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VALUATION OF TWO-FACTOR OPTIONS UNDER THE MERTON JUMP-DIFFUSION MODEL USING ORTHOGONAL SPLINE WAVELETS

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Abstract: This paper addresses the two-asset Merton model for option pricing represented by non-stationary integro-differential equations with two state variables. The drawback of most classical methods for solving these types of equations is that the matrices arising from discretization are full and ill-conditioned. In this paper, we first transform the equation using logarithmic prices, drift removal, and localization. Then, we apply the Galerkin method with a recently proposed orthogonal cubic spline-wavelet basis combined with the Crank–Nicolson scheme. We show that the proposed method has many benefits. First, as is well-known, the wavelet-Galerkin method leads to sparse matrices, which can be solved efficiently using iterative methods. Furthermore, since the basis functions are cubic splines, the method is higher-order convergent. Due to the orthogonality of the basis functions, the matrices are well-conditioned even without preconditioning, computation is simplified, and the required number of iterations is less than for non-orthogonal cubic spline-wavelet bases. Numerical experiments are presented for European-style options on the maximum of two assets.

Keywords: wavelet-Galerkin method, Crank–Nicolson scheme, orthogonal spline wavelets

MSC: 65T60, 65M60, 47G20, 60G51

1. Introduction

Numerous models have been developed for the fair pricing of options. These models include the famous Black–Scholes and stochastic volatility models, which assume that the underlying asset price is a continuous function of time. This assumption, however, is not always consistent with the behavior of real market prices. Therefore, several models have been developed which allow for jumps in the price of the underlying. This paper focuses on one of these models, the Merton jump-diffusion model

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with two assets, represented by a nonstationary partial integro-differential equation (PIDE) with two state variables. From the mathematical point of view, it is not straightforward to solve this model numerically due to several difficulties. First, the integral term results in linear systems with full matrices for many standard methods, such as the finite difference and finite element methods. Moreover, the integral term requires the computation of four-dimensional integrals. Furthermore, the differential operator is degenerate, and functions representing the initial conditions are typically not smooth.

Option pricing is a central topic in financial mathematics, and there is a vast amount of literature concerning the numerical valuation of options. However, when it comes to multi-dimensional jump-diffusion models, due to the difficulties mentioned above, there are only a few studies on numerical methods for their solution. Thus, this remains an important and active field of research. An implicit finite difference scheme combined with fixed-point iterations was proposed for two-asset jump-diffusion models in [9]. Operator-splitting methods and various-time stepping schemes were studied in [4]. Wavelet-based methods have also been employed for multi-dimensional models, for example, in [7, 11, 13]. In [7], the wavelet-Galerkin method was used for the two-asset Merton model. Compared with [7], the method proposed in this article uses orthogonal wavelet bases and includes transformation into logarithmic prices and drift removal, resulting in a different variational problem.

As already mentioned, the standard methods used for PIDEs typically lead to full matrices. In contrast, the Galerkin method with a wavelet basis leads to matrices that can be closely approximated by sparse matrices, as discussed in [2, 6, 11]. This paper uses the Galerkin method with orthogonal cubic spline wavelets combined with the Crank–Nicolson scheme. The aim is to show that this method is suitable and efficient for the two-asset Merton model. Its advantages are that the resulting system's matrices are sparse and uniformly conditioned, higher-order convergence is achieved, and a small number of iterations is needed to solve the system to the required accuracy.

2. The two-asset Merton model

The two-asset Merton model is a generalization of the original Merton jump-diffusion model developed in [12]. The model assumes that the price S_τ^i of the asset i at time τ follows the jump-diffusion process

$$\ln\left(\frac{S_\tau^i}{S_0^i}\right) = \left(r - \frac{\sigma_i^2}{2} - \lambda\kappa_i\right)\tau + \sigma_i W_\tau^i + \sum_{k=1}^{N_\tau} Y_k^i, \quad i = 1, 2, \tag{1}$$

see [3, 4]. The parameters in the model have the following interpretation. The parameter r represents the risk-free interest rate, and σ_i is the volatility of asset i corresponding to the diffusion part of the process. The processes W_τ^1 and W_τ^2 are Wiener processes with correlation coefficient ρ. The number of price jumps is represented by the Poisson process N_τ with intensity λ. The random variables Y_k^i are independent and identically distributed for a given i. The parameter κ_i is the expected relative jump size, $\kappa_i = E(\mathrm{e}^{Y_k^i} - 1)$.

