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THERMAL STRESSES IN AN INITIALLY STRESSED CIRCULAR CYLINDER WITH A SMOOTH RIGID INSULATING COVER ON THE CURVED SURFACE

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1. INTRODUCTION

While the solution of problems of thermal stresses for isotropic and transversely isotropic bodies has been considered by several research workers in great details, comparatively little work has been done on similar problems when the body is initially stressed. GREEN, RIVLIN and SHIELD (1) and Green and ZERNA [2] considered small deformations superposed on large deformations, all at a constant temperature. By an extension of the above work ENGLAND and Green [3] obtained the general solution of the equations for the small superposed deformation and a steady-state-temperature distribution in a compressible as well as incompressible body in terms of three stress functions. They considered the body to be initially isotropic and small deformations were assumed to be superposed on large deformations, at constant temperature, and obtained the general solutions in the special case when two extension ratios parallel to two rectangular certesian coordinates are equal.

The object of the present paper, though a similar type of problem for an aeolotropic materials has been considered by Das [5], is to find the thermal stresses when the isotropic finite cylinder is initially stressed. The curved lateral surface of the finite cylinder is enclosed in a smooth rigid insulating cover while the plane ends have a prescribed distribution of temperature. Numerical results for the stress $(t_{\theta\theta})_{r=1}$ on the curved surface of the cylinder have been obtained for the particular material known as Mooney-type material when the temperature distributions on the plane ends are either constant or paraboloidal.

2. METHODS OF SOLUTION:

INCOMPRESSIBLE CASE:

We consider the deformation of the body which is such that the state of stress, strain and temperature differs slightly from the state in a known finite deformation of uniform extensions parallel to a set of rectangular cartesian co-ordinate axes.

We suppose that the displacement vector, the stress components and the temperature have the forms

$$V' + \varepsilon V, \quad T' + \varepsilon T, \quad t'_{ij} + \varepsilon t_{ij}$$

where V' is the displacement vector, t'_{ij} are the stress components of the known large deformation, T' is the temperature corresponding to that, ε is a constant small enough for square and higher powers to be neglected; and V, T, t_{ij} are the corresponding vector quantities defining the superposed small deformations. The stress strain relations obtained by Green and England (1961) and Green and Zerna (1954) in the special case where the initial deformation has two equal extension ratios $\lambda_1 = \lambda_2$, the other λ_3 and the body is incompressible, are given by

$$(2) \quad t_{11} = p' + a\frac{\partial u}{\partial x} + b\frac{\partial v}{\partial y} + \omega_1 T, \qquad t_{12} = \tfrac{1}{2}(a-b)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$t_{22} = p' + b\frac{\partial u}{\partial x} + a\frac{\partial v}{\partial y} + \omega_1 T, \qquad t_{23} = d_{44}\frac{\partial v}{\partial z} + d_{55}\frac{\partial w}{\partial y},$$

$$t_{33} = p' + c\frac{\partial w}{\partial z} + \omega_3 T, \qquad t_{31} = d_{44}\frac{\partial u}{\partial z} + d_{55}\frac{\partial w}{\partial x}.$$

where $a, b, c, \ldots$ are functions of t'_{ij}, extension ratios, W and I_i, and p' is an arbitrary scalar function. The functions are given by the relations

$$(3) \quad a = 2t'_{11} - 2\Psi\lambda_1^2\lambda_3^2 - 2p + 2\lambda_1^2(\lambda_1^2 - \lambda_3^2)\{A + B\lambda_1^2(\lambda_1^2 + \lambda_3^2) + F(2\lambda_1^2 + \lambda_3^2)\},$$

$$b = 2\lambda_1^2(\lambda_1^2 - \lambda_3^2)\{\Psi + A + B\lambda_1^2(\lambda_1^2 + \lambda_3^2) + F(2\lambda_1^2 + \lambda_3^2)\},$$

$$c = 2t'_{33} - 2\Psi\lambda_1^2\lambda_3^2 - 2p + 2\lambda_3^2(\lambda_3^2 - \lambda_1^2)\{A + 2B\lambda_1^4 + 3F\lambda_1^2\},$$

$$d_{44} = \lambda_3^2(\Phi + \lambda_1^2\Psi); \quad d_{55} = \lambda_1^2(\Phi + \lambda_1^2\Psi),$$

$$\omega_1 = L\lambda_1^2 + M\lambda_1^2(\lambda_1^2 + \lambda_3^2); \quad \omega_3 = L\lambda_3^2 + 2M\lambda_3^2\lambda_1^2.$$

