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Czechoslovak Mathematical Journal, Vol. 24 (1974), No. 4, 522–527

Persistent URL: http://dml.cz/dmlcz/101271

Czechoslovak Mathematical Journal, 24 (99) 1974, Praha

QUASI-REFLECTIONS AND LIMITS

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(Received November 10, 1972)

0. Introduction. In the theory of reflections the following theorems are well known (HERRLICH [1], 9.1 and 9.2; the basis of the theory of reflections: MITCHELL [2], Chap. V, paragraph 5):

Let $\mathfrak{A}$ be a full, replete and reflective subcategory of the category $\mathfrak{C}$, $R : \mathfrak{C} \to \mathfrak{A}$ a reflector, $E : \mathfrak{A} \to \mathfrak{C}$ the inclusion functor and $D : \mathfrak{M} \to \mathfrak{A}$ a diagram. Then it holds:

0.1. *If $(L, l_M)_{M \in |\mathfrak{M}|}$ is a limit of $E \circ D$, then $L \in |\mathfrak{U}|$ and $(L, l_M)_{M \in |\mathfrak{M}|}$ is a limit of D.*

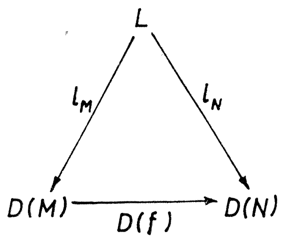
0.2. *The functor E is a limit preserving functor.*

0.3. *The functor R is a colimit preserving functor.*

In this paper the concept of the reflection is generalized to the concept of quasi-reflection and it is shown that in case of $\mathfrak{A}$ being quasi-reflective, Theorems 0.1, 0.2 and 0.3 hold if we consider only the λ-, λ_l- and λ_c-diagrams. (Theorem 0.3 will change only in the case when the domain of the diagram having a colimit in $\mathfrak{C}$ is contained in $\mathfrak{A}$.) In the 3rd paragraph we show that further weakening of the supposition concerning the λ-, λ_l- and λ_c-diagrams is in a certain sense impossible.

We recall the fundamental notions and notation: Let $\mathfrak{C}$ be a category. The class of objects of $\mathfrak{C}$ will be denoted by $|\mathfrak{C}|$. For $X, Y \in |\mathfrak{C}|$ the set of morphisms from X to Y is denoted by $(X, Y)_{\mathfrak{C}}$, 1_X denotes the identity of X.

By a *functor* we shall mean a covariant functor. A functor $D : \mathfrak{M} \to \mathfrak{C}$ is said to be a *diagram* if $\mathfrak{M}$ is a small category. A pair $(L, l_M)_{M \in |\mathfrak{M}|}$ is called a *lower bound* of a diagram $D : \mathfrak{M} \to \mathfrak{C}$ if $l_M \in (L, D(M))_{\mathfrak{C}}$ for each $M \in |\mathfrak{M}|$ and if for each $\mathfrak{M}$-morphism $f \in (M, N)_{\mathfrak{M}}$ the diagram



commutes.

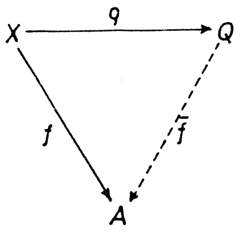
A lower bound $(L, l_M)_{M\in|\mathfrak{M}|}$ is called a *limit* of a diagram $D:\mathfrak{M}\to\mathfrak{C}$ if for each lower bound $(L', l'_M)_{M\in|\mathfrak{M}|}$ of D there exists a unique morphism $l\in(L', L)_{\mathfrak{C}}$ such that for each $M\in|\mathfrak{M}|$ we have $l_M\circ l = l'_M$.

The dual notions: *upper bound and colimit* of D will be denoted by the pair $(l_M, L)_{M\in|\mathfrak{M}|}$.