The process represented by (1) is a general jump-diffusion process. The Merton model further assumes that $\mathrm{e}^{Y_k^1}$ and $\mathrm{e}^{Y_k^2}$ have the bivariate log-normal distribution with density

$$f(y_1, y_2) = \frac{K}{y_1 y_2} \exp\left(-\frac{\left(\frac{\ln y_1 - \gamma_1}{\delta_1}\right)^2 + \left(\frac{\ln y_2 - \gamma_2}{\delta_2}\right)^2 - 2\hat{\rho}\left(\frac{\ln y_1 - \gamma_1}{\delta_1}\right)\left(\frac{\ln y_2 - \gamma_2}{\delta_2}\right)}{2(1 - \hat{\rho}^2)} \right), \quad (2)$$

where $K = 1/2\pi\delta_1\delta_2\sqrt{1 - \hat{\rho}^2}$. Let T be the maturity date, and let S_i be the price of asset i. Then, $t = T - \tau$ is the time to maturity and the option value $V(S_1, S_2, t)$ satisfies [4, 9, 12]

$$\frac{\partial V}{\partial t} - \mathcal{L}_D(V) - \mathcal{L}_I(V) = 0, \quad S_1, S_2 \in (0, \infty), \, t \in (0, T), \quad (3)$$

where $\mathcal{L}_D$ is a degenerate differential operator defined as

$$\begin{aligned} \mathcal{L}_D(V) = & \frac{\sigma_1^2 S_1^2}{2}\frac{\partial^2 V}{\partial S_1^2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + \frac{\sigma_2^2 S_2^2}{2}\frac{\partial^2 V}{\partial S_2^2} \\ & + (r - \lambda\kappa_1) S_1 \frac{\partial V}{\partial S_1} + (r - \lambda\kappa_2) S_2 \frac{\partial V}{\partial S_2} - (r + \lambda) V \end{aligned} \quad (4)$$

and $\mathcal{L}_I$ is an integral operator given by

$$\mathcal{L}_I(U) = \lambda \int_0^\infty \int_0^\infty V(S_1 y_1, S_2 y_2, t) f(y_1, y_2) \, \mathrm{d}y_1 \, \mathrm{d}y_2. \quad (5)$$

The degeneracy means that for $S_1 = 0$ and $S_2 = 0$, some second-order terms of the differential operator $\mathcal{L}_D$ vanish. The first-order terms of $\mathcal{L}_D$ represent drift.

The initial and boundary conditions depend on the type of option. We consider a European option on the maximum of two assets as an example. This option gives its holder the right, but not the obligation, to sell (for a put option) or buy (for a call option) the most expensive of two underlying assets at the strike price K at maturity T. In this case, the initial condition representing the value of an option at maturity is

$$V(S_1, S_2, 0) = \begin{cases} \max(K - \max(S_1, S_2), 0) & \text{for a put option,} \\ \max(\max(S_1, S_2) - K, 0) & \text{for a call option.} \end{cases} \quad (6)$$

3. Transformation and variational formulation

Two main approaches are typically employed for the numerical solution of PDEs and PIDEs representing option-pricing problems. The first approach moves directly to a variational formulation of the equation with the degenerate differential operator. In this case, the analysis has to be carried out using weighted Sobolev spaces, and error estimates are available only in the norms of these spaces. This approach has been studied for PDE models, for example, in [1], and for the Merton model in [7].

