Zdeněk Frolík

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ON PARACOMPACT UNIFORM SPACES

ZDENĚK FROLÍK, Praha

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1. Introduction. By a space we always mean a uniform Hausdorff space. Following [F–H] a space X will be called *paracompact* if each open cover has a σ-discrete (in the uniform sense) refinement. It should be noted that paracompact spaces were introduced in [Fe] under the name "spaces of paracompact type". In [F–H] paracompact spaces were introduced to express an important property o analytic spaces, see 1.4 and 1.6 below. In [Fe] one of the reasons for introduction of paracompact spaces was the following easy but useful observation: if a space is the union of a countable family of its paracompact subspaces then the space itself is paracompact. The aim of this note is to characterize paracompact and separable paracompact spaces in the spirit of the famous Tamano characterization of paracompactness of a topological space X by normality of the product space $X \times \beta X$, see Theorems 1 and 2 in § 2. In this paragraph we recall several facts from [F–H] and add a few observations. Note that by a theorem of A. H. Stone, every metric uniform space is paracompact.

Proposition 1. *A space X is paracompact iff each countably additive open cover has a σ-discrete refinement.*

Proof. "Only if" is self-evident. For "if", let $\mathscr{U}$ be an open cover and let $\mathscr{V}$ be the smallest countably additive collection containing $\mathscr{U}$, and let $\{X_a \mid a \in A\}$ be a σ-discrete refinement of $\mathscr{V}$. For each a in A choose V_a in V containing X_a, and a sequence $\{U_n^a \mid n \in \omega\}$ in $\mathscr{U}$ such that $V_a = \bigcup\{U_n^a \mid n \in \omega\}$. Then $\{X_a \cap U_n^a \mid a \in A, n \in \omega\}$ is a σ-discrete refinement of $\mathscr{U}$.

Proposition 2. *Let X be a uniform space, and let Y be a Lindelöf topological space containing X as a topological subspace. Then X is paracompact iff for each closed set C in Y, $C \cap X = \emptyset$, there exists a σ-discrete cover $\mathscr{M}$ of X such that $\overline{M}^Y \cap C = \emptyset$ for each M in $\mathscr{M}$.*

Proof. For "only if", if C is given, take a σ-discrete refinement $\mathscr{M}$ of the open cover of X consisting of all $U \cap X$ where U is open in Y, and $\overline{U} \cap C = \emptyset$. For "if", given an open countably additive cover $\mathscr{U}$ of X, let

$$C = Y \setminus \bigcup \{U' \mid U \in \mathscr{U}\},$$

where U' is the largest open set in Y intersecting X in U, and take a σ-discrete cover $\mathscr{M}$ of X such that $\overline{M}^Y \cap C = \emptyset$ for each M in $\mathscr{M}$; clearly $\mathscr{M}$ refines $\mathscr{U}$.

Corollary. *A topological space X is Lindelöf iff each compatible uniformity is paracompact.*

Proof. "Only if" is evident from Proposition 2, and for "if" observe that a precompact uniform space is paracompact iff the underlying topology is Lindelöf (in a precompact space, or more generally, in a separable uniform space, σ-discrete means just countable).

Recall that $\operatorname{coz}(X)$ stands for the collection of all cozero sets in X, i.e. the collection of all $\operatorname{coz}(f) = \{x \mid fx \neq 0\}$ with $f \in U(X)$ which is the set of all uniformly continuous functions (real valued) on X.

Proposition 3. *A space X is paracompact iff each open cover of X has a σ-discrete (in the uniform sense!) refinement ranging in* $\operatorname{coz}(X)$.

Proof. "If" is self-evident, and to check "only if" take any open cover $\mathscr{U}$ of X; since $\operatorname{coz}(X)$ is a base for the topology there exists a refinement $\mathscr{V} \subset \operatorname{coz}(X)$ of $\mathscr{U}$. Let $\mathscr{M} = \bigcup\{\mathscr{M}_n\}$ be a refinement of $\mathscr{V}$ with each $\mathscr{M}_n$ discrete. For each n choose a discrete family $\{G_M \mid M \in \mathscr{M}_n\}$ ranging in $\operatorname{coz}(X)$ with $G_M \supset M$ for each M. For each $M \in \mathscr{M}$ choose $V \in \mathscr{V}$ with $M \subset V_M$. Then $\{V_M \cap G_M \mid M \in \mathscr{M}\}$ has the required property.

