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INFINITE STRIP IN PLANE THEORY OF ELASTICITY

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1. Introduction. Biharmonic problems can be solved effectively by reducing them to a Hilbert problem which can be solved by using MUSKHELISHVILI'S [1] method. But it is difficult to have a solution from this method when the region under consideration is multiply connected or an infinite strip. Starting with Muskhelishvili's general equation a method was developed by SHERMAN (c.f. [1], § 102) to solve the plane elastic problem in a multiply connected region. Later MILNE-THOMSON [2] further extended Muskhelishvili's method to solve biharmonic problems in a region bounded by concentric circles. BUCHWALD [3] used Milne-Thomson's idea and applied it

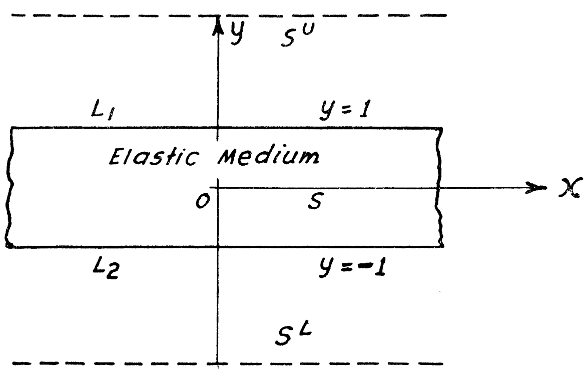

Fig. 1.

to infinite strip. He also mentioned in this paper in detail the references of the work investigated by different authors. The strip problem can also be solved by using Fourier Integral method. HOWLAND [4] used this method under different boundary conditions. A useful account of the Fourier method can be had in SNEDDON'S [5] book.

In the present paper we have developed Sherman's and Milne-Thomson's idea to solve a Hilbert problem, the solution of which depends on the solution of a difference-differential equation which can be solved by using Fourier transform in a complex plane. In the last part of our work we have applied this method to solve strip problem under three types of different boundary conditions.

2. In this section we shall discuss some elementary properties of the reflection principle. Let S be the region bounded by the lines $y = \pm 1$ the lines $y = 1$ and $y = -1$ are denoted by L_1 and L_2 and they do not belong to the region S. These lines are the boundaries of S. Here we also define two regions S^U and S^L by $1 < y < 3$ and $-3 < y < -1$ respectively.

A function $\varphi(z)$ is given to be homolorphic in S. From this function a function $\varphi^U(z)$ can be constructed for z in S^U by the following manner:

$$\varphi^U(z) = \overline{\varphi(z_1)} = \bar{\varphi}(z - 2i) \tag{2.1}$$

where z_1, is the reflection point of z by the line $y = 1$ and these two points are related by the equation

$$z - \bar{z}_1 = 2i \tag{2.2}$$

Further as z moves in S^U, z_1 generates the region S. The function $\varphi^U(z)$ defined by (2.1) is holomorphic in S^U, in fact

$$\begin{aligned}
\overline{\varphi(z_1)} &= u(x, y_1) - iv(x, y_1) \\
&= u(x, 2 - y) - iv(x, 2 - y) \\
&= u_1(x, y) + iv_1(x, y)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u_1}{\partial x} &= \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y_1} \\
&= -\frac{\partial v_1(x, y)}{\partial y_1} = +\frac{\partial v_1}{\partial y}
\end{aligned}$$

Similarly

$$\frac{\partial u_1}{\partial y} = -\frac{\partial v_1}{\partial x}$$

It is also clear from (2.1) that as $z \to t$ on L_1 from S^U, $\varphi^U(z)$ and $\bar{\varphi}(z - 2i)$ approach the same boundary value (provided such value exists), this former from S^U and the latter from S. Conversely if $\varphi^U(z)$ is a function holomorphic in S^U and a function $\varphi(z)$ defined in S by

$$\varphi(z) = \bar{\varphi}^U(z - 2i)$$

for z in S is a holomorphic in S.

In a similar manner in the region S^L a function $\varphi^L(z)$ may be defined by

$$\varphi^L(z) = \bar{\varphi}(z + 2i) \tag{2.3}$$

The function $\varphi^L(z)$ defined by (2.3) is holomorphic in S^L provided $\varphi(z)$ is holomorphic in S. Clearly, part played by $\varphi^L(z)$ and $\varphi(z)$ can be interchanged.