This paper focuses on the second approach, which is based on transformation into logarithmic prices. This has the advantage that it removes the degeneracy of the differential operator. Therefore, standard Sobolev spaces are used for the analysis and error estimates. Various papers have studied this approach, but mainly for PDE models. For PIDEs, we refer to [11, 13].

Hence, we first adjust (3). The degeneracy and drifts are removed using the substitution $U(x_1, x_2, t) = V(S_1, S_2, t)$, where $x_i = \log S_i - (\sigma_i^2/2 + \lambda\kappa_i - r)\, t$ for $i = 1, 2$. Then, the unbounded domain $\mathbb{R}^2$ for (x_1, x_2) is approximated by a bounded domain $\Omega = I \times I$, where I is a chosen finite interval. Finally, as in [11], we set U to zero outside Ω, that is,

$$U(x_1, x_2, t) = 0, \quad (x_1, x_2) \in \mathbb{R}^2 \backslash \Omega, \quad t \in (0, T). \tag{7}$$

Note that this homogeneous Dirichlet boundary condition is artificial and does not describe the actual situation. However, setting this condition simplifies the method and does not affect the solution significantly in the parts of Ω which are not close to the boundary, when Ω is large enough, see [11].

After these adjustments, we obtain an elliptic differential operator

$$\mathcal{D}(U) = \frac{\sigma_1^2}{2}\frac{\partial^2 U}{\partial x_1^2} + \rho\sigma_1\sigma_2\frac{\partial^2 U}{\partial x_1 \partial x_2} + \frac{\sigma_2^2}{2}\frac{\partial^2 U}{\partial x_2^2} - (r + \lambda) U \tag{8}$$

and an integral operator

$$\mathcal{I}(U) = \lambda \iint_{\Omega} U(t_1, t_2, t)\, g(t_1 - x_1, t_2 - x_2)\, \mathrm{d}t_1\, \mathrm{d}t_2, \tag{9}$$

where $g(x_1, x_2) = f(\mathrm{e}^{x_1}, \mathrm{e}^{x_2})\, \mathrm{e}^{x_1}\mathrm{e}^{x_2}$. The transformed equation has the form

$$\frac{\partial U}{\partial t} = \mathcal{D}(U) + \mathcal{I}(U). \tag{10}$$

Let $\langle \cdot, \cdot \rangle$ denote the L^2 inner product. To derive a variational formulation, we define a bilinear form $a = a_D - a_I$, where

$$\begin{aligned} a_D(u, v) = &\frac{\sigma_1^2}{2}\left\langle \frac{\partial u}{\partial x_1}, \frac{\partial v}{\partial x_1} \right\rangle + \rho\sigma_1\sigma_2 \left\langle \frac{\partial u}{\partial x_1}, \frac{\partial v}{\partial x_2} \right\rangle \\ &+ \frac{\sigma_2^2}{2}\left\langle \frac{\partial u}{\partial x_2}, \frac{\partial v}{\partial x_2} \right\rangle + (r + \lambda)\langle u, v \rangle \end{aligned} \tag{11}$$

and $a_I(u, v) = \langle \mathcal{I}(u), v \rangle$. Let $U_0 \in L^2(\Omega)$ be the transformed payoff function. The variational formulation consists in determining the function $U \in L^2(0, T; H_0^1(\Omega))$ such that $\frac{\partial U}{\partial t} \in L^2(0, T; H^{-1}(\Omega))$ and

$$\left\langle \frac{\partial U}{\partial t}, v \right\rangle + a(U, v) = 0 \quad \forall v \in V, \text{ a.e. in } (0, T), \quad U(x_1, x_2, 0) = U_0(x_1, x_2). \tag{12}$$

4. The orthogonal cubic spline-wavelet basis

There is not a universally accepted definition of a wavelet basis in the mathematical literature. Here, we consider a wavelet basis of the space $L^2(I)$, where I is a bounded interval, in the following sense. Let $\mathcal{J}$ be an index set such that $\lambda \in \mathcal{J}$ takes the form $\lambda = (j, k)$ and $|\lambda| = j$ denotes the level. Then, $\Psi = \{\psi_\lambda, \lambda \in \mathcal{J}\}$ is a wavelet basis of $L^2(I)$ if it satisfies the following four conditions:

(i) The set Ψ is an orthogonal basis for $L^2(I)$.