We also know that, for a material for which the Helmholtz's free-energy function is given by

$$(4) \qquad W = W(I_1, I_2, I_3, T').$$

where T' is the temperature and I_1, I_2, I_3 are the three strain invariants, $A, B, F, \ldots$ $\ldots, \Phi, \Psi, \ldots, t'_{11}, \ldots$ are given by

$$(5) \qquad \Phi = \frac{2}{\sqrt{I_3}}\frac{\partial W}{\partial I_1}; \quad \Psi = \frac{2}{\sqrt{I_3}}\frac{\partial W}{\partial I_2},$$

$$A = \frac{2}{\sqrt{I_3}}\frac{\partial^2 W}{\partial I_1^2}, \quad B = \frac{2}{\sqrt{I_3}}\frac{\partial^2 W}{\partial I_2^2}, \quad C = \frac{2}{\sqrt{I_3}}\frac{\partial^2 W}{\partial I_1\,\partial I_2},$$

$$L = \frac{2}{\sqrt{I_3}}\frac{\partial^2 W}{\partial T'\,\partial I_1}, \quad M = \frac{2}{\sqrt{I_3}}\frac{\partial^2 W}{\partial T'\,\partial I_2}$$

with $I_3 = 1$ for an incompressible body in all above relations. Also

(6) $$t'_{11} = \Phi\lambda_1^2 + \Psi\lambda_1^2(\lambda_1^2 + \lambda_3^2) + p\,, \quad t'_{33} = \Phi\lambda_3^2 + 2\Psi\lambda_3^2\lambda_1^2 + p\,,$$

where p is an arbitrary scalar representing a hydrostatic tension.

Since the deformated body is assumed to be in equilibrium under a steady – state distribution of temperature, the temperature equation is given by

(7) $$\nabla_1^2 T + k^2 \frac{\partial^2 T}{\partial z^2} = 0\,, \quad \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\,.$$

where $k = \sqrt{(\gamma_3/\gamma_1)}$, γ_1, γ_3 are functions of the extension ratios and strain invariants.

When the body is incompressible with two equal extension ratios, we have

(8) $$\lambda_1^2\lambda_3 = 1 \quad \text{and} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

and in this case the general solutions are known to be (cf. 6)

(9) $$u = \frac{\partial}{\partial x}(\varphi_1 + \varphi_2 + \overline{T})\,; \quad v = \frac{\partial}{\partial y}(\varphi_1 + \varphi_2 + \overline{T})\,,$$

$$w = \frac{\partial}{\partial z}(n_1^2\varphi_1 + n_2^2\varphi_2 + k^2\overline{T})\,; \quad p' = \frac{\partial^2}{\partial z^2}(l_1\varphi_1 + l_2\varphi_2) + \mu T,$$

where φ_1, φ_2 are the solutions of

(10) $$\left(\nabla_1^2 + n_i^2 \frac{\partial^2}{\partial z^2}\right)\varphi_i = 0 \quad (i = 1, 2)\,,$$

n_1^2, n_2^2 being the roots of the quadratic equation,

(11) $$n^4 d_{55} + n^2(d_{44} + d_{55} - a - c) + d_{44} = 0\,.$$

Also l_1, l_2 are the roots of the quadratic equation,

(12) $$l^2 d_{55} + l[d_{44} - c - (a - d_{55})^2 + 2d_{44}d_{55}] + \\ + d_{44}[d_{44} - c + d_{44}d_{55}] = 0\,.$$

All the above results have been obtained under the assumption that there is an axisymmetric temperature distribution.