3. Basic equations in plane theory elasticity are given by (cf. [1], p. 451).

$$X_x + Y_y = 2[\Phi(z) + \overline{\Phi(z)}] \tag{3.1}$$

$$Y_y - X_x + 2iX_y = 2[\bar{z}\, \Phi'(z) + \Psi(z)] \tag{3.2}$$

$$2\mu(u + iv) = \varkappa\, \varphi(z) - z\, \overline{\varphi'(z)} - \overline{\psi(z)}\,. \tag{3.3}$$

where $\Phi(z) = \varphi'(z)$, $\Psi(z) = \psi'(z)$ are functions holomorphic in S. In this discussion it will be assumed that the resultant vector (X, Y) of the external forces acting on the boundaries are finite and that the stressed and rotation vanishes at infinity. Under these assumptions for large $|z|$

$$\Phi(z) = \frac{\gamma}{z} + o(1/z) \tag{3.4}$$

$$\Phi'(z) = -\frac{\gamma}{z^2} + o(1/z^2) \tag{3.5}$$

$$\Psi(z) = \frac{\gamma'}{z} + o(1/z) \tag{3.6}$$

$$\varphi(z) = \gamma \log z + o(1) + \text{const}\,. \tag{3.7}$$

$$\psi(z) = \gamma' \log z + o(1) + \text{const}\,. \tag{3.8}$$

where $o(1/z)$ and $o(1)$ are quantities such that

$$|o(1/z)| < \varepsilon/|z|\,, \quad |o(1)| < \varepsilon$$

where ε depends on $|z|$ and tends to zero as $|z| \to \infty$. From (3.1), (3.2) and (3.3) it can be shown that

$$Y_y - iX_y = \Phi(z) + \overline{\Phi(z)} + z\overline{\Phi'(z)} + \overline{\Psi(z)} \tag{3.9}$$

and

$$2\mu(u' + iv') = \varkappa\, \Phi(z) - \overline{\Phi(z)} - z\, \overline{\Phi'(z)} - \overline{\Psi(z)} \tag{3.10}$$

where,

$$u' = \frac{\partial u}{\partial x}\,, \quad v' = \frac{\partial v}{\partial x}$$

If the resultant forces $X = Y = 0$, then $\gamma = \gamma' = 0$ in (3.4) and (3.6).

The function $\Phi(z)$ is holomorphic only in S and is not defined in S^U and S^L. Thus $\Phi(z)$ can be defined in S^U and S^L in an arbitrary manner but here we shall adopt the Muskhelishvili's approach to define $\Phi(z)$ in S^U and S^L viz. $\Phi^U(z)$ is the analytic continuation of $\Phi(z)$ across the boundary L_1 for the unloaded portion of the boundary provided such boundary exists.

Considering (3.9) one can define $\Phi^U(z)$ in S^U by the following equation

$$\begin{aligned}\Phi^U(z) &= -\overline{\Phi(\bar{z}+2i)} - z\,\overline{\Phi'(\bar{z}+2i)} - \overline{\Psi(\bar{z}+2i)} \\ &= -\bar{\Phi}(z-2i) - z\,\bar{\Phi}'(z-2i) - \bar{\Psi}(z-2i)\end{aligned} \tag{3.11}$$

Now if we assume that z is in S, then $\bar{z}+2i$ is in S^U and the above expression can be written as

$$\begin{aligned}\Phi^U(\bar{z}+2i) &= -\overline{\Phi(\bar{\bar{z}})} - (\bar{z}+2i)\,\overline{\Phi'(\bar{\bar{z}})} - \overline{\Psi(\bar{\bar{z}})} \\ &= -\bar{\Phi}(\bar{z}) - (\bar{z}+2i)\,\bar{\Phi}'(\bar{z}) - \bar{\Psi}(\bar{z})\end{aligned}$$

Taking complex conjugate of both sides of the above equation we get

$$\Psi(z) = -\Phi(z) - (z-2i)\,\Phi'(z) - \bar{\Phi}^U(z-2i) \tag{3.12}$$

It is clear from (3.11) and (2.1) that $\Phi^U(z)$ is holomorphic in S^U. Complex conjugate of (3.12) and (3.9) give

$$Y_y - iX_y = \Phi(z) - \Phi^U(\bar{z}+2i) + (z-\bar{z}-2i)\,\overline{\Phi'(z)} \tag{3.13}$$