Corollary. *A topologically fine space is paracompact iff the induced topological space is paracompact.*

Proof. It is well-known that a topological space is paracompact if every open cover has a σ-discrete (in the topological sense) open refinement.

Important examples of paracompact uniform space are obtained from the following (see also [F–H], Proposition 4).

Recall that a family $\{M_a \mid a \in A\}$ is σ-discretely decomposable (abbr. σ-dd) if there exists a family $\{M_{an} \mid a \in A, n \in \omega\}$ such that $M_a = \bigcup\{M_{an} \mid n \in \omega\}$ for each a, and $\{M_{an} \mid a \in A\}$ is discrete for each n. A correspondence $f: X \to Y$ is called σ-dd-preserving if for each σ-dd family $\{X_a\}$ in X, the family $\{f[X_a]\}$ is σ-dd in Y. For the properties of the two notions see [F–H], § 1, § 2. Observe that each σ-discrete family is σ-dd.

Proposition 4. *Let f be an upper semi-continuous σ-dd-preserving correspondence of X onto Y, and let the values of f be Lindelöf. If X is paracompact then so is Y.*

Proof. Let $\mathscr{U}$ be a countably additive open cover of Y, and let $\mathscr{V}$ be the collection of all $V_u = \{x \mid fx \subset U\}$, $U \in \mathscr{U}$. Clearly $\mathscr{V}$ is an open cover of X (each fx is Lindelöf); choose a σ-discrete refinement $\mathscr{M}$ of $\mathscr{V}$. Then $f[\mathscr{M}]$ is a σ-dd refinement of $\mathscr{U}$, and any "σ-dd decomposition" of $f[\mathscr{M}]$ is a σ-discrete refinement of $\mathscr{U}$.

Remark. The assumption that f is σ-dd-preserving may be weakened to σ-dr-preserving (see [F–H], § 2).

Corollary. *If X is paracompact, and if K is compact then $X \times K$ is paracompact.*

Remark. In topology the product of a paracompact space by a discrete space is paracompact. It is a good exercise for the reader to show that this is not true in uniform spaces.

Proof. Take the inverse of the projection $X \times K \to X$ for f in Proposition 4.

The union of a discrete family of cozero sets does not need to be a cozero set. Following [F_3] we define by induction $\mathrm{h}^0 \operatorname{coz}(X) = \operatorname{coz}(X)$, and $\mathrm{h}^\alpha \operatorname{coz}(X)$ consists of σ-discrete unions of members of $\bigcup\{\mathrm{h}^\beta \operatorname{coz}(X) \mid \beta < \alpha\}$. The elements of $\mathrm{h} \operatorname{coz}(X) = \bigcup\{\mathrm{h}^\alpha \operatorname{coz}(X)\}$ are called *hyper-cozero* sets, and the elements of $\mathrm{h}^\alpha \operatorname{coz}(X)$ are called *hyper-cozero sets of class* α. The following result corresponds to the fact from topology that every paracompact space is normal. Of course, by a $\mathrm{h}^\alpha \operatorname{coz}(X)$ function we mean a function on X such that the preimages of open sets are in $\mathrm{h}^\alpha \operatorname{coz}(X)$.

Proposition 5. (a) *If X is paracompact, then*

$$\mathrm{h}^1 \operatorname{coz}(X) = \mathrm{h} \operatorname{coz}(X) = \operatorname{coz}(\mathrm{t_f}X),$$

($\mathrm{t_f}$ denotes the topologically fine coreflection) *and any two disjoint closed sets are separated by a* $\mathrm{h}^1 \operatorname{coz}(X)$ – *function.*

(b) *If X is a separable paracompact space then*

$$\operatorname{coz}(X) = \operatorname{coz}(\mathrm{t_f}X),$$

and any two disjoint closed sets in X are separated by a $\operatorname{coz}(X)$ – *function.*

Proof. (a) It is enough to show that each open F_σ-set G is in $\mathrm{h}^1 \operatorname{coz}(X)$. Let $G = \bigcup\{F_n \mid n \in \omega\}$ with all F_n closed. We shall express G as a σ-discrete union of sets in $\operatorname{coz}(X)$ as follows: For each n let $\mathscr{U}_n \subset \operatorname{coz}(X)$ be a σ-discrete refinement of $\{G, X \setminus F_n\}$ (Proposition 3). Then

$$G = \bigcup\{U \mid U \in \bigcup\{\mathscr{U}_n\},\ U \subset G\}.$$

(b) If X is separable then σ-discrete means just countable, and hence $\mathrm{h} \operatorname{coz}(X) = \operatorname{coz}(X)$.