(ii) The basis functions are local, i.e., diam supp $\psi_\lambda \leq C 2^{-|\lambda|}$ for all $\lambda \in \mathcal{J}$.

(iii) The set Ψ has a hierarchical structure,

$$\Psi = \Phi_{j_0} \cup \bigcup_{j=j_0}^{\infty} \Psi_j, \quad \Phi_{j_0} = \{\phi_{j_0,k}, k \in \mathcal{I}_{j_0}\}, \quad \Psi_j = \{\psi_{j,k}, k \in \mathcal{J}_j\}. \tag{13}$$

The functions $\phi_{j_0,k}$ are called scaling functions and the functions $\psi_{j,k}$ are called wavelets.

(iv) The wavelets have vanishing moments, that is, $\langle p, \psi_{j,k} \rangle = 0$, $k \in \mathcal{J}_j$, $j \geq j_0$, for any polynomial p of degree less than $L \geq 1$, where L depends on the wavelet type.

The method in this paper uses orthogonal cubic spline wavelets on the interval with four vanishing moments, recently constructed in [8] using general principles from [10]. The scaling functions in the inner part of the interval are defined as translations and dilations of six generators, which are illustrated in Fig. 1. In addition, boundary functions are constructed near the endpoints of the interval.

Similarly, wavelets in the inner part of the interval are constructed as translations and dilations of six generators, and several boundary functions need to be added. Plots of the wavelet generators are shown in Fig. 2. Since all the basis functions are cubic splines, they are given in closed form and can be handled easily. The resulting basis satisfies the conditions $i) - iv)$ above.

The two-dimensional wavelet basis is constructed using so-called anisotropic tensor products of these one-dimensional bases see [6, 8], that is, it contains the functions $\psi_\lambda = \psi_{\lambda_1} \otimes \psi_{\lambda_2}$, where ψ_{λ_1} and ψ_{λ_2} are univariate basis functions. Then $|\lambda| = \max(\lambda_1, \lambda_2)$ is a level of ψ_λ. Furthermore, we denote $[\lambda] = \min(\lambda_1, \lambda_2)$.

5. The orthogonal wavelet method

Let Ψ^k contain all basis functions up to level k and let $X_k = \operatorname{span} \Psi^k$. Let $U_{k,0}$ be an approximation of $U_0 \in L^2(\Omega)$ in X_k. The wavelet-Galerkin method consists in finding a solution $U_k \in L^2(0, T; X_k)$ such that $\frac{\partial U_k}{\partial t} \in L^2(0, T; X_k')$ and the equation

$$\left\langle \frac{\partial U_k}{\partial t}, v_k \right\rangle + a(U_k, v_k) = 0, \quad U_k(x_1, x_2, 0) = U_{k,0}(x_1, x_2) \tag{14}$$

is satisfied for all $v_k \in X_k$ and almost everywhere in $(0, T)$.