1. Quasi-reflection. Definition. Let $\mathfrak{A}$ be any subcategory of a category $\mathfrak{C}$ and let $X\in|\mathfrak{C}|$. A morphism $q\in(X, Q)_{\mathfrak{C}}$ is called an *$\mathfrak{A}$-quasi-reflection of X* if it holds:

(a) $Q\in|\mathfrak{A}|$,

(b) for each $A\in|\mathfrak{A}|$ and each $f\in(X, A)_{\mathfrak{C}}$ there exists at least one morphism $\bar{f}\in(Q, A)_{\mathfrak{A}}$ such that the diagram



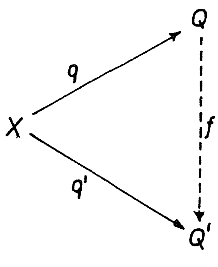
commutes,

(c) $g = 1_Q$ for each $g\in(Q, Q)_{\mathfrak{A}}$ such that $g\circ q = q$.

If for each $X\in|\mathfrak{C}|$ there exists an $\mathfrak{A}$-quasi-reflection of X, then $\mathfrak{A}$ is called a *quasi-reflective subcategory of* $\mathfrak{C}$.

Evidently, it holds:

1.1. *Let $q\in(X, Q)_{\mathfrak{C}}$ and $q'\in(X, Q')_{\mathfrak{C}}$ be $\mathfrak{A}$-quasi-reflections of X. Then there exists a unique isomorphism $f\in(Q, Q')_{\mathfrak{A}}$ such that the diagram*



commutes.

1.2. *Let f be an isomorphism from X to Y where $Y\in|\mathfrak{A}|$. Then f is an $\mathfrak{A}$-quasi-reflection of X.*

1.3. Examples. a) The full subcategory $\mathfrak{A}$ of all complete spaces in the category $\mathfrak{C}$ of all uniform spaces with uniformly continuous mappings is a quasi-reflective subcategory of $\mathfrak{C}$. This subcategory $\mathfrak{A}$ is not a reflective subcategory of $\mathfrak{C}$.

b) Let $\mathfrak{C}$ be the category of all (partially) ordered sets with order-preserving maps and let $\mathfrak{A}$ be the full subcategory of all complete lattices. Then the embedding of each ordered set X in its Mc Neille completion is an $\mathfrak{A}$-quasi-reflection of X that is not an $\mathfrak{A}$-reflection of X.

c) In the category of all semigroups with homomorphisms, the full subcategory of all semigroups with identity element is a quasi-reflective subcategory that is not a reflective one.

d) Similarly, the full subcategory of all ordered sets with the least element of the category of all ordered sets is a quasi-reflective subcategory in this category that is not a reflective one.

2. λ-Diagram. In what follows $\mathfrak{M}$ will denote a small category, $I : \mathfrak{M} \to \mathfrak{M}$ the inclusion functor. For $M_1 \in |\mathfrak{M}|$, $M_2 \in |\mathfrak{M}|$ we put $M_1 \lesssim M_2$ if there exists a morphism $f \in (M_1, M_2)_{\mathfrak{M}}$. The relation $\lesssim$ is a quasi-ordering on the set $|\mathfrak{M}|$. *A component of the category* $\mathfrak{M}$ will mean a component of the connection of $(|\mathfrak{M}|, \lesssim)$ taken as a full subcategory of $\mathfrak{M}$. The inclusion functor from a component $\mathfrak{S}$ of $\mathfrak{M}$ to $\mathfrak{M}$ will be denoted by $I_{\mathfrak{S}}$ $(I_{\mathfrak{S}} : \mathfrak{S} \to \mathfrak{M})$.

Definition. The category $\mathfrak{M}$ will be called a *λ-category* if for each component $\mathfrak{S}$ of $\mathfrak{M}$ the diagram $I_{\mathfrak{S}}$ has a lower bound. If the diagram I has a lower (an upper) bound, we shall call $\mathfrak{M}$ a *λ_l-category* (*λ_c-category*). A diagram $D : \mathfrak{M} \to \mathfrak{C}$ will be called a *λ-diagram*, a *λ_l-diagram*, a *λ_c-diagram*, if the category $\mathfrak{M}$ is a *λ-category*, a *λ_l-category* or a *λ_c-category*, respectively.