Let us consider a finite cylinder which was initially isotropic and let the plane faces be defined by $z = \pm h$, the curved lateral surface defined by $r = a_1$ being enclosed in a smooth rigid insulating cover while the plane ends have a prescribed temperature distribution. The axis of Z being the axis of symmetry, the position of a typical point may be expressed in terms of cylindrical coordinates (r, θ, Z) and in the case of axis symmetry, the displacement vectors have components (u, o, w) and the non-vanishing components of the stress-tensor will be t_{rr}, $t_{\theta\theta}$, t_{zz} and t_{rz} which are given by

$$(13)\quad t_{rr} = \left(a \frac{\partial^2}{\partial r^2} + b \frac{1}{r}\frac{\partial}{\partial r}\right)(\Phi_1 + \Phi_2 + \overline{T}) + \frac{\partial^2}{\partial z^2}(l_1\Phi_1 + l_2\Phi_2) + (\omega_1 + \mu)\, T,$$

$$t_{\theta\theta} = \left(b \frac{\partial^2}{\partial r^2} + a \frac{1}{r}\frac{\partial}{\partial r}\right)(\varphi_1 + \varphi_2 + \overline{T}) + \frac{\partial^2}{\partial z^2}(l_1\Phi_1 + l_2\Phi_2) + (\omega_1 + \mu)\, T,$$

$$t_{zz} = \frac{\partial^2}{\partial z^2}\{(cn_1^2 + l_1)\,\varphi_1 + (cn_2^2 + l_2)\,\varphi_2 + ck^2\overline{T}\} + (\omega_3 + \mu)\, T,$$

$$t_{rz} = d_{44}\frac{\partial^2}{\partial r\,\partial z}(\varphi_1 + \varphi_2 + \overline{T}) + d_{55}\frac{\partial^2}{\partial r\,\partial z}(n_1^2\varphi_1 + n_2^2\varphi_2 + k^2\overline{T})\,.$$

In the case of axis symmetry, the temperature $T = T(r, z)$ which satisfies the differential equation (7) can be assumed to be in the form

$$(14)\qquad T = \sum_{n=1}^{\infty} A_n \frac{\operatorname{ch} \frac{\alpha_n}{k} z}{\operatorname{ch} \frac{\alpha_n}{k} h} J_0(\alpha_n r)\,,$$

having the following boundary conditions for T:

$$(15)\qquad T = \begin{cases} f(r) & \text{when} \quad 0 < r < b_1\,, \quad z = \pm h\,, \\ 0 & \text{when} \quad b_1 < r < a_1\,, \quad z = \pm h\,, \end{cases}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{when} \quad r = a_1$$

where b_1 denotes the radius of the circle of the heat exposure on $z = \pm h$.

The constants A_n in (14) are the expression coefficients of the function $f(r)$ in a series of Bessel functions of zero order defined by

$$(16)\qquad f(r) = \sum_{n=1}^{\infty} A_n\, J_0(\alpha_n r)$$

assuming that the quantities $\alpha_n a_1$ are the roots of the equation

$$(17)\qquad J_1(\alpha_n a_1) = 0\,.$$

Let us take

$$\bar{T} = \sum_{n=1}^{\infty} A_n F_1(\alpha_n) \frac{\operatorname{ch} \frac{\alpha_n}{k} z}{\operatorname{ch} \frac{\alpha_n}{k} h} J_0(\alpha_n r)\,, \tag{18}$$

where $F_1(\alpha_n)$ is an arbitrary function of α_n given by

$$\alpha_n^2 F_1(\alpha_n) = \frac{k^2(\omega_3 - \omega_1)}{d_{55}(n_1^2 - k^2)(n_2^2 - k^2)} = \text{const} = E' \text{ (say)} \tag{19}$$

and correspondingly

$$\mu = \frac{\omega_1 k^2[(c - d_{44}) - k^2 d_{55}] + \omega_3[k^2(a - d_{55}) - d_{44}}{d_{55}(n_1^2 - k^2)(n_2^2 - k^2)}\,. \tag{20}$$