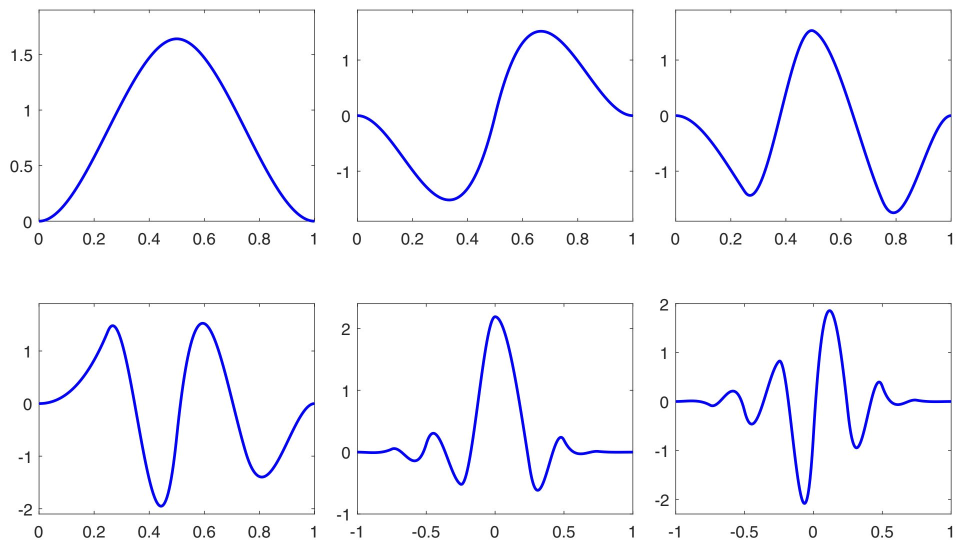

Figure 1: Generators of inner orthogonal cubic spline scaling functions.

The Crank–Nicolson scheme is used for temporal discretization to obtain a fully discrete scheme. Let $M \in \mathbb{N}$, $\tau = T/M$, $t_l = l\tau$ for $l = 0, \ldots, M$, and let $U_k^l(x_1, x_2) = U_k(x_1, x_2, t_l)$. The aim is to find a solution U_k^l of the equation

$$\frac{\langle U_k^{l+1}, v_k \rangle}{\tau} - \frac{\langle U_k^l, v_k \rangle}{\tau} + \frac{a\left(U_k^{l+1}, v_k\right)}{2} + \frac{a\left(U_k^l, v_k\right)}{2} = 0, \quad U_k^0 = U_{k,0} \tag{15}$$

for all $v_k \in X_k$.

Now, we expand the solution U_k^l in the basis Ψ^k,

$$U_k^l = \sum_{\psi_\lambda \in \Psi^k} \left(c_k^l\right)_\lambda \psi_\lambda, \tag{16}$$

set $v_k = \psi_\mu$, and substitute it into (15).

Let $\mathbf{G}^k$ and $\mathbf{K}^k$ be matrices corresponding to the differential and integral terms, respectively, defined as

$$\mathbf{G}_{\mu,\lambda}^k = \frac{\langle \psi_\lambda, \psi_\mu \rangle}{\tau} + a_D(\psi_\lambda, \psi_\mu), \quad \mathbf{K}_{\mu,\lambda}^k = a_I(\psi_\lambda, \psi_\mu), \quad \psi_\lambda, \psi_\mu \in \Psi^k. \tag{17}$$

Furthermore, let the vector $\mathbf{f}_k^l$ be defined as

$$\left(\mathbf{f}_k^l\right)_\mu = \frac{\left(U_k^l, \psi_\mu\right)}{\tau} - \frac{a\left(U_k^l, \psi_\mu\right)}{2}, \quad \psi_\mu \in \Psi^k. \tag{18}$$

Then, for $l = 1, \ldots, M$, the column vector $\mathbf{c}_k^{l+1}$ of coefficients $\left(c_k^{l+1}\right)_\lambda$ is a solution of the linear system $\mathbf{A}^k \mathbf{c}_k^{l+1} = \mathbf{f}_k^l$, where $\mathbf{A}^k = \mathbf{G}^k - \mathbf{K}^k$.