Further, let $\mathfrak{A}$ be a full subcategory of $\mathfrak{C}$, $E : \mathfrak{A} \to \mathfrak{C}$ the inclusion functor and $D : \mathfrak{M} \to \mathfrak{A}$ a diagram.

2.1. Theorem. *Let $(L, l_M)_{M \in |\mathfrak{M}|}$ be a limit of the diagram $E \circ D$, let D be a λ-diagram and $q \in (L, Q)_{\mathfrak{C}}$ an $\mathfrak{A}$-quasi-reflection of L. Then L and Q are isomorphic objects in $\mathfrak{C}$.*

Proof. For each component $\mathfrak{S}$ of $\mathfrak{M}$ the diagram $I_{\mathfrak{S}}$ has a lower bound $(S(\mathfrak{S}), h_X)_{X \in |\mathfrak{S}|}$. Further, there exists a morphism $r_{\mathfrak{S}} \in (Q, D(S(\mathfrak{S})))_{\mathfrak{A}}$ such that $r_{\mathfrak{S}} \circ q = l_{S(\mathfrak{S})}$. For $M \in |\mathfrak{S}|$ we put $k_M = D(h_M) \circ r_{\mathfrak{S}}$. Evidently $(Q, k_M)_{M \in |\mathfrak{M}|}$ is a lower bound of D. Therefore there exists $g \in (Q, L)_{\mathfrak{C}}$ such that $l_M \circ g = k_M$ for each $M \in |\mathfrak{M}|$. For $M \in |\mathfrak{M}|$ it holds $l_M = D(h_M) \circ l_{S(\mathfrak{S})} = D(h_M) \circ r_{\mathfrak{S}} \circ q = k_M \circ q = l_M \circ g \circ q$, hence $g \circ q = 1_L$. Then $(q \circ g) \circ q = q$, hence $q \circ g = 1_Q$.

This Theorem implies

Corollary. *Let $\mathfrak{A}$ be a replete, quasi-reflective subcategory of a category $\mathfrak{C}$. Then $\mathfrak{A}$ is product stable (i.e., closed under products) in $\mathfrak{C}$.*

2.2. Theorem. *Let $(L, l_M)_{M \in |\mathfrak{M}|}$ be a limit of the diagram D and let D be a λ_l-diagram. Then $(L, l_M)_{M \in |\mathfrak{M}|}$ is a limit of the diagram $E \circ D$.*

Proof. Let $(X, x_M)_{M \in |\mathfrak{M}|}$ be a lower bound of the diagram I. Then $(D(X), D(x_M))_{M \in |\mathfrak{M}|}$ is a lower bound of the diagram D. Hence there exists $v \in (D(X), L)_{\mathfrak{A}}$

such that $l_M \circ v = D(X_M)$ for each $M \in |\mathfrak{M}|$. Since $l_M \circ (v \circ l_X) = (l_M \circ v) \circ l_X = D(x_M) \circ l_X = l_M$, it holds $v \circ l_X = 1_L$.

Let $(L', l'_M)_{M \in |\mathfrak{M}|}$ be a lower bound of the diagram $E \circ D$. We put $u = v \circ l'_X$. Then we have $l_M \circ u = (l_M \circ v) \circ l'_X = D(x_M) \circ l'_X = l'_M$. If $u' \in (L', L)_{\mathfrak{C}}$ and $l_M \circ u' = l'_M$ for each $M \in |\mathfrak{M}|$, then we have $l_X \circ u' = l'_X$, hence $u' = (v \circ l_X) \circ u' = v \circ (l_X \circ u') = v \circ l'_X = u$.