A space X is called *σ-dd-simple* if $\{\{M_{ab} \mid b \in B_a\}\ a \in A\}$ is a family of σ-dd families, and if the family $\{\bigcup\{M_{ab} \mid b \in B_a\} \mid a \in A\}$ is σ-dd then so is the family

$\{M_{ab} \mid a \in A,\ b \in B_a\}$. Similarly, one defines *discrete-simple* and *σ-discrete-simple.* It is routine to check that X is σ-dd-simple iff $\{\{M_{ab} \mid b \in B_a\}\ a \in A\}$ is a family of discrete families, and if the family $\{\bigcup\{M_{ab} \mid b \in B_a\} \mid a \in A\}$ is discrete then $\{M_{ab}\}$ is σ-dd. Recall from [F−H] the following result. The proof is natural.

Proposition 6. *In a paracompact space each topologically discrete family is σ-dd (but need not be σ-discrete even if the space is metrizable). Every paracompact space is σ-dd simple.*

Remark. Note that a uniform space X is paracompact iff the induced topology is paracompact (i.e. $t_f X$ is paracompact by Corollary to Prop. 3) and X is σ-dd-equivalent to $t_f X$. Since $t_f X$ is finer than X we can say that X is paracompact iff the induced topology is paracompact and each discrete family in $t_f X$ (or in the topology, this is equivalent if the topology is paracompact) is σ-dd in X.

§2. Main results. The classical Tamano Theorem [T] says that each of the following two conditions is necessary and sufficient for a completely regular topological space X to be paracompact:

(i) $X \times \beta X$ is normal.

(ii) For each compact $C \subset \beta X \setminus X$ there exists a continuous function on $X \times \beta X$ which is 0 on $X \times C$ and 1 on the diagonal Δ_X $(= \{\langle x, x\rangle \mid x \in X\})$.

The point of the theorem was that (ii) is sufficient. The aim of this note is to prove a similar characterization of paracompact uniform spaces. The use of βX in Tamano Theorem is not important; it can be replaced by any compactification of X. In what follows by a compactification of a uniform space X we mean any compactification K of the induced topological space; for convenience we always assume $X \subset K$.

First observe that the use of uniformly continuous functions is too restrictive: Denote by $\vee Y$ the Samuel compactification of Y (i.e. the completion of the precompact reflection pY of Y).

Theorem 0. *A uniform space X is compact if (and only if) for some compactification K of X the following holds: for each compact $C \subset K \setminus X$ there exists a uniformly continuous function on $X \times K$ which is 0 on $X \times C$, and 1 on Δ_X.*

Proof. Assume that the condition holds for some K. Take any compact $C \subset K \setminus X$ and show that $C = \emptyset$. It is well-known that (see [Č],

$$\vee(X \times K) = \vee X \times K$$

because K is precompact. Since $X \times C$ and Δ_X are separated by a uniformly continuous function, necessarily

$$\overline{X \times C} \cap \overline{\Delta_X} = \emptyset$$

where the closures are taken in $\vee X \times K$. But $\overline{X \times C} = \vee X \times C$, and the projection of Δ_X into K is K. Thus $C = \emptyset$.

The main results are:

Theorem 1. *A space X is separable and paracompact if (and only if) there exists a compactification K of X such that the following holds:*
for each compact $C \subset K \setminus X$ there exists a

$$G \in \operatorname{coz}(X \times K) \quad \textit{with} \quad \Delta_X \subset G \subset X \times K \setminus X \times C .$$

Theorem 2. *A space X is paracompact if (and only if) there exists a compactification K of X such that:*
for each compact $C \subset K \setminus X$ there exists

$$G \in \mathrm{h}^1 \operatorname{coz}(X \times K) \quad \textit{with} \quad \Delta_X \subset G \subset X \times K \setminus X \times C .$$

For convenience of the reader we list several other characterizations in the following two theorems.

Theorem 1′. *Each of the following properties of a space X is necessary and sufficient for X to be paracompact and separable (i.e. the underlying topology is Lindelöf by Remark to Corollary to Proposition* 1.2)

(a) *For some compactification K of X the following holds:*
if $C \subset K \setminus X$ is compact then there exists a cozero set G in $X \times K$ such that $\Delta_X \subset G \subset (X \times K) \setminus X \times C$.