Similarly from (3.10) and (3.12) we have

$$2\mu(u'+iv') = \varkappa\,\Phi(z) + \Phi^U(\bar{z}+2i) - (z-\bar{z}-2i)\,\overline{\Phi'(z)} \tag{3.14}$$

The above reasoning can be applied to the region S^L to define Φ^L. In the region S^L, $\Phi^L(z)$ can be defined as

$$\Phi^L(z) = -\overline{\Phi(\bar{z}-2i)} - z\,\overline{\Phi'(\bar{z}-2i)} - \overline{\Psi(\bar{z}-2i)}$$

Hence

$$\Psi(z) = -\Phi(z) - (z+2i)\,\Phi'(z) - \bar{\Phi}^L(z+2i) \tag{3.15}$$

and proceeding as before, from (3.9) and (3.10) we get

$$Y_y - iX_y = \Phi(z) - \Phi^L(\bar{z}-2i) + (z-\bar{z}+2i)\,\overline{\Phi'(z)} \tag{3.16}$$

$$2\mu(u'+iv') = \varkappa\,\Phi(z) + \Phi^L(\bar{z}-2i) - (z-\bar{z}+2i)\,\overline{\Phi'(z)} \tag{3.17}$$

Since stresses and displacements given by (3.13), (3.16) and (3.14) and (3.17) are identical. $\Psi(z)$ given by (3.12) and (3.15) must give the same holomorphic function in S.

$$\Phi^U(\bar{z}+2i) - \Phi^L(\bar{z}-2i) + 4i\,\overline{\Phi'(z)} = 0 \tag{3.18}$$

The above difference equation was obtained by Buchwald [3].

4. Application. The result obtained in section 3 will be applied in this section. Here we shall discuss the first and the second fundamental problems of the infinite strip and a third type of problem in which stresses are given in L_1 and displacements are prescribed in L_2.

(a) First fundamental problem. Let

$$Y_y - iX_y = f_1(x + i) \text{ on } L_1 \tag{4a.1}$$

$$= f_2(x - i) \text{ on } L_2 \tag{4a.2}$$

Here it will be assumed that the real and imaginary part of $f_1(x + i)$ and $f_2(x - i)$ satisfy Holder condition on L_1 and L_2 respectively, including the point at infinity and they vanish at infinity like (3.4) when stresses are given on the finite part of the boundaries and if t_j $(j = 1, 2, \ldots, n)$ be the end points, then near the points t_j,

$$|\Phi(z)| < \frac{A}{|z - t_j|^{\alpha}}, \quad 0 \leqq \alpha < 1$$

and A is a constant. Further on L_1 and L_2

$$\lim_{y \to 1} (z - \bar{z} - 2i)\overline{\Phi'(z)} \to 0$$

and

$$\lim_{y \to -1} (z - \bar{z} + 2i)\overline{\Phi'(z)} \to 0$$

Equations (3.13) and (3.16) with the help of (4a.1) and (4a.2) can be written as

$$\Phi(x + i) - \Phi^U(x + i) = f_1(x + i) \text{ on } L_1 \tag{4a.3}$$

and

$$\Phi(x - i) - \Phi^L(x - i) = f_2(x - i) \text{ on } L_2 \tag{4a.4}$$

Here the problem is to find the holomorphic function $\Phi(z)$ which satisfies the boundary conditions (4a.3) and (4a.4) on L_1 and L_2 respectively and on S it satisfies the difference-differential equation (3.18). Our method of approach will be to solve the non-homogeneous Hilbert problem (4a.3) and (4a.4) first and then to determine the unknown holomorphic function we shall use (3.18).

Now, solution of (4a.3) is given by

$$\Phi(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_1(x + i)}{x + i - z}\, d(x + i) + \Phi_1(z) \text{ for } z \text{ in } S, \tag{4a.5}$$

$$\Phi^U(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_1(x + i)}{x + i - z}\, d(x + i) + \Phi_1(z) \text{ for } z \text{ in } S^U \tag{4a.6}$$

where $\Phi_1(z)$ is holomorphic in $S^U + L_1 + S$ but may not be continued analytically across the boundary L_2.