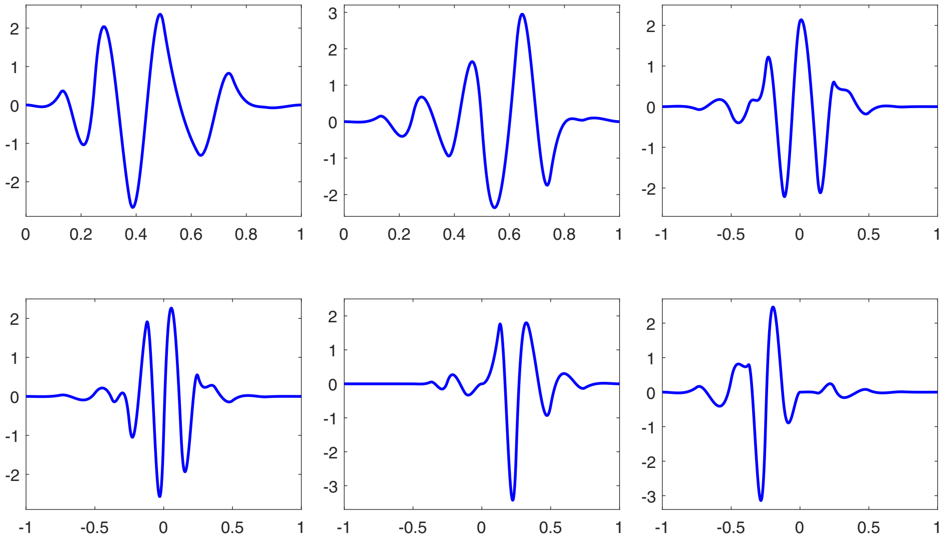

Figure 2: Generators of inner orthogonal cubic spline wavelets.

For the numerical solution of this system, the GMRES method is used. No preconditioning of the system is necessary because the use of orthogonal wavelets ensures that the system is well-conditioned similarly as in [8].

Since the matrix $\mathbf{G}^k$ corresponds to the differential operator, it is sparse. The GMRES method requires multiplying the matrix $\mathbf{G}^k$ with a vector. This can be realized using the Kronecker product of matrices corresponding to one-dimensional differential operators, as detailed in [8]. Due to the L^2 orthogonality of the basis, some of these matrices are identity matrices, which positively affects the resulting condition number of the matrix $\mathbf{G}^k$ and greatly simplifies the computation.

The next theorem, for which a proof can be found in [6], yields a decay estimate for the entries of the matrix $\mathbf{K}^k$.

Theorem 1. *Let ψ_λ and ψ_μ be wavelets with $L = 4$ vanishing moments, as defined in Section 4. Then there exists a real constant C such that*

$$|a_I(\psi_\lambda, \psi_\mu)| \leq C 2^{-(L+1)([\lambda]+[\mu])}. \tag{19}$$

By Theorem 1, the entries of the matrix $\mathbf{K}^k$ decrease exponentially. Thus, many of them are very small and can be set to zero. This process is called compression of the matrix $\mathbf{K}^k$. For the compression strategy and the effect of compression, we refer to [6].

6. Numerical example

Numerical results are presented for a benchmark example from [4, 7]. The market values of European put and call options on the maximum of two assets are evaluated using the proposed method. The advantage of considering options on the maximum of two assets is that, in this special case, the analytic solution is known [3], which enables us to compute the errors of the numerical solution.

The parameters for the options are set as in [4, 7]. The strike price is $K = 100$, the risk-free interest rate is $r = 0.05$, the volatilities of the asset prices are $\sigma_1 = 0.12$ and $\sigma_2 = 0.15$, and the correlation coefficient for the asset prices is $\rho = 0.3$. The parameters for the jump part of the process are $\lambda = 0.6$, $\gamma_1 = -0.1$, $\gamma_2 = 0.1$, $\hat{\rho} = -0.20$, $\delta_1 = 0.17$, and $\delta_2 = 0.13$. The time to maturity is $T = 1$ year. A sufficiently large domain for (S_1, S_2) has to be chosen, therefore we set it to be $(0.1, 5K)^2$. Plots of the resulting functions representing prices of put and call options are shown in Fig. 3. Since artificial boundary conditions are used, the plot of the put option price is shown only in the region $(1, 200)^2$ and the plot of the call option price is shown only in the region $(1, 150)^2$, to avoid the area near $S_1 = 0$ and $S_2 = 0$.

