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Numerical methods for MHD problems

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NUMERICAL METHODS FOR MHD PROBLEMS

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1. Introduction

Utilization of the magnetohydrodynamic (MHD) phenomena - mutual influence of electromagnetic field and dynamics of motion of electrically conductive fluids - represents one of very prospective technologies in the area of modern metallurgy. This paper deals with numerical modelling of the stirring of molten metal in a cylindrical crucible furnace by time-varying electromagnetic field. This time-varying electromagnetic field is supposed to generate such eddy currents in the charge that the heat produced by the corresponding Joule loss is approximately equal to the heat loss due to convection to the neighbourhood and, moreover, these currents produce also Lorentz forces making the liquid in the furnace move.

2. Formulation of the problem

The task is to model steady-state stirring of a molten metal by time-varying electromagnetic field in a device for induction stirring (Fig. 1). Particular components influencing the process that have to be taken into account are:

- molten metal (aluminium) **1** (characterised by a given mass density ρ_{Al}, specific heat c_{Al}, thermal conductivity λ_{Al}, dynamic viscosity η_{Al}, electric conductivity γ_{Al} and permeability μ_{Al}),
- cylindrical fire-clay crucible **2** and lid **3** (thermal conductivity λ_F, electric conductivity γ_F and permeability μ_F),
- field coil **5** with N hollow turns made from material with electric conductivity γ_{Cu} and permeability μ_{Cu}. The coil carries sinusoidal current of amplitude I_{ext} and angular frequency ω.
- Space **4** above the molten metal is supposed to be filled with air.
- Ambient air **6**.

This task is formulated as a weakly coupled electromagnetic-hydrodynamic problem. The first step of solution is therefore to find the electromagnetic field and corresponding distribution of the Lorentz forces in the liquid (that is considered incompressible). The second step is to find out consequent field of velocities. Moreover, this task can be formulated as an axisymmetric problem.

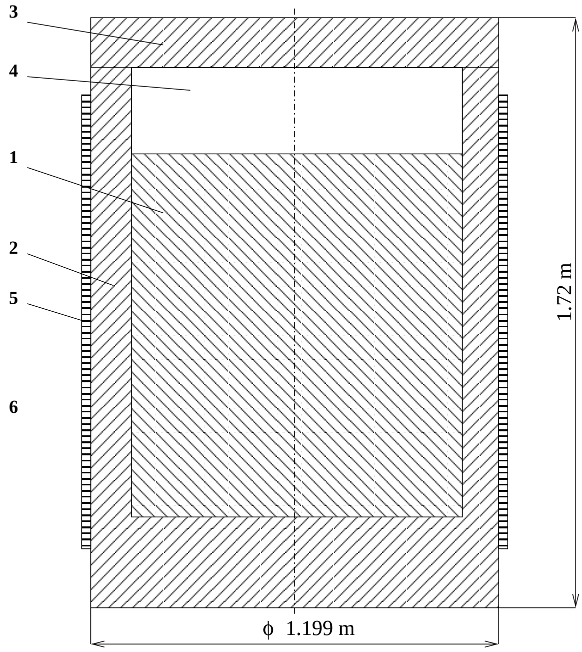

Fig. 1: *Crucible with the molten metal.*

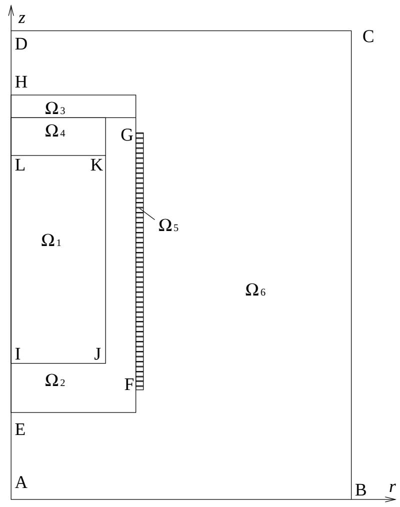

Fig. 2: *Definition areas for particular fields.*

3. Mathematical model of the problem

3.1. Electromagnetic field

Its definition area is bounded by line ABCDA (Fig. 2). Line AD denotes the axis, line ABCD represents a fictitious boundary characterised by conditions for open boundary problems.

