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In memoriam RNDr. František Zítek, CSc

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NEWS AND NOTICES

IN MEMORIAM RNDr. FRANTIŠEK ZÍTEK, CSc.

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It is with deep regret that we record the sudden and premature death of RNDr. František Zítek, CSc., head of the Department of Probability Theory and Mathematical Statistics of the Mathematical Institute of the Czechoslovak Academy of Sciences, who died in Prague on November 18, 1988.

F. Zítek was born on July 19, 1929 at Velké Hamry, a village in Northern Bohemia. After attending the elementary and secondary school in Chotěboř he continued his studies at Lycée Carnot in Dijon (France) in the years 1945–1947. His interests were then divided among philosophy, linguistics and mathematics. He made his first choice by matriculating at Philosophical Faculty (Faculty of Humanities) of Charles University – subjects philosophy and French. Nevertheless, his interests at the Faculty were not restricted to these two fields. His desire for exactness in thinking and expressing himself was not always satisfied, which led him finally to the decision to join the Faculty of Science of Charles University in 1949. However, he did not give up his interest in philosophy and, the more so, in linguistics. He majored in mathematical statistics and probability theory. Immediately after graduating from the University in 1952 he won a scholarship of the Czechoslovak Ministry of Education for a one-year stay in Wroclaw (Poland). He worked there in the seminars of Prof. Steinhaus (applied mathematics) and Prof. Marczewski (stochastic processes). Thanks to his talent for languages, he managed to learn Polish so well that our common Polish friends found his Polish perfect and free of any accent.

After returning from Poland, Zítek started to work in the Mathematical Institute of the Czechoslovak Academy of Sciences in Prague. He defended his CSc. thesis *Random functions with independent increments and stochastic differential equations* in 1957. His supervisor was Academician Josef Novák. However, let us adjourn the description of his contribution to mathematics for a moment. F. Zítek devoted much energy to activities connected with the administration and organization of the Mathematical Institute. He became head of the Department of Probability Theory and Mathematical Statistics in 1972, holding the office till his unexpected death. Moreover, he was Vice-Director of the Institute for the period 1981–1984, member of the Scientific Board for Mathematics of the Academy and of several other committees. In particular, we should mention his work as Chief Editor of the Časopis pro pěstování matematiky (Journal for Cultivation of Mathematics), which he was doing for many years.

Dr. Zítek was also a very good teacher. Under his supervision, more than thirty students graduated from the University, and five prepared their CSc. theses under his direction.

In spite of his modesty, F. Zítek was awarded several distinctions. Let us mention only the Silver Bernard Bolzano Plaque of the Czechoslovak Academy of Sciences for merits in mathematical sciences (1979) and the medal of the Faculty of Mathematics and Physics, Charles University (1978).

The scientific work of František Zítek concerned predominantly three fields of mathematics – stochastic differential equations, queueing theory and weak convergence of probability measures. His interpretation of the notion of a stochastic differential equation stemmed from the approach of P. Lévy: it consisted in determining the distributions of the corresponding random function avoiding the investigation of its trajectories. The crucial point here is the definition of a stochastic derivative. In [10] the derivative of a random function $X = \{X(t)\}$ is considered to be a random function $Z = \{Z(t)\}$ such that $(\partial/\partial t)\,\psi_X(t, s) = \psi_Z(t, s)$, where the ψ-function of the random function X or Z is the logarithm of the characteristic function with the argument s of the random variable $X(t)$ or $Z(t)$, respectively. Another way to define the stochastic differential equation led to the consideration of random functions of an interval and to the theory of the Burkill integral of these functions. A random function X of an interval is a transformation which assigns a random variable $X(I)$ to each interval $I \subset K$, where K is a given interval on the real line. Attention is paid mainly to the case that the variables $X(I)$ have infinitely divisible distributions and $X(I_j)$ are independent for an arbitrary finite system of disjoint intervals $\{I_j\}$. Each division $\mathscr{D} = \{I_j\}$ of the interval K into intervals I_j generates a sum $S(\mathscr{D}, K, X) = \sum_{I_j \in \mathscr{D}} X(I_j)$. The corresponding integral $\int_K X$ can be defined in the usual way as a certain limit of the random variables $S(\mathscr{D}_n, K, X)$ for $n \to \infty$. Zítek studied convergences in distribution and in probability and considered all sequences $\{\mathscr{D}_n\}$ such that the norms of the divisions $\mathscr{D}_n$ tend to zero as $n \to \infty$ [12] or such that (roughly speaking) $\mathscr{D}_{n+1}$ is a refinement of $\mathscr{D}_n$ [27].