2.3. Theorem. *Let $(l_M, L)_{M \in |\mathfrak{M}|}$ be a colimit of the diagram $E \circ D$, D a λ_c-diagram, $q \in (L, Q)_{\mathfrak{C}}$ an $\mathfrak{A}$-quasi-reflection of L. Then $(q \circ l_M, Q)_{M \in |\mathfrak{M}|}$ is a colimit of the diagram D.*

Proof. Let $(x_M, X)_{M \in |\mathfrak{M}|}$ be an upper bound of the diagram $I : \mathfrak{M} \to \mathfrak{M}$. Then $(D(x_M), D(X))_{M \in |\mathfrak{M}|}$ is an upper bound of $E \circ D$. Hence there exists $w \in (L, D(X))_{\mathfrak{C}}$ such that $w \circ l_M = D(x_M)$ for each $M \in |\mathfrak{M}|$. We have $(l_X \circ w) \circ l_M = l_X \circ (w \circ l_M) = l_X \circ D(x_M) = l_M$, therefore $l_X \circ w = 1_L$. There exists $\bar{w} \in (Q, D(X))_{\mathfrak{A}}$ such that $\bar{w} \circ q = w$. We put $p = l_X \circ \bar{w}$. Then we have $p \circ q = l_X \circ \bar{w} \circ q = l_X \circ w = 1_L$, hence $(q \circ p) \circ q = q \circ (p \circ q) = q$, consequently $q \circ p = 1_Q$. L and Q are isomorphic objects in $\mathfrak{C}$.

3. Some extensions of $\mathfrak{M}$. Let $\mathfrak{M}$ be a small category. There exist different symbols L, Q, P for which $L \notin |\mathfrak{M}|$, $Q \notin |\mathfrak{M}|$, $P \notin |\mathfrak{M}|$. We shall define three categories $\mathfrak{J} = \mathfrak{J}(\mathfrak{M})$, $\mathfrak{K} = \mathfrak{K}(\mathfrak{M})$ and $\mathfrak{L} = \mathfrak{L}(\mathfrak{M})$ such that $\mathfrak{M}$ is a full subcategory of theirs and $|\mathfrak{J}| = |\mathfrak{M}| \cup \{L, Q\}$, $|\mathfrak{K}| = |\mathfrak{L}| = |\mathfrak{M}| \cup \{L, Q, P\}$.

3.1. The extension $\mathfrak{J}(\mathfrak{M})$. There exist symbols 1_L, 1_Q, q, w and l_M for each $M \in |\mathfrak{M}|$. For $M \in |\mathfrak{M}|$ we put $(L, M)_{\mathfrak{J}} = \{l_M\}$, $(Q, M)_{\mathfrak{J}} = \{[f, w] : f \in (N, M)_{\mathfrak{M}}, N \in |\mathfrak{M}|\}$, $(M, L)_{\mathfrak{J}} = (M, Q)_{\mathfrak{J}} = \emptyset$. Further, we put $(L, Q)_{\mathfrak{J}} = \{q\}$, $(L, L)_{\mathfrak{J}} = \{1_L\}$, $(Q, Q)_{\mathfrak{J}} = \{1_Q\}$, $(Q, L)_{\mathfrak{J}} = \emptyset$. The symbols $1_L, \ldots$ are chosen so that it holds $X, Y, U, V \in |\mathfrak{J}|$, $(X, Y)_{\mathfrak{J}} \cap (U, V)_{\mathfrak{J}} \neq \emptyset \Rightarrow X = U, Y = V$.

We shall define the composition $\circ$ in $\mathfrak{J}$ in the following way: let $M, N, O \in |\mathfrak{M}|$, $f \in (M, N)_{\mathfrak{M}}$, $g \in (N, O)_{\mathfrak{M}}$. We put $f \circ l_M = l_N$, $g \circ [f, w] = [g \circ f, w]$, $[f, w] \circ q = l_N$. The operation $\circ$ for $1_L(1_Q)$ will be defined so that $1_L(1_Q)$ will be the identity of $L(Q)$.