Similarly the solution of the Hilbert problem (4a.4) can be written as

$$\Phi(z) = \frac{1}{2\pi i}\int_{L_2} \frac{f_2(x-i)}{x-i-z}\, \mathrm{d}(x-i) + \Phi_2(z) \text{ for } z \text{ in } S\,, \tag{4a.7}$$

$$\Phi^L(z) = \frac{1}{2\pi i}\int_{L_2} \frac{f_2(x-i)}{x-i-z}\, \mathrm{d}(x-i) + \Phi_2(z) \text{ for } z \text{ in } S^L \tag{4a.8}$$

where $\Phi_2(z)$ is holomorphic in $S + L_2 + S^L$ but may not be analytic across L_1. Since $\Phi(z)$ given by (4a.5) and (4a.7) are identical, we can write

$$\Phi(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_1(x+i)}{x+i-z}\, \mathrm{d}(x+i) + \frac{1}{2\pi i}\int_{L_2} \frac{f_2(x-i)}{x-i-z}\, \mathrm{d}(x-i) + f(z) \quad \text{for } z \text{ in } S\,, \tag{4a.9}$$

$$\Phi^U(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_1(x+i)}{x+i-z}\, \mathrm{d}(x+i) + \frac{1}{2\pi i}\int_{L_2} \frac{f_2(x-i)}{x-i-z}\, \mathrm{d}(x-i) + f(z) \quad \text{for } z \text{ in } S^U\,, \tag{4a.10}$$

$$\Phi^L(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_1(x+i)}{x+i-z}\, \mathrm{d}(x+i) + \frac{1}{2\pi i}\int_{L_2} \frac{f_2(x-i)}{x-i-z}\, \mathrm{d}(x-i) + f(z) \quad \text{for } z \text{ in } S^L \tag{4a.11}$$

where $f(z)$ is holomorphic in $S^U + L_1 + S + L_2 + S^L$. Now in (4a.9), (4a.10) and (4a.11) excepting the only function of $f(z)$, the remaining expressions are known. To determine this unknown holomorphic function $f(z)$ we substitute (4a.9), (4a.10) and (4a.11) in (3.18),

$$f(\bar{z} + 2i) - f(\bar{z} - 2i) + 4i f'(\bar{z}) = X(\bar{z})$$

where $X(\bar{z})$ is a known function of $\bar{z}$. The above expression can also be written as

$$f(z + 2i) - f(z - 2i) + 4i f'(z) = X(z) \tag{4a.12}$$

where z is in S. Now $f(z)$ is regular in the strip $-3 < y < 3$ and $|f(z)| \to 0$ uniformly as $|z| \to \infty$ in the strip $-3 + \varepsilon < y < 3 - \varepsilon$ where ε is an arbitrary positive number, $f(z)$ can be determined from (4a.12) by taking Fourier transform of (4a.12) in the complex plane (cf. Noble [6]). Let

$$F(\alpha) = \frac{1}{(2\pi)^{1/2}}\int_{ic-\infty}^{ic+\infty} f(z)\, e^{-iz\alpha}\, \mathrm{d}z \tag{4a.13}$$

for a given c, $-3 < c < 3$ and any given real α, then we have

$$f(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} F(\alpha)\, e^{iz\alpha}\, d\alpha \tag{4a.14}$$

Hence from (4a.12), we get

$$F(\alpha)\,(e^{-2\alpha} - e^{2\alpha}) - \bar{F}(-\alpha)\,.\,4\alpha = -2X_1(\alpha)$$

where

$$X_1(\alpha) = \frac{-1}{2(2\pi)^{1/2}} \int_{ic-\infty}^{ic+\infty} X(z)\, e^{-iz\alpha}\, dz$$

is known.

The above expression can be written as

$$F(\alpha) \sinh 2\alpha + \bar{F}(-\alpha)\,.\,2\alpha = X_1(\alpha) \tag{4a.15}$$

Taking complex conjugate on both sides and replacing α by $-\alpha$ we have

$$\bar{F}(-\alpha) \sinh 2\alpha + F(\alpha)\,.\,2\alpha = -\bar{X}_1(\alpha) \tag{4a.16}$$

Elimination of $\bar{F}(-\alpha)$ between (4a.15) and (4a.16) yields

$$F(\alpha) = \frac{X_1(\alpha) \sinh 2\alpha + \bar{X}_1(-\alpha)\,.\,2\alpha}{\sinh^2 2\alpha - 4\alpha^2} = \frac{1}{2}\,\frac{X_1(\alpha) + \bar{X}_1(-\alpha)}{\sinh 2\alpha - 2\alpha} + \frac{1}{2}\,\frac{X_1(\alpha) - \bar{X}_1(-\alpha)}{\sinh 2\alpha + 2\alpha}$$

