving the equation

$$\sin\theta + \frac{K_{II}}{K_I}(3\cos\theta - 1) = 0 \tag{8}$$

with the conditions

$$\theta \in (-\pi,\pi); \qquad K_I > 0; \qquad K_{II}\sin\frac{\theta}{2} < 0, \tag{9}$$

where the ratio K_{II}/K_I is obtained by fitting (1)–(3) to the values of the stress field in a number of Gauss points near the crack tip.

Another approach (referred to as MCSC2) is based on substituting the values of the stress field at Gauss points into the original definition of circumferential stress (5). After smooth of these data by a polynomial function, the angle θ that maximizes this function can be found.

Both approaches give almost the same results; see Figure 4. The first method (MCSC1) turned out to be numerically preferable and therefore is used in the following examples.

4.2. Maximum strain energy release rate criterion (MSERRC)

This criterion determines the crack propagation in the direction that leads to the maximum strain energy release rate defined in (4). Numerical realization consists in simulation of a number of sufficiently small crack extensions in several different directions. For each direction, the strain energy release rate is evaluated by subtracting the final strain energy from the original one and dividing by the increment of the crack area. The obtained values are smoothed using a polynomial function, for which the maximum is then found.

This criterion predicts, in most cases, similar crack trajectories to the MCSC. However, application to the three-point bending test with an eccentric initial crack leads to a different crack path; see Figure 4.

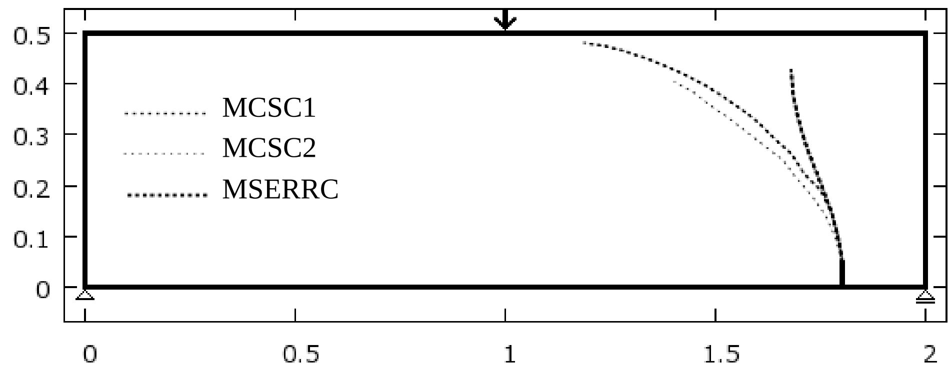

Figure 4: Comparison of crack paths according to different criteria for the three-point bending test with an eccentric initial crack

4.3. Comparative example

The last example is taken from [1]. It is a rectangular panel with two holes and two initial cracks subjected to tension in the vertical direction. In this example, both criteria lead to almost the same crack paths and the results are similar to those from [1]; see Figure 5. The load-displacement diagrams are depicted in Figure 6. Both criteria predict the same behaviour but the curve obtained with MCSC is not smooth. This means that MCSC is less accurate when used to decide whether the crack is propagating or not.

5. Conclusion

In the opening mode, the best results are obtained by the global energy method. In the mixed mode that arises in the three-point bending test, the MSERRC criterion

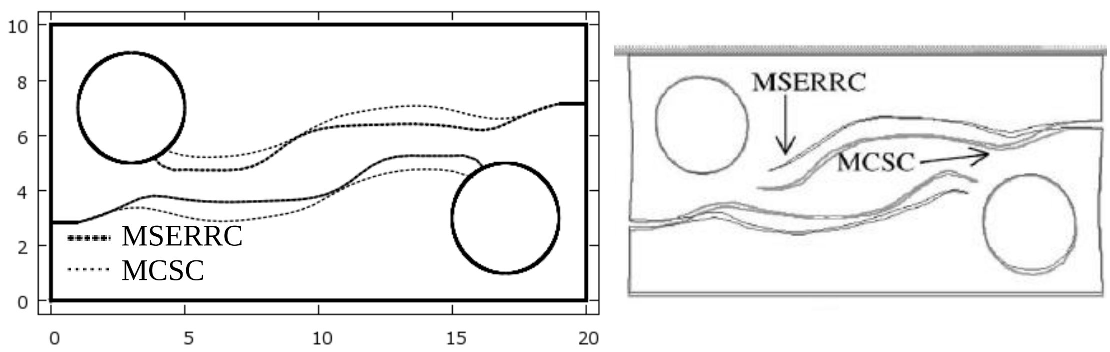

Figure 5: Comparison of crack paths according to different criteria in a vertical tensile test. Simulation in OOFEM (left) against results taken from [1] (right).

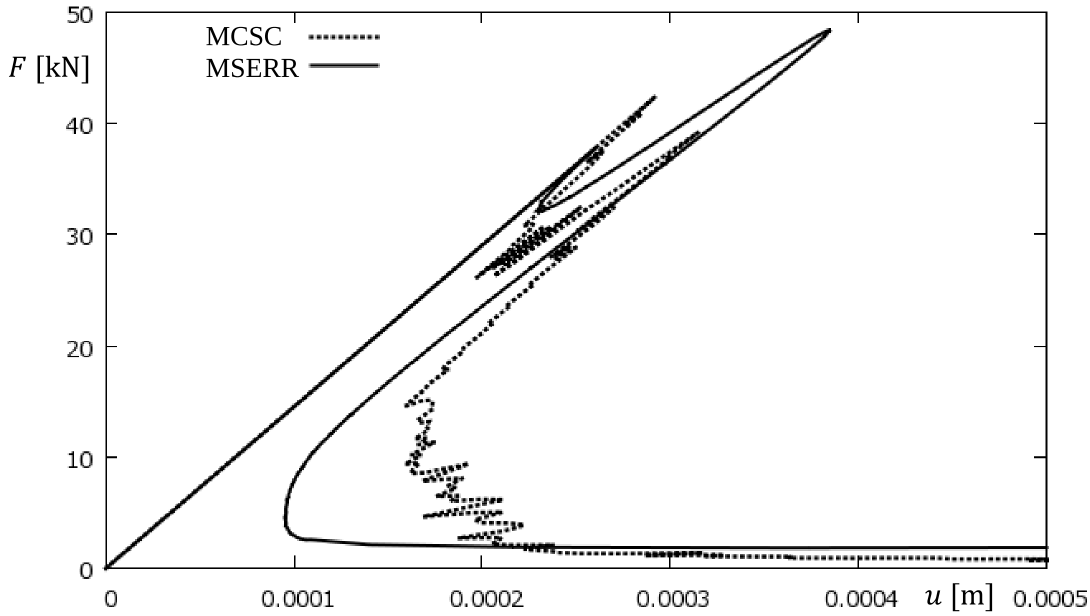

Figure 6: Comparison of load-displacement diagrams for different criteria

based on this method leads to a different crack path than the MCSC criterion using the circumferential stress. But both MSERRC and MCSC give similar results in other examples in mixed mode. Both criteria seem to be accurate in prediction of the propagation angle, but to determine whether the crack propagates it is more appropriate to use MSERRC.

Acknowledgements

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