In this way we obtain four types of the integral. The papers by Zítek concern mainly the one which corresponds to the case of the convergence of $S(\mathscr{D}_n, K, X)$ in distribution provided the norms of the divisions $\mathscr{D}_n$ tend to zero. This integral is called the (BB)-*integral* (Bernoulli-Burkill). The papers [12], [13], [17] and [20] investigate conditions for the integrability of the function X on K (i.e. the existence of $Z(I) = \int_I X$ for all intervals $I \subset K$), properties of the indefinite integral Z of X, connections with the Burkill integral (under certain conditions the mathematical expectation of the (BB)-integral of the function X equals the Burkill integral of a non-random function which attains at each $I \subset K$ the value of the mathematical expectation of the random variable $X(I)$) and relations between the integrability of X

and its continuity and differentiability. We have arrived at the second approach to the notion of a stochastic differential equation $\delta X(t) = Z(t)$. For each $t \in K$ and $h > 0$ such that $t + h \in K$, let a random variable $Z([t; t + h))$ be given and let Z be (BB)-integrable on K. A solution to the stochastic differential equation is defined to be a random function $X = \{X(t); t \in K\}$ such that $X(t)$ has the same distribution as (BB)- $\int_{[0;t)} Z$ for all $t \in K$. Relations between the two approaches to the stochastic differential equations discussed above can be found in [20].

The paper [9] represents a link with the second (and probably the most important) field of Zítek's research – the queueing theory. Namely, the functions of an interval are a suitable tool for the description of the process of arrivals of customers (called a stream) – e.g. the average number $M(I)$ of customers arriving within a time interval I or the probability $\lambda(I)$ that at least one customer arrives during I. For regular streams (i.e. if $M([0; t))$ is continuous in t) it is shown in [9] that the equality of $M(K)$ and the Burkill integral $\int_K \lambda$ for each $K \subset [0; \infty)$ is the necessary and sufficient condition for customers to arrive one by one. The analysis of the so-called singular streams is given in [7]. They concern the case when customers can arrive only at certain discrete instants. This situation is frequently met in the everyday life – e.g. at exit escalators from underground stations which are exploited just after arrivals of trains.

Let us now discuss the concrete systems studied in [32], [33], [35], [36], [37] and [40]. Their common feature is an investigation of the steady-state behaviour of systems with Poisson stream of customers, exponentially distributed service time (except for [35] where a general distribution of service time is considered) and finite number of servers. Zítek paid attention to the cases when the mutual order of customers at the output of the system differs from that at the input. It is so in [32] where it comes true – even when the FIFO (first in first out) discipline is applied – if the service time of a customer is much shorter than the service times of those who came before him. Another example can be found in [35], [36], [37] and [40] where an arriving customer is sent to the front of the queue with probability p or to its rear with probability $1 - p$. Moreover, it is assumed in [36] that the number of waiting customers is limited. Hence, a possibility arises that a customer may never be served because when he occupies the last place in the queue a new customer can come and the former is displaced from the system without being served.

The book *Ztracený čas* (*Lost time*) [30] deserves a particular note. It was the first book on queueing theory written in Czech and made this subject rather popular. Moreover, its chapter 2 can be regarded as an introduction to the theory of stochastic processes, in particular of homogeneous Markov processes with discrete state-space. In the rest of the book, the reader can find the analysis of several models of queueing systems.

Zítek found a fruitful inspiration for the research in the third field mentioned above, i.e. the study of weak convergence of probability measures, in the book by H. Bergström *Limit Theorems for Convolutions*, Almqvist & Wiksell, Stockholm, and J. Wiley & Sons, New York (1963). Consider a double sequence of random variables $\{X_{nk}\}_{k=1}^{k_n}$ having distribution functions F_{nk}. Let $X_{n1}, X_{n2}, \ldots, X_{nk_n}$ be independent for each n and let $\lim_{n\to\infty} k_n = \infty$. Put

$$X_n = \sum_{k=1}^{k_n} X_{nk} .$$

Denote by F_n and E respectively the distribution functions of X_n and of a random variable Y equal to zero with probability 1. H. Bergström proved that the problem of weak convergence of distribution functions $\{F_n\}_{n=1}^{\infty}$ can be converted into the investigation of the convergence of functions

$$H_n(x) = \sum_{k=1}^{k_n} F_{nk}(x) - k_n E(x) .$$

Namely, conditions for the weak convergence of $\{F_n\}_{n=1}^{\infty}$ can be expressed in terms of the so-called Gaussian norm of functions H_n and $H_{nk} = F_{nk} - E$. The Gaussian norm $\{{}_G\|f\|_\sigma;\ \sigma > 0\}$ is defined on the set of functions of bounded variation on R by