Therefore

$$\begin{aligned} f(z) &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} F(\alpha)\, e^{iz\alpha}\, d\alpha = \frac{1}{2(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{X_1(\alpha) + \bar{X}_1(-\alpha)}{\sinh 2\alpha - 2\alpha}\, e^{iz\alpha}\, d\alpha \\ &\quad + \frac{1}{2(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{X_1(\alpha) - \bar{X}_1(-\alpha)}{\sinh 2\alpha + 2\alpha}\, e^{iz\alpha}\, d\alpha \end{aligned} \tag{4a.17}$$

Thus $\Phi(z)$ is completely known from (4a.17) and (4a.9). Consequently $\Psi(z)$ can be determined from (3.12), (4a.9), (4a.10) and (4a.17). Hence the problem is soluble in case of first fundamental boundary value problem for infinite strip.

(b) Second Fundamental Problem. Let

$$2\mu(u' + iv') = g_1(x + i) \text{ on } L_1 \tag{4b.1}$$

$$= g_2(x - i) \text{ on } L_2 \tag{4b.2}$$

where g_1 and g_2 satisfy same conditions on L_1 and L_2 respectively as f_1 and f_2 do. Moreover the order and behaviours of $\Phi(z)$ and $\Psi(z)$ will be same as before. Now (3.14) and (3.17) with the help of (4b.1) and (4b.2) can be written as

$$\Phi^U(x + i) + \varkappa\, \Phi(x + i) = g_1(x + i) \text{ on } L_1 \tag{4b.3}$$

and

$$\varkappa\, \Phi(x-i) + \Phi^L(x-i) = g_2(x-i) \text{ on } L_2 \tag{4b.4}$$

Solutions of (4b.3) and (4b.4) are given by

$$\Phi^U(z) = \frac{1}{2\pi i}\int_{L_1} \frac{g_1(x+i)}{x+i-z}\,\mathrm{d}(x+i) + \Phi_3(z) \quad \text{for } z \text{ in } S^U\,, \tag{4b.5}$$

$$-\varkappa\, \Phi(z) = \frac{1}{2\pi i}\int_{L_1} \frac{g_1(x+i)}{x+i-z}\,\mathrm{d}(x+i) + \Phi_3(z) \quad \text{for } z \text{ in } S \tag{4b.6}$$

and

$$\varkappa\, \Phi(z) = \frac{1}{2\pi i}\int_{L_2} \frac{g_2(x-i)}{x-i-z}\,\mathrm{d}(x-i) + \Phi_4(z) \text{ for } z \text{ in } S\,, \tag{4b.7}$$

$$-\Phi^L(z) = \frac{1}{2\pi i}\int_{L_2} \frac{g_2(x-i)}{x-i-z}\,\mathrm{d}(x-i) + \Phi_4(z) \text{ for } z \text{ in } S^L \tag{4b.8}$$

where $\Phi_3(z)$ is holomorphic in S and S^U including L_1 and $\Phi_4(z)$ is holomorphic in S and S^L including the boundary L_2. Strictly speaking $\Phi_3(z)$ and $\Phi_4(z)$ are not analytic across the boundaries L_2 and L_1 respectively. Comparing (4b.6) and (4b.7) we can write

$$\varkappa\, \Phi(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{g_1(x+i)}{x+i-z}\,\mathrm{d}(x+i) + \frac{1}{2\pi i}\int_{L_2} \frac{g_2(x-i)}{x-i-z}\,\mathrm{d}(x-i) + g(z) \quad \text{for } z \text{ in } S\,, \tag{4b.9}$$

$$\Phi^U(z) = \frac{1}{2\pi i}\int_{L_1} \frac{g_1(x+i)}{x+i-z}\,\mathrm{d}(x+i) - \frac{1}{2\pi i}\int_{L_2} \frac{g_2(x-i)}{x-i-z}\,\mathrm{d}(x-i) - g(z) \quad \text{for } z \text{ in } S^U\,, \tag{4b.10}$$