The harmonic magnetic field is described by the *Helmholtz equation* for the phasor of vector potential $\boldsymbol{A}$

$$\triangle \underline{\boldsymbol{A}} - \mathrm{j}\,\omega\,\gamma\,\mu\,\underline{\boldsymbol{A}} = -\mu\,\underline{\boldsymbol{J}}_{ext}, \tag{1}$$

where j denotes the imaginary unit and $\underline{\boldsymbol{J}}_{ext}$ the phasor of the current density of the external sources. Particular forms of this equation for the subregions Ω_1 to Ω_6 can be obtained after substituting correct parameters in accordance with Par. 2. The boundary conditions have the form

$$\underline{\boldsymbol{A}} = \underline{\boldsymbol{0}} \quad \text{on line AD (antisymmetry)}, \tag{2}$$

$$\underline{\boldsymbol{A}} = \underline{\boldsymbol{0}} \quad \text{on line ABCD (zero force line)}. \tag{3}$$

Phasor $\underline{\boldsymbol{f}}_L$ of the Lorentz force producing motion of the liquid is given as

$$\underline{\boldsymbol{f}}_L = \underline{\boldsymbol{J}}_{eddy} \times \mathrm{rot}\,\underline{\boldsymbol{A}}, \tag{4}$$

where $\underline{\boldsymbol{J}}_{eddy}$ (phasor of the eddy current density) follows from the relation

$$\underline{\boldsymbol{J}}_{eddy} = \mathrm{j}\,\omega\,\gamma\,\underline{\boldsymbol{A}}. \tag{5}$$

3.2. Steady-state velocity field of the molten metal

The definition area for the considered velocity field is bounded by line IJKLI. We suppose that swelling of level KL can be neglected.

The stationary laminar flow of the incompressible liquid is described by the *Navier-Stokes equation* and *continuity equation*

$$\begin{aligned} \rho\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} &= -\mathrm{grad}\,p+\eta\,\triangle\boldsymbol{v}+\rho\,\boldsymbol{g}+\boldsymbol{f}_L, &(6)\\ \mathrm{div}\,\boldsymbol{v} &= 0, &(7)\end{aligned}$$

where p denotes pressure, $\boldsymbol{v}=v_r\,\mathbf{r}_0+v_z\,\mathbf{z}_0$ velocity of the fluid and $\boldsymbol{g}=-g\,\mathbf{z}_0$; g is free-fall acceleration. The boundary conditions have the form

$$v_r=0,\quad \frac{\partial v_z}{\partial r}=0 \quad \text{on line IL (symmetry)}, \tag{8}$$

$$v_r=0,\quad v_z=0 \quad \text{on line IJKL (laminar flow along solid wall)}. \tag{9}$$

3.3. Steady-state temperature field

Its definition area is bounded by line EFGHE. This field is described by the equation

$$\mathrm{div}\ (\lambda\ \mathrm{grad}\ T)-\rho\,c\,\boldsymbol{v}\cdot\mathrm{grad}\ T=-w_J, \tag{10}$$

where T is temperature and w_J the density of the Joule loss that may be expressed as

$$w_J=\frac{\underline{\boldsymbol{J}}_{eddy}\times\underline{\boldsymbol{J}}^*_{eddy}}{\gamma}, \tag{11}$$

$\underline{\boldsymbol{J}}^*_{eddy}$ being the complex conjugate to $\underline{\boldsymbol{J}}_{eddy}$. Special forms of (10) for particular subregions Ω_1 to Ω_3 may be obtained by substituting suitable parameters in accordance with Par. 2. The boundary conditions have the form

$$\begin{aligned} \frac{\partial T}{\partial \boldsymbol{n}} &= 0 &&\text{on line EH (symmetry)}, &(12)\\ -\lambda\frac{\partial T}{\partial \boldsymbol{n}} &= \alpha(T-T_0) &&\text{on line EF, GH (ambient air)}, &(13)\\ T &= T_c &&\text{on line FG (field coil)}, &(14)\end{aligned}$$

where α denotes the coefficient of the convective heat transfer, T_0 the temperature of the ambient air, T_c the temperature of the field coil, $\boldsymbol{n}$ the outward normal.