(1) $$_G\|f\|_\sigma = \sup_{x\in R} \left| \lim_{y\to-\infty} f(y) + \int_{-\infty}^{\infty} \Phi\left(\frac{x-t}{\sigma}\right) \mathrm{d}f(t) \right| ,$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\tfrac{1}{2}t^2\right) \mathrm{d}t$$

is the distribution function of the normal distribution $N(0, 1)$ with zero mean and variance equal to 1. It is clear that the formula (1), containing a convolution of the function f and the distribution function of $N(t, \sigma^2)$, is not easy to handle. That was why Zítek introduced another norm called a Fourier norm which was based on a standard and well-known notion of the characteristic function of a distribution. In [28] he defined

$$_F\|f\|_\sigma = \left| \lim_{y\to-\infty} f(y) \right| + \sup_{|s|\leqq 1/\sigma} \left| \int_{-\infty}^{\infty} \mathrm{e}^{ist} \, \mathrm{d}f(t) \right|$$

on the set of functions of bounded variation on R and showed that the weak convergence of distribution functions $f_n \to f$ is equivalent to the equality $\lim_{n\to\infty} {}_F\|f_n - f\|_\sigma = 0$ for all positive σ. Other properties of the Fourier norm, analogous to those found by H. Bergström in case of the Gaussian norm, are given in [41]. An important question is whether the Gaussian and Fourier norms are equivalent. It is proved in [31] that there exists a positive constant C such that for all functions f of bounded variation on R which are non-decreasing on the intervals $(-\infty; 0)$ and $(0; \infty)$ (note that these demands are met e.g. by the functions H_n and H_{nk} introduced above) the inequalities ${}_F\|f\|_\sigma \leqq C \, {}_G\|f\|_\sigma$ and ${}_G\|f\|_\sigma \leqq C \, {}_F\|f\|_\sigma$ are true for each positive σ. On the other hand, an example in [28] shows that these norms are not equivalent on the set of all functions of bounded variation on R. Indeed, there exists a sequence $\{f_n\}_{n=1}^{\infty}$ of such functions for which $\lim_{n\to\infty} {}_F\|f_n\|_\sigma = 0$ while $\{{}_G\|f_n\|_\sigma\}_{n=1}^{\infty}$ is bounded from below by a positive constant. Let us mention that Zítek inspired some of his colleagues to do the research in this field.

A survey of Zítek's research interests would not be complete without mentioning his work in linguistics – [24], [25], [26], graph theory [38] and mathematical statistics [1], [2], [3], [43] and [44].

Mathematical Olympiad, the competition of secondary-school students in mathematics, was an integral part of Zítek's activity for more than two decades. It could even be said that it was his hobby. He was the author of several problems used in Mathematical Olympiads, mainly those of geometric nature. He contributed to the edition Škola mladých matematiků (School of young mathematicians) by the booklet [34] and took part in preparation of several year-books of the Mathematical Olympiads. At the post of Chairman of the Central Committee of the Czechoslovak Mathematical Olympiad (since 1983), he paid much attention to the organization of this competition also in elementary schools and to the establishment of its new branch oriented to programming.

Zítek's activity in the Mathematical Olympiad was not restricted only to the national level. Since 1963 he was almost every year Chairman or Vice-Chairman of the Czechoslovak team at the International Mathematical Olympiads held in various countries in Europe, America and Australia. At the 25th Olympiad that took place in Prague in 1984 he served as Chairman of the international jury. His diplomatic tact together with his extensive knowledge of languages contributed considerably to the smooth course of the competition.

Dr. Zítek was a remarkable man in many ways. As we have tried to show, he made a substantial contribution to Czechoslovak mathematics. Neither can we disregard his part in forming the scientific profile of the Mathematical Institute of the Czechoslovak Academy of Sciences. F. Zítek will be remembered for his enthusiasm in mathematical research, the wide scope of his knowledge, as well as for his ability of communicating with other people and understanding both their professional and personal problems.

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