(b) *For each compactification K of X the condition in* (a) *is satisfied.*

(c) *For some compactification K of X the following holds:*
if $C \subset K \setminus X$ is compact then there exists a coz-*function which is* 1 *on Δ_X and* 0 *on $X \times C$.*

(d) *For each compactification K of X the condition in* (c) *is satisfied.*

(e) *For each compact space K any two disjoint closed sets in $X \times K$ are separated by a* coz-*function.*

Theorem 2′. *Each of the following conditions is necessary and sufficient for a space X to be paracompact:*

(a) *For some compactification K of X the following condition holds:*
if $C \subset K \setminus X$ is compact then there exists a $G \in \mathrm{h}^1 \operatorname{coz}(X \times K)$ such that $\Delta_X \subset$ $\subset G \subset (X \times K) \setminus (X \times C)$.

(b) *For each compactification K of X the condition in* (a) *holds:*

(c) *For some compactification K of X the following condition holds:*
if $C \subset K \times X$ is compact then there exists a h^1 coz-*function which is* 0 *on $X \times C$ and* 1 *on Δ_X.*

(d) *For each compactification K of X the condition in* (c) *holds:*

(e) *For each compact space K, any two disjoint closed sets in $X \times K$ can be separated by a* h^1 coz-*function.*

Beginning of the proof of Theorem 2′. If X is paracompact then (e) holds by Corollary to Proposition 1.4 and by Proposition 1.5 (a). Clearly (e) $\Rightarrow$ (d) $\Rightarrow$ (c) $\Rightarrow$ $\Rightarrow$ (a), and (e) $\Rightarrow$ (b) $\Rightarrow$ (a). It remains to show that (a) is sufficient. This will be done in § 3.

The beginning of the proof of Theorem 1′ is similar.

§3. Proofs. For the proof of Theorem 1 we only need the following

Lemma 1. *If S is separable, then for any space X, $G \in \operatorname{coz}(X \times S)$ if (and only if) G is the union of countably many rectangles $U \times V$, $U \in \operatorname{coz}(X)$, $V \in \operatorname{coz}(S)$.*

Proof of Theorem 1 (the rest of the proof of Theorem 1′). Assume that $G \in$ $\in \operatorname{coz}(X \times K)$, $\Delta_X \subset G \subset X \times K \setminus X \times C$. Write G as the union of a sequence $\{G_n\}$ in $\operatorname{coz}(X \times K)$ such that $\bar{G}_n \subset G$ for each n. By Lemma 1 each G_n is the union of a sequence $\{U_n^k \times V_n^k \mid k \in \omega\}$. Since $\bar{G}_n \cap X \times C = \emptyset$, also $\bar{V}_n^k \cap C = \emptyset$. By Proposition 1.1, X is paracompact and separable.

For the proof of Lemma 1, and also for the proof of Lemma 3 which is used to prove Theorem 2 we state the following useful fact. Recall that a family is called completely $\mathcal{M}$-additive if the union of each subfamily is in $\mathcal{M}$.

Lemma 2. *Let α be a basis for uniform covers of X consisting of completely $\operatorname{coz}(X)$-additive covers. Then $G \in \operatorname{coz}(X)$ iff there exists a sequence $\{\mathscr{V}_n\}$ in α such that*

$$G = \bigcup\{\bigcup\{V \mid V \in \mathscr{V}_n,\ V \subset G\} \mid n \in \omega\}\,.$$

Proof of Lemma 1. The space S has a basis β consisting of countable covers ranging in $\operatorname{coz}(S)$. Given $G \in \operatorname{coz}(X \times S)$, by Lemma 2 there exists a sequence $\{\mathscr{U}_n\}$ of completely $\operatorname{coz}(X)$-additive covers of X, and a sequence $\{\mathscr{W}_n\}$ in β such that G is obtained from $\{\mathscr{U}_n \times \mathscr{W}_n\}$ by the formula in Lemma 2. For each $W \in \mathscr{W}_n$ put

$$\bigcup(W) = \bigcup\{U \mid U \in \mathscr{U}_n,\ U \times W \subset G\}\,.$$

Thus G is the union of all $U(W) \times W$, $W \in U\{\mathscr{W}_n\}$.

Remark. It is easy to see that in a product space $X \times Y$ each cozero set is a countable union of rectangles iff one of the spaces is separable.