$$\Phi^L(z) = \frac{1}{2\pi i}\int_{L_1} \frac{g_1(x+i)}{x+i-z}\,\mathrm{d}(x+i) - \frac{1}{2\pi i}\int_{L_2} \frac{g_2(x-i)}{x-i-z}\,\mathrm{d}(x-i) - g(z) \quad \text{for } z \text{ in } S^L \tag{4b.11}$$

where $g(z)$ is holomorphic in S, S^U and S^L including the boundaries L_1 and L_2. This unknown function $g(z)$ can be found out by substituting (4b.9), (4b.10) and (4b.11) in (3.18) and the required difference-differential equation is

$$-g(\bar{z}+2i) + g(\bar{z}-2i) + \frac{4i}{\varkappa}\,\bar{g}'(\bar{z}) = X_2(\bar{z})$$

where $X_2(\bar{z})$ is a known function of $\bar{z}$. The above expression can be written as

$$-g(z+2i) + g(z-2i) + \frac{4i}{\varkappa}\,\bar{g}'(z) = X_2(z) \tag{4b.12}$$

Multiplying both sides of (4b.12) by $e^{-iz\alpha}$ and integrating we get

$$G(\alpha)\,(e^{2\alpha} - e^{-2\alpha}) - \bar{G}(-\alpha)\,.\,\frac{4\alpha}{\varkappa} = 2X_3(\alpha) \tag{4b.13}$$

where

$$G(\alpha) = \frac{1}{(2\pi)^{1/2}} \int_{ic-\infty}^{ic+\infty} g(z)\, e^{-iz\alpha}\, dz$$

and

$$2X_3(\alpha) = \frac{1}{(2\pi)^{1/2}} \int_{ic-\infty}^{ic+\infty} X_2(z)\, e^{-iz\alpha}\, dz$$

which is a known function of α. The path of integration here is same as before. (4b.13) can be written as

$$G(\alpha) \sinh 2\alpha - \bar{G}(-\alpha)\,.\,\frac{2\alpha}{\varkappa} = X_3(\alpha) \tag{4b.14}$$

complex conjugate of (4b.14) is

$$\bar{G}(-\alpha) \sinh 2\alpha - G(\alpha)\,\frac{2\alpha}{\varkappa} = -\bar{X}_3(-\alpha) \tag{4b.15}$$

The above two equations give

$$G(\alpha) = \frac{1}{2}\,.\,\frac{X_3(\alpha) + \bar{X}_3(-\alpha)}{\sinh 2\alpha + \dfrac{2\alpha}{\varkappa}} + \frac{1}{2}\,.\,\frac{X_3(\alpha) - \bar{X}_3(-\alpha)}{\sinh 2\alpha - \dfrac{2\alpha}{\varkappa}} \tag{4b.16}$$

$$2g(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{X_3(\alpha) + \bar{X}_3(-\alpha)}{\sinh 2\alpha + \dfrac{2\alpha}{\varkappa}}\, e^{i\alpha z}\, d\alpha + \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{X_3(\alpha) - \bar{X}_3(-\alpha)}{\sinh 2\alpha - \dfrac{2\alpha}{\varkappa}}\, e^{iz\alpha}\, d\alpha$$

Hence in this case also the problem is solved completely.

(c) Stresses are given on one boundary and the displacement on the other.

Let

$$Y_y - iX_y = f_3(x + i) \text{ on } L_1 \tag{4c.1}$$

$$2\mu(u' + iv') = g_3(x - i) \text{ on } L_2 \tag{4c.2}$$

The corresponding Hilbert problems are

$$\Phi(x + i) - \Phi^U(x + i) = f_3(x + i) \text{ on } L_1 \tag{4c.3}$$

$$\varkappa\Phi(x - i) + \Phi^L(x - i) = g_3(x - i) \text{ on } L_2 \tag{4c.4}$$

Solutions of the above equations are

$$\Phi^U(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_3(x+i)}{x+i-z}\, d(x+i) + \Phi_5(z) \text{ for } z \text{ in } S^U, \tag{4c.5}$$

$$\Phi(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_3(x+i)}{x+i-z}\, d(x+i) + \Phi_5(z) \text{ for } z \text{ in } S, \tag{4c.6}$$

and

$$\varkappa\, \Phi(z) = \frac{1}{2\pi i}\int_{L_2} \frac{g_3(x-i)}{x-i-z}\, d(x-i) + \Phi_6(z) \text{ for } z \text{ in } S, \tag{4c.7}$$

$$-\Phi^L(z) = \frac{1}{2\pi i}\int_{L_2} \frac{g_3(x-i)}{x-i-z}\, d(x-i) + \Phi_5(z) \text{ for } z \text{ in } S^L. \tag{4c.8}$$