4. Numerical solution and results of the problem

4.1. Electromagnetic field

Computation of the electromagnetic field was performed by the finite element method, see, e. g., [1], [2]. Discretization of the domain Ω was constructed by triangulation τ_h, which was chosen strongly non-uniform, because we assumed big changes

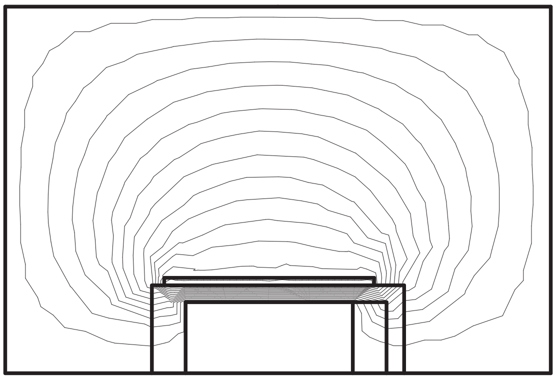

Fig. 3: *Distribution of magnetic field.*

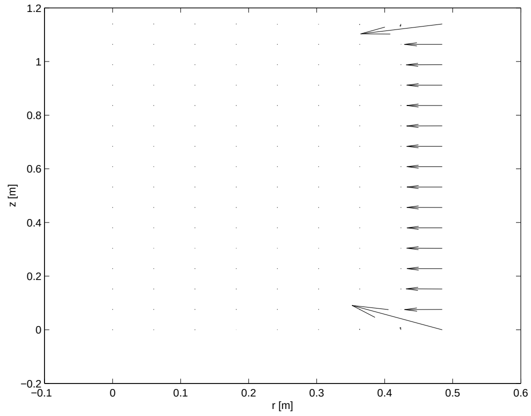

Fig. 4: *Distribution of Lorentz forces in the charge (max. value* $2.7\ 10^5\ N \cdot m^{-3}$*).*

in the solution. The mesh was very fine in subregions $\Omega_1, \Omega_2, \Omega_5$. On the contrary, near line ABCD the triangulation was chosen rougher. The mesh with 130,000 nodes was applied. Distribution of the electromagnetic field was obtained by using piecewise linear finite elements. The convergence of the solution was tested by total Lorentz forces in the whole volume of furnace: $\int_\Omega |\boldsymbol{f}_L| \mathrm{d}V$. Some results are presented in several following figures. Fig. 3 shows the distribution of force lines. Obvious is increased density of the force line near the surface of the charge that is caused by very high density of eddy currents in well electrical conductive liquid metal. Fig. 4 depicts the distribution of the Lorentz forces in the charge. Their distribution is strongly non-uniform and they fast decrease towards the axis of the crucible. This is caused by non-uniformity of the magnetic field in the charge.

4.2. Steady-state velocity field of the molten metal

Computation of the steady-state velocity field was also performed by the finite element method. Discretization of the domain Ω was constructed by uniform rectangular mesh (3,000 nodes). We used piecewise quadratic finite elements for approximation of the velocity and piecewise linear finite elements for approximation of the pressure. Because the Navier-Stokes equations are nonlinear, we used the following iterative process for the linearization of nonlinear terms (the solution of the Navier-Stokes problem is approximated by a sequence of linear Oseen problems, see [3], [4]):

1. Let $\tilde{v_r}^0, \tilde{v_z}^0$ be guesses of $v_r,\ v_z$, respectively, and $\varepsilon > 0$ a given toleration.

2. For $n = 0, 1, 2, \ldots$

   (a) By the solution of Oseen problem we receive v_r^{n+1}, v_z^{n+1}.

   (b) If $\frac{v_r^{n+1} - \tilde{v_r}^n}{v_r^{n+1}} < \varepsilon$, $\frac{v_z^{n+1} - \tilde{v_z}^n}{v_z^{n+1}} < \varepsilon$, END.