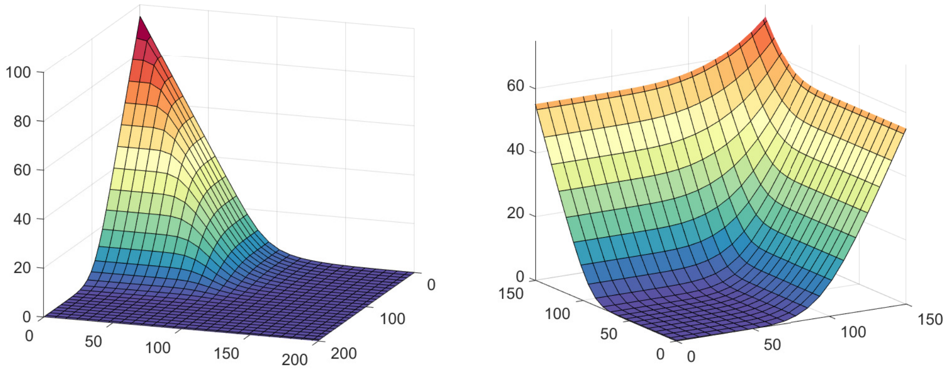

Figure 3: The functions representing prices of European put (left) and call (right) options at one year to maturity.

Table 1 lists the resulting values of options, errors, and numbers of iterations. In this table, N denotes the number of basis functions and M denotes the number of time steps. The values V_P represent the computed prices of options for asset prices (S_1, S_2) equal to $P = (100, 100)$. The corresponding pointwise error is denoted by ρ_P. We set a region of interest as $\text{ROI} = (K/2, 3K/2)^2$ and compute the L^∞ (ROI) error ρ_∞ and the L^2 (ROI) error ρ_2. For the numerical solution, the GMRES method without restart is used. The GMRES iterations are set to stop when the relative residual is less than 10^{-9}. The number of iterations is denoted by it.

Conclusions

A wavelet-based method is proposed for pricing European-style two-factor options under the Merton jump-diffusion model. The first important step of the method is adjusting the original integro-differential equation, including transformation into logarithmic prices, drift removal, and localization. After these adjustments, it is possible

type	N	M	V_P	ρ_P	ρ_∞	ρ_2	it
put	144	2	2.98889	-1.46e0	6.55e0	2.30e-1	7
	576	8	1.30073	-1.19e-1	1.29e0	4.14e-2	7
	2304	32	1.19320	-1.11e-2	2.90e-1	4.61e-3	6
	9216	128	1.18752	-5.46e-3	2.67e-2	6.30e-4	6
call	144	2	20.6315	-4.23e0	2.63e1	3.37e-1	8
	576	8	15.7047	6.93e-1	2.73e0	3.29e-2	7
	2304	32	16.3922	5.51e-3	1.50e-1	1.30e-3	6
	9216	128	16.3923	5.46e-3	6.30e-2	4.99e-4	6

Table 1: Option values V_P for $P = (100, 100)$, pointwise errors ρ_P, L^∞ errors ρ_∞, L^2 errors ρ_2, and numbers of GMRES iterations it.

to remove the degeneracy of the differential operator and derive a variational formulation using standard Sobolev spaces. The variational problem is solved by the Galerkin method with an orthogonal cubic spline-wavelet basis combined with the Crank-Nicolson scheme. It is shown that the method is suitable for the given equation and has many advantages. Due to the vanishing moments of the wavelets, the matrix corresponding to the integral term can be efficiently represented by a sparse matrix, which is not the case in many standard methods. Furthermore, the L^2 orthogonality of the basis results in matrices with uniformly bounded condition numbers even without any preconditioning. Therefore, a sufficiently accurate solution can be obtained using a small number of iterations. Since the basis functions are cubic splines, the method is higher-order convergent. The proposed method could be used to price various options and could be generalized to other jump-diffusion models and options with more assets.

Acknowledgements

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