Let $\mathfrak{B}$ be a full subcategory of $\mathfrak{J}$, $|\mathfrak{B}| = |\mathfrak{M}| \cup \{Q\}$, and let $E : \mathfrak{B} \to \mathfrak{J}$, $A : \mathfrak{M} \to \mathfrak{B}$ be the inclusion functors.

Clearly:

3.1.1. *$(L, l_M)_{M \in |\mathfrak{M}|}$ is a lower bound of the diagram $E \circ A$, $q \in (L, Q)_{\mathfrak{J}}$ is a $\mathfrak{B}$-quasi-reflection of L, L and Q are not isomorphic objects in $\mathfrak{J}$.*

3.1.2. *The following conditions are equivalent*:

(a) *for each $M \in |\mathfrak{M}|$ there exists $k_M \in (Q, M)_{\mathfrak{J}}$ such that $(Q, k_M)_{M \in |\mathfrak{M}|}$ is a lower bound of $E \circ A$,*

(b) *A is a λ-diagram,*

(c) *$(L, l_M)_{M\in|\mathfrak{M}|}$ is not a limit of the diagram $E \circ A$.*

Proof. I. If $(L, l_M)_{M\in|\mathfrak{M}|}$ is not a limit of $E \circ A$, then for each $M \in |\mathfrak{M}|$ there exists $k_M \in (Q, M)_{\mathfrak{Z}}$ such that $(Q, k_M)_{M\in|\mathfrak{M}|}$ is a lower bound of $E \circ A$ or there exists $N \in |\mathfrak{M}|$ and for each $M \in |\mathfrak{M}|$ there exists $h_M \in (N, M)_{\mathfrak{M}}$ such that $(N, h_M)_{M\in|\mathfrak{M}|}$ is a lower bound of $E \circ A$. In this latter case we put $k_M = [h_M, w]$. Then $(Q, k_M)_{M\in|\mathfrak{M}|}$ is a lower bound of $E \circ A$. Therefore, it holds (c) → (a).

II. Let (a) hold. Then for each $M \in |\mathfrak{M}|$ there exist $N(M) \in |\mathfrak{M}|$ and $f_M \in \in (N(M), M)_{\mathfrak{M}}$ such that $k_M = [f_M, w]$. For $X, Y \in |\mathfrak{M}|, X \leqq Y$ we have $N(X) = N(Y)$. Therefore, for each component $\mathfrak{S}$ of $\mathfrak{M}$ there exists $N_{\mathfrak{S}}$ such that for $S \in \mathfrak{S}$ we obtain $N(S) = N_{\mathfrak{S}}$. $(N_{\mathfrak{S}}, f_S)_{S\in|\mathfrak{S}|}$ is a lower bound of $I_{\mathfrak{S}} : \mathfrak{S} \to \mathfrak{M}$. Hence (b) holds.

III. The implication (b) → (c) follows from 2.1.

3.1.1 and 3.1.2 imply

3.1.3. *Let $\mathfrak{M}$ be not a λ-category. Then there exist a category $\mathfrak{C}$, a full, replete subcategory $\mathfrak{A}$ of $\mathfrak{C}$, a functor $D : \mathfrak{M} \to \mathfrak{A}$, $L \in |\mathfrak{C}|$, $Q \in |\mathfrak{A}|$, $q \in (L, Q)_{\mathfrak{C}}$, $l_M \in \in (L, D(M))_{\mathfrak{C}}$ for each $M \in |\mathfrak{M}|$ with the following properties:*

a) *$(L, l_M)_{M\in|\mathfrak{M}|}$ is a limit of diagram $E \circ D$ ($E : \mathfrak{A} \to \mathfrak{C}$ is the inclusion functor),*