The solutions of equations (10) are taken in the form

$$\varphi_1 = \sum_{n=1}^{\infty} A_n \left[C_1(\alpha_n) \operatorname{ch} \frac{\alpha_n}{n_1} z + D_1(\alpha_n) \operatorname{sh} \frac{\alpha_n}{n_1} z \right] J_0(\alpha_n r)\,, \tag{21}$$

$$\varphi_2 = \sum_{n=1}^{\infty} A_n \left[M_1(\alpha_n) \operatorname{ch} \frac{\alpha_n}{n_2} z + N_1(\alpha_n) \operatorname{sh} \frac{\alpha_n}{n_2} z \right] J_0(\alpha_n r)\,,$$

where $C_1(\alpha_n)$, $D_1(\alpha_n)$, $M_1(\alpha_n)$ and $N_1(\alpha_n)$ are functions of α_n to be determined from the consideration that the plane faces $Z = \pm h$ should be stress free and, since the curved lateral surface is enclosed by a smooth rigid cover, the tangential stress t_{rz} must vanish on $r = a_1$. Thus the above constants are determined from the boundary conditions,

$$t_{zz} = t_{rz} = 0 \quad \text{when} \quad z = \pm h\,, \tag{22}$$

$$t_{rz} = u = 0 \quad \text{when} \quad r = a_1\,. \tag{23}$$

Inserting T, $\bar{T}$ from (14) and (18) and φ_1, φ_2 from (21) into (13) we have

$$t_{rr} = \sum_{n=1}^{\infty} \left\{ (a - b) \left(C_1 \operatorname{ch} \frac{\alpha_n}{n_1} z + D_1 \operatorname{sh} \frac{\alpha_n}{n_1} z + M_1 \operatorname{ch} \frac{\alpha_n}{n_2} z + \right.\right. \tag{24}$$

$$\left. + N_1 \operatorname{sh} \frac{\alpha_n}{n_2} z + F_1 \frac{\operatorname{ch} \frac{\alpha_n}{k} z}{\operatorname{ch} \frac{\alpha_n}{k} h} \right) \frac{\alpha_n}{r} J_1(\alpha_n r) +$$

$$+\,\alpha_n^2\left[\left(\frac{l_1}{n_1^2}-a\right)\left(C_1\,\mathrm{ch}\,\frac{\alpha_n}{n_1}\,z+D_1\,\mathrm{sh}\,\frac{\alpha_n}{n_1}\,z\right)+\right.$$

$$\left.+\left(\frac{l_2}{n_2^2}-a\right)\left(M_1\,\mathrm{ch}\,\frac{\alpha_n}{n_2}\,z+N_1\,\mathrm{sh}\,\frac{\alpha_n}{n_2}\,z\right)-aF_1\,\frac{\mathrm{ch}\,\frac{\alpha_n}{k}\,z}{\mathrm{ch}\,\frac{\alpha_n}{k}\,h}\right]J_0(\alpha_n r)+$$

$$\left.+\,(\omega_1+\mu)\,\frac{\mathrm{ch}\,\frac{\alpha_n}{k}\,z}{\mathrm{ch}\,\frac{\alpha_n}{k}\,h}\,J_0(\alpha_n r)\right\},$$

$$t_{\theta\theta}=\sum_{n=1}^{\infty}A_n\left\{(b-a)\left(C_1\,\mathrm{ch}\,\frac{\alpha_n}{n_1}\,z+D_1\,\mathrm{sh}\,\frac{\alpha_n}{n_1}\,z+M_1\,\mathrm{ch}\,\frac{\alpha_n}{n_2}\,z+\right.\right.$$

$$\left.+N_1\,\mathrm{sh}\,\frac{\alpha_n}{n_2}\,z+F_1\,\frac{\mathrm{ch}\,\frac{\alpha_n}{k}\,z}{\mathrm{ch}\,\frac{\alpha_n}{k}\,h}\right)\frac{\alpha_n}{r}\,J_1(\alpha_n r)+$$

$$+\,\alpha_n^2\left[\left(\frac{l_1}{n_1^2}-b\right)\left(C_1\,\mathrm{ch}\,\frac{\alpha_n}{n_1}\,z+D_1\,\mathrm{sh}\,\frac{\alpha_n}{n_1}\,z\right)+\right.$$

$$\left.+\left(\frac{l_2}{n_2^2}-b\right)\left(M_1\,\mathrm{ch}\,\frac{\alpha_n}{n_2}\,z+N_1\,\mathrm{sh}\,\frac{\alpha_n}{n_2}\,z\right)-bF_1\,\frac{\mathrm{ch}\,\frac{\alpha_n}{k}\,z}{\mathrm{ch}\,\frac{\alpha_n}{k}\,h}\right]J_0(\alpha_n r)+$$