Lemma 3. *Assume that S is separable, and X is any space. Then $G \in$ $\in \mathrm{h}^1 \operatorname{coz}(X \times S)$ iff G is the union of a family $\{U_a \times W_a \mid a \in A\}$ such that $\{U_a\}$ is a σ-discrete family ranging in $\operatorname{coz}(X)$, and $\{W_a\}$ ranges in $\operatorname{coz}(S)$.*

Proof of Theorem 2 (the rest of the proof of Theorem 2′). Assume that K is compactification of X such that the condition in Theorem 2 holds. Let $C \subset K \setminus X$ be compact, and choose $G \in \mathrm{h}^1 \operatorname{coz}(X \times K)$ with $\Delta_X \subset G \subset X \times K \setminus X \times C$. By Lemma 3 we can express G as the union of a family $\{U_a \times W_a \mid a \in A\}$ with properties from Lemma 3. For each a express W_a as the union of a sequence $\{W_a^n \mid n \in \omega\}$ such that the closure (in K) of each W_a^n is contained in W_a. Then

$$\{U_a \cap W_a^n \mid a \in A,\ n \in \omega\}$$

is σ-discrete, and the closure of each member is disjoint to C. By Proposition 1.2 the space X is paracompact.

Proof of Lemma 3. Self-evidently "if" holds. Assume that $G \in \mathrm{h}^1 \operatorname{coz}(X \times S)$. Hence $G = \bigcup\{G_b \mid b \in B\}$ where $\{G_b\}$ is a σ-discrete (in $X \times S$) family ranging in $\operatorname{coz}(X \times S) \setminus \{\emptyset\}$. By Lemma 1, each G_b can be written as the union of a sequence $\{U_{bn} \times W_{bn}\}$ with $U_{bn} \in \operatorname{coz}(X)$, $W_{bn} \in \operatorname{coz}(S) \setminus \{\emptyset\}$. Thus $\{U_{bn} \times W_{bn}\}$ is σ-discrete in $X \times S$, and $\bigcup\{U_{bn} \times W_{bn}\} = G$. It remains to show that $\{U_{bn} \mid b \in B,\ n \in \omega\}$ is σ-discrete in X, and this follows from the following general result.

Lemma 4. *Assume that X and S are spaces, and S is separable. If $\{M_a \times N_a \mid a \in A\}$ is discrete in $X \times S$, and $N_a \neq \emptyset$ for each a then $\{M_a\}$ is σ-discrete in X.*

Proof. Take a uniform cover $\mathscr{U} \times \mathscr{V}$ of $X \times S$ which witnesses discreteness of $\{M_a \times N_a\}$ such that $\mathscr{V}$ is countable, say $\mathscr{V} = \{V_n \mid n \in \omega\}$.

Put

$$A_n = \{a \mid N_a \cap V_n \neq \emptyset\}\,.$$

Clearly $\mathscr{U}$ witnesses discreteness of $\{M_a \mid a \in A_n\}$.

§4. λ-paracompact spaces. If Φ is a functor of the category of uniform spaces into itself then a space X is called Φ-paracompact if ΦX is paracompact.

Proposition 1. *If Φ preserves the topology then if X is Φ-paracompact then so is $X \times K$ for any compact space K.*

Proof. The identity mapping $\Phi(X \times K) \to \Phi X \times K$ is uniformly continuous. By Corollary to Proposition 1.4 the product space $\Phi X \times K$ is paracompact, and hence $\Phi(X \times K)$ is paracompact.

We are interested just in the case when Φ is the locally fine coreflection λ because, in that case, λ-paracompact spaces are characterized by separation by hyper-cozero sets in the spirit of Tamano Theorem.

Proposition 2. *If X is λ-paracompact then any two disjoint closed sets in X can be separated by a* h coz*-function on X.*

The proof follows immediately from Proposition 1.5 (a) and the following fact from [F₂, F₃].

Lemma 1. *For any space we have*

$$\mathrm{h} \operatorname{coz}(X) = \operatorname{coz}(\lambda X) = \mathrm{h} \operatorname{coz}(\lambda X)\,.$$

From Proposition 1 and 2 it follows immediately that the condition in the following result is necessary.

Theorem 3. *In order that a space X be λ-paracompact it is necessary and sufficient that there exists a compactification K of X such that the following holds:*

For each compact $C \subset K \setminus X$ there exists a $G \in \mathrm{h}\,\mathrm{coz}(X \times K)$ such that $\Delta_X \subset \subset G \subset X \times K \setminus X \times C$.

For the proof of sufficiency we shall need the following result.