b) *q is an $\mathfrak{A}$-quasi-reflection of L,*

c) *L and Q are not isomorphic objects in $\mathfrak{C}$.*

3.2. The extension $\mathfrak{K}(\mathfrak{M})$. There exist different symbols 1_L, 1_Q, 1_P, q, r, s, u, v. For each $M \in |\mathfrak{M}|$ let p_M, r_M, s_M, l_M denote any different symbols. For $M \in |\mathfrak{M}|$ we put $(L, M)_{\mathfrak{K}} = \{l_M\}$, $(Q, M)_{\mathfrak{K}} = \{r_M, s_M\}$, $(P, M)_{\mathfrak{K}} = \{p_M\}$, $(M, L)_{\mathfrak{K}} = (M, Q)_{\mathfrak{K}} = = (M, P)_{\mathfrak{K}} = \emptyset$. Further, we put $(Q, L)_{\mathfrak{K}} = \{r, s\}$, $(P, Q)_{\mathfrak{K}} = \{q\}$, $(P, L)_{\mathfrak{K}} = \{u, v\}$, $(L, Q)_{\mathfrak{K}} = (Q, P)_{\mathfrak{K}} = (L, P)_{\mathfrak{K}} = \emptyset$, $(L, L)_{\mathfrak{K}} = \{1_L\}$, $(Q, Q)_{\mathfrak{K}} = \{1_Q\}$ and $(P, P)_{\mathfrak{K}} = = \{1_P\}$. The symbols $1_L, \ldots$ are chosen so that $X, Y, U, V \in |\mathfrak{K}|$, $(X, Y)_{\mathfrak{K}} \cap (U, V)_{\mathfrak{K}} \neq \neq \emptyset \Rightarrow X = U$, $Y = V$.

We shall define the composition $\circ$ of morphisms in $\mathfrak{K}$ in the following way: let $M, N \in |\mathfrak{M}|$, $f \in (M, N)_{\mathfrak{K}}$. We put $f \circ l_M = l_N$, $f \circ r_M = r_N$, $f \circ s_M = s_N$, $f \circ p_M = = p_N$, $l_M \circ r = r_M$, $l_M \circ s = s_M$, $l_M \circ u = l_M \circ v = p_M$, $r \circ q = u$, $s \circ q = v$, $r_M \circ q = = s_M \circ q = p_M$. The operation $\circ$ for $1_L(1_Q, 1_P)$ will be defined so that $1_L(1_Q, 1_P)$ is the identity of $L(Q, P)$.

Let $\mathfrak{B}_l$ be a full subcategory of $\mathfrak{K}$, $|\mathfrak{B}_l| = |\mathfrak{M}| \cup \{L, Q\}$, let $E : \mathfrak{B}_l \to \mathfrak{K}$, $B : \mathfrak{M} \to \to \mathfrak{B}_l$ be the inclusion functors.

Clearly it holds:

3.2.1. *q is a $\mathfrak{B}_l$-quasi-reflection of P, therefore $\mathfrak{B}_l$ is a quasi-reflective subcategory of $\mathfrak{K}$. $(L, l_M)_{M\in|\mathfrak{M}|}$ is not a limit of the diagram $E \circ B$. $(L, l_M)_{M\in|\mathfrak{M}|}$ is a limit of the diagram B iff the diagram B is not a λ_l-diagram.*

3.2.1 implies

3.2.2. *Let $\mathfrak{M}$ be not a λ_1-category. Then there exist a category $\mathfrak{C}$, a full, replete quasi-reflective subcategory $\mathfrak{A}$ of $\mathfrak{C}$, a functor $D : \mathfrak{M} \to \mathfrak{A}$, $L \in |\mathfrak{A}|$, $l_M \in (L, D(M))_{\mathfrak{A}}$ for each $M \in |\mathfrak{M}|$ so that $(L, l_M)_{M \in |\mathfrak{M}|}$ is a limit of the diagram D but not a limit of the diagram $E \circ D(E : \mathfrak{A} \to \mathfrak{C}$ denotes the inclusion functor).*