(4c.6) and (4c.7) show that

$$\Phi(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_3(x+i)}{x+i-z}\, d(x+i) + \frac{1}{2\pi i\varkappa}\int_{L_2} \frac{g_3(x-i)}{x-i-z}\, d(x-i) + h(z) \quad \text{for } z \text{ in } S \tag{4c.9}$$

Therefore (4c.5) and (4c.8) can be expressed as

$$\Phi^U(z) = \frac{-1}{2\pi i}\int_{L_1} \frac{f_3(x+i)}{x+i-z}\, d(x+i) + \frac{1}{2\pi i\varkappa}\int_{L_2} \frac{g_3(x-i)}{x-i-z}\, d(x-i) + h(z) \quad \text{for } z \text{ in } S^U \tag{4c.10}$$

and

$$\Phi^L(z) = \frac{\varkappa}{2\pi i}\int_{L_1} \frac{f_3(x+i)}{x+i-z}\, d(x+i) - \frac{1}{2\pi i}\int_{L_2} \frac{g_3(x-i)}{x-i-z}\, d(x-i) - \varkappa\, h(z) \quad \text{for } z \text{ in } S^L \tag{4c.11}$$

The corresponding difference-differential equation is

$$h(\bar{z}+2i) + \varkappa\, h(\bar{z}-2i) + 4i\,\bar{h}'(\bar{z}) = \Psi_1(\bar{z}) \tag{4c.12}$$

where $\Psi_1(\bar{z})$ is the known function of $\bar{z}$

$$H(\alpha)\,(\varkappa e^{2\alpha} + e^{-2\alpha}) - 4\alpha\,.\,\bar{H}(-\alpha) = \Psi_2(\alpha) \tag{4c.13}$$

where

$$H(\alpha) = \frac{1}{(2\pi)^{1/2}}\int_{ic-\infty}^{ic+\infty} h(z)\, e^{-i\alpha z}\, dz$$

$$\Psi_2(\alpha) = \frac{1}{(2\pi)^{1/2}}\int_{ic-\infty}^{ic+\infty} \Psi_1(z)\, e^{-iz\alpha}\, dz$$

complex conjugate of (4c.13) is given by

$$\bar{H}(-\alpha)\,(e^{2\alpha} + \varkappa e^{-2\alpha}) + 4\alpha\,.\,H(\alpha) = \bar{\Psi}_2(-\alpha) \tag{4c.14}$$

Eliminating $\overline{H}(-\alpha)$ between (4c.13) and (4c.14),

$$H(\alpha) = \frac{\Psi_2(\alpha)\,(e^{2\alpha} + \varkappa e^{-2\alpha}) + 4\alpha\,.\,\overline{\Psi}_2(-\alpha)}{(e^{2\alpha} + \varkappa e^{-2\alpha})\,(e^{-2\alpha} + \varkappa e^{2\alpha}) + 16\alpha^2}$$

Therefore,

$$h(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} H(\alpha)\, e^{+iz\alpha}\, d\alpha \tag{4c.15}$$

Thus, $\Phi(z)$ and $\Psi(z)$ are determined by using (4c.15). The procedure is the same as in case (a) and therefore does not need further explanation.

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Souhrn

NEKONEČNÝ PÁS V ROVINNÉ TEORII ELASTICITY

Prasanta Kumar Chaudhuri

V článku je řešena prvá, druhá a smíšená úloha teorie rovinné pružnosti na nekonečném pásu S, ohraničeném přímkami $y = \pm i$. Pomocí analytického prodloužení hledaných funkcí Φ, Ψ do pásů symetrických s S podle přímek $y = \pm$ převádí se úloha na řešení speciální Riemann-Hilbertovy úlohy pro funkci Φ. Pomocí Fourierovy transformace se pak najde explicitní vzorec pro neznámou holomorfní funkci, figurující v obvyklém řešení hořejší okrajové úlohy pomocí Cauchyho integrálu a tím se dostávají explicitní vzorce pro Φ, Ψ a tedy X_x, Y_y, X_y.