Lemma 2. *If X is σ-dd-simple and if S is separable then*

$$\mathrm{h}\,\mathrm{coz}(X \times S) = \mathrm{h}^1\,\mathrm{coz}(X \times S).$$

Proof. It is enough to check that $\mathrm{h}^1\,\mathrm{coz}(X \times S)$ is closed under the formation of the unions of discrete families. Let $\{U_a \mid a \in A\}$ be a discrete family ranging in $\mathrm{h}^1\,\mathrm{coz}(X \times S)$. By Lemma 3.1, each U_a can be written as a countable union of sets of the form

$$U'_a = \bigcup\{G^a_b \times H^a_b \mid b \in B_a\}$$

where $\{G^a_b \mid b \in B_a\}$ is discrete in X, $G^a_b \in \mathrm{coz}(X)$, $H^a_b \in \mathrm{coz}(S)$. Hence it is enough to show that

$$U' = \bigcup\{U'_a \mid a \in A\} \in \mathrm{h}^1\,\mathrm{coz}(X \times S)$$

whenever $\{U'_a\}$ is discrete, and each U'_a is of the form described above. Take a uniform cover $\mathscr{U} \times \mathscr{V}$ of $X \times S$ which witnesses discreteness of $\{U'_a\}$ in $X \times S$, and let $\mathscr{V}$ be countable, say $\mathscr{V} = \{V_n \mid n \in \omega\}$ and $\mathscr{V} \subset \mathrm{coz}(S)$. Put

$$U'_{an} = U'_a \cap (X \times V_n).$$

It is enough to show that $U'_{an} \in \mathrm{h}^1\,\mathrm{coz}(X \times S)$ for each n. Hence we may and shall assume that $H^a_b \subset V_n$ where $n \in \omega$ is fixed. Also we may and shall assume that $H^a_b \neq \emptyset$ for each a and b. Then it is easy to check that $\mathscr{U}$ witnesses discreteness of

$$\{\bigcup\{G^a_b \mid b \in B_a\} \mid a \in A\}.$$

Since also each $\{G^a_b \mid b \in B_a\}$ is discrete, and X is σ-dd-simple, necessarily $\{G^a_b \mid a \in A, b \in B\}$ is σ-dd, and hence the union $\bigcup\{G^a_b \times H^a_b\}$ is in $\mathrm{h}^1\,\mathrm{coz}(X \times S)$ by Lemma 3.3.

The proof of sufficiency in Theorem 3. Assume that K is a compactification of X such that the condition holds. Take any compact $C \subset K \setminus X$ and the corresponding $G \in \mathrm{h}\,\mathrm{coz}(X \times K)$ with $\Delta_X \subset G \subset X \times K \setminus X \times C$. By Lemma 1

$$G \in \mathrm{h}\,\mathrm{coz}(\lambda X \times K)$$

and by Lemma 2

$$G \in \mathrm{h}^1\,\mathrm{coz}(\lambda X \times K).$$

Hence λX is paracompact by Theorem 2.

For the convenience of the reader let us state a longer list of characterizations of λ-paracompact spaces.

Theorem 3'. *Each of the following conditions is necessary and sufficient for X be λ-paracompact:*

(1) *For each compactification K of X the conditions in Theorem 3 holds.*

(2) *For some, and then each, compactification K of X, any two disjoint closed sets in $X \times K$ are separated by a* h *coz-function.*

(3) *For each compact space K the space $X \times K$ has the property in* (2).

The class of all discrete-simple spaces is coreflective [F_2]; this is easy to show by using l_∞-partitions of the unity. Denote by δ the coreflection on discrete-simple spaces. It is shown in [F_2, F_3] that the identity maps

$$\operatorname{coz} \delta(X) \to \lambda X \to \delta X$$

are uniformly continuous, where coz is the metric fine coreflection. Since a family is σ-dd in Y iff it is σ-dd in coz Y, we obtain immediately:

Proposition 3. *A space is λ-paracompact iff it is δ-paracompact.*

Using the results of [F_2] one could perhaps describe the spaces characterized by separation of h^α coz-functions. Here we were using the categorial product. Very interesting situation appears if we consider the Semi-uniform product of Isbell. Then the conditions (i) and (ii) in Tamano Theorem are usually not equivalent. This situation will be considered elsewhere. For a survey see the lecture on Leningrad international topological Conference 1982, to appear in Lecture notes in Mathematics. In particular, uniformly paracompact spaces introduced in [R] are considered, and a space X is paracompact iff $m_1 X$ is uniformly paracompact. Here $m_1 X$ is the space which has all σ-discrete coz (X)-covers for a basis of all uniform covers.

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Author's address: 115 67 Praha 1, Žitná 25, ČSSR (Matematický ústav ČSAV).