(c) Repeat (a) with the new guesses

$$\tilde{v_r}^{n+1} = \frac{v_r^{n+1} + v_r^n}{2}, \quad \tilde{v_z}^{n+1} = \frac{v_z^{n+1} + v_z^n}{2}. \tag{15}$$

3. $v_r^{n+1} \approx v_r, \quad v_z^{n+1} \approx v_z.$

But our problem is that this iterative process converges only for low Reynolds number (Re < 1000), on the contrary, for a higher Reynolds number spurious oscillations appear and the flow of molten metal in the furnace has Reynolds number around 10^7. So we started to solve the task with smaller Lorentz forces. For example, in Fig. 5 there is another distribution of forces in the charge and in Fig. 6 there is consequent velocity field of the molten metal. This result was received by using the above mentioned iterative process. For the successful solution of this problem some stabilization method for axisymmetric flow has to be found.

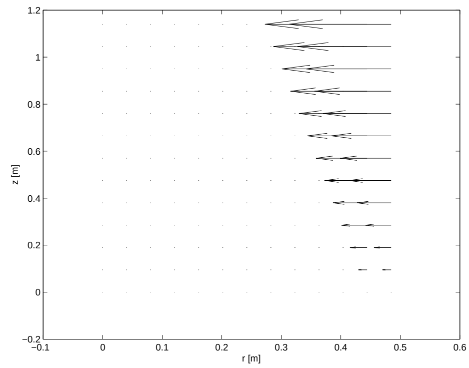

Fig. 5: *Distribution of forces (max. value 200 $N \cdot m^{-3}$).*

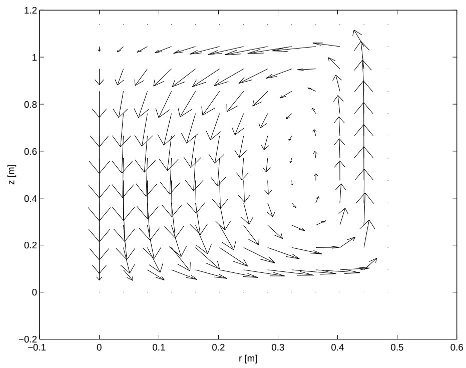

Fig. 6: *Consequent distribution of velocities.*

4.3. Steady-state temperature field

Computation of the temperature field was performed by the finite element method, see, e. g., [3]. Discretization of the domain Ω was constructed by rectangular mesh, which was chosen strongly non-uniform, because we assumed again big changes in the solution. The mesh with 10,000 nodes was applied. Distribution of the temperature field was obtained by using piecewise quadratic finite elements. Fig. 7 shows distribution of the temperature in the molten metal. While the temperature distribution in the molten metal is almost uniform, high gradient of temperature is apparent on the wall of the crucible. This fact following from very different thermal conductivities of the metal and fire clay is desirable particularly from the viewpoint of the operation of the device.

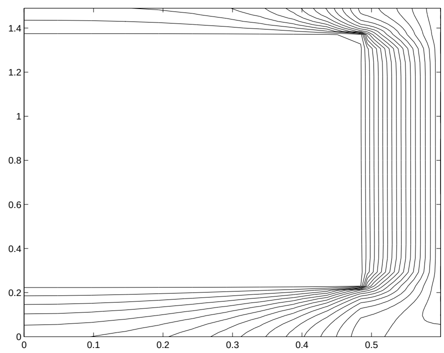

Fig. 7: *Temperature field (max. value* 630 ^{o}C*).*

5. Conclusion

The presented electromagnetic-thermal-hydrodynamic problem was solved as a weakly coupled task, but with respecting the influence of motion of the metal on the distribution of the temperature field. In order to obtain the solution for flow with a high Reynolds number it is necessary to find some stabilization scheme. Next work will be aimed at the including of swelling of the charge level.

References

[1] J. Polák: *Variační principy a metody teorie elektromagnetického pole.* Praha, Academia 1988.

[2] M. Křížek, K. Segeth: *Numerické modelování problémů elektrotechniky.* Praha, Nakladatelství Karolinum 2001.

[3] S. S. Rao: *The finite element method in engineering.* Oxford, Pergamon Press 1989.

[4] M. Feistauer: *Mathematical methods in fluid dynamics.* Essex, U.K., Longman Scientific and Technical 1993.