3.3. The extension $\mathfrak{L}(\mathfrak{M})$. There exist different symbols $1_L, 1_Q, 1_P, q, r, u, v$. For each $M \in |\mathfrak{M}|$ let l_M, p_M, q_M denote any different symbols. For $M \in |\mathfrak{M}|$ we put $(M, L)_{\mathfrak{L}} = \{l_M\}$, $(M, Q)_{\mathfrak{L}} = \{q_M\}$, $(M, P)_{\mathfrak{L}} = \{p_M\}$, $(L, M)_{\mathfrak{L}} = (Q, M)_{\mathfrak{L}} = (P, M)_{\mathfrak{L}} = \emptyset$. Further, $(L, Q)_{\mathfrak{L}} = \{q\}$, $(L, P)_{\mathfrak{L}} = \{r\}$, $(Q, P)_{\mathfrak{L}} = \{u, v\}$, $(P, Q)_{\mathfrak{L}} = (P, L)_{\mathfrak{L}} = (Q, L)_{\mathfrak{L}} = \emptyset$, $(L, L)_{\mathfrak{L}} = \{1_L\}$, $(Q, Q)_{\mathfrak{L}} = \{1_Q\}$, $(P, P)_{\mathfrak{L}} = \{1_P\}$. The symbols $1_L, \ldots$ are chosen so that $X, Y, U, V \in |\mathfrak{L}|$, $(X, Y)_{\mathfrak{L}} \cap (U, V)_{\mathfrak{L}} \neq \emptyset \Rightarrow X = U$, $Y = V$.

The composition $\circ$ for the morphisms of $\mathfrak{L}$ is defined in the following way: for $f \in (M, N)_{\mathfrak{L}}$, $M, N \in |\mathfrak{M}|$ we put $l_N \circ f = l_M$, $p_N \circ f = p_M$, $q_N \circ f = q_M$. Further, we put $q \circ l_M = q_M$, $r \circ l_M = p_M$, $u \circ q_M = v \circ q_M = p_M$ for each $M \in |\mathfrak{M}|$ and $u \circ q = v \circ q = r$. The operation $\circ$ for $1_L(1_Q, 1_P)$ will be defined so that $1_L(1_Q, 1_P)$ is identity of $L(Q, P)$.

Let $\mathfrak{B}_c$ be a full subcategory of $\mathfrak{L}$, $|\mathfrak{B}_c| = |\mathfrak{M}| \cup \{P, Q\}$, let $E : \mathfrak{B}_c \to \mathfrak{L}$, $C : \mathfrak{M} \to \mathfrak{B}_c$ be the inclusion functors.

Then the following assertion holds:

3.3.1. *$q \in (L, Q)_{\mathfrak{B}}$ is a $\mathfrak{B}_c$-quasi-reflection of L, therefore $\mathfrak{B}_c$ is a quasi-reflective subcategory of $\mathfrak{L}$. $(q \circ l_M, Q)_{M \in |\mathfrak{M}|} = (q_M, Q)_{M \in |\mathfrak{M}|}$ is not a colimit of the diagram C. $(l_M, L)_{M \in |\mathfrak{M}|}$ is a colimit of the diagram $E \circ C$ iff the diagram C is not a λ_c-diagram.*

From 3.3.1 we obtain

3.3.2. *Let $\mathfrak{M}$ be not a λ_c-category. Then there exist a category $\mathfrak{C}$, a full, replete subcategory $\mathfrak{A}$ of $\mathfrak{C}$, a functor $D : \mathfrak{M} \to \mathfrak{C}$, $L \in |\mathfrak{C}|$, $Q \in |\mathfrak{A}|$, $q \in (L, Q)_{\mathfrak{C}}$, $l_M \in (D(M), L)_{\mathfrak{C}}$ for each $M \in |\mathfrak{M}|$ with the following properties:*

a) *$(l_M, L)_{M \in |\mathfrak{M}|}$ is a colimit of the diagram $E \circ D$ ($E : \mathfrak{A} \to \mathfrak{C}$ is the inclusion functor),*

b) *q is an $\mathfrak{A}$-quasi-reflection of L,*

c) *$(q \circ l_M, Q)_{M \in |\mathfrak{M}|}$ is not a colimit of the diagram D.*

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