$$\left.+\,(\omega_1+\mu)\,\frac{\mathrm{ch}\,\frac{\alpha_n}{k}\,z}{\mathrm{ch}\,\frac{\alpha_n}{k}\,h}\,J_0(\alpha_n r)\right\}.$$

$$t_{zz}=\sum_{n=1}^{\infty}A_n\left[\alpha_n^2\left\{\left(\frac{cn_1^2+l_1}{n_1^2}\right)\left(C_1\,\mathrm{ch}\,\frac{\alpha_n z}{n_1}+D_1\,\mathrm{sh}\,\frac{\alpha_n z}{n_1}\right)+\right.\right.$$

$$+\left(\frac{cn_2^2+l_2}{n_2^2}\right)\left(M_1\operatorname{ch}\frac{\alpha_n z}{n_2}+N_1\operatorname{sh}\frac{\alpha_n z}{n_2}\right)+cF_1\frac{\operatorname{ch}\dfrac{\alpha_n z}{k}}{\operatorname{ch}\dfrac{\alpha_n h}{k}}+$$

$$\left.\left.+(\omega_3+\mu)\frac{\operatorname{ch}\dfrac{\alpha_n z}{k}}{\operatorname{ch}\dfrac{\alpha_n h}{k}}\right\}\right]J_0(\alpha_n r)\,.$$

$$t_{rz}=-\sum_{n=1}^{\infty}A_n\left[\alpha_n^2\left\{\left(\frac{d_{44}+n_1^2 d_{55}}{n_1}\right)\left(C_1\operatorname{ch}\frac{\alpha_n}{n_1}z+D_1\operatorname{sh}\frac{\alpha_n}{n_1}z\right)+\right.\right.$$

$$+\left(\frac{d_{44}+n_2^2 d_{55}}{n_2}\right)\left(M_1\operatorname{ch}\frac{\alpha_n z}{n_2}+N_1\operatorname{sh}\frac{\alpha_n z}{n_2}\right)+$$

$$\left.\left.+\left(\frac{d_{44}+k^2 d_{55}}{k}\right)F_1\frac{\operatorname{sh}\dfrac{\alpha_n}{k}z}{\operatorname{ch}\dfrac{\alpha_n}{k}h}\right\}\right]J_1(\alpha_n r)\,,$$

$$u=-\sum_{n=1}^{\infty}A_n\alpha_n\left(C_1\operatorname{ch}\frac{\alpha_n z}{n_1}+D_1\operatorname{sh}\frac{\alpha_n z}{n_1}+M_1\operatorname{ch}\frac{\alpha_n z}{n_2}+N_1\operatorname{sh}\frac{\alpha_n z}{n_2}+\right.$$

$$\left.+F_1\frac{\operatorname{ch}\dfrac{\alpha_n}{k}z}{\operatorname{ch}\dfrac{\alpha_n}{k}h}\right)J_1(\alpha_n r)\,,$$

$$w=\sum_{n=1}^{\infty}A_n\alpha_n\left\{n_1\left(C_1\operatorname{ch}\frac{\alpha_n z}{n_1}+D_1\operatorname{sh}\frac{\alpha_n z}{n_1}\right)+n_2\left(M_1\operatorname{ch}\frac{\alpha_n z}{n_2}+N_1\operatorname{sh}\frac{\alpha_n z}{n_2}\right)+\right.$$

$$\left.+kF_1\frac{\operatorname{sh}\dfrac{\alpha_n}{k}z}{\operatorname{ch}\dfrac{\alpha_n}{k}h}\right\}J_0(\alpha_n r)\,.$$

The boundary conditions (23) are satisfied identically, and the conditions (22) are satisfied if

$$D_1 = N_1 = 0 \tag{25}$$

and

$$\left(\frac{cn_1^2 + l_1}{n_1^2}\right) C_1 \operatorname{ch} \frac{\alpha_n h}{n_1} + \left(\frac{cn_2^2 + l_2}{n_2^2}\right) M_1 \operatorname{ch} \frac{\alpha_n h}{n_2} + cF_1 + \omega_3 + \mu = 0\,, \tag{26}$$

$$\left(\frac{d_{44} + n_1^2 d_{55}}{n_1}\right) C_1 \operatorname{ch} \frac{\alpha_n h}{n_1} + \left(\frac{d_{44} + n_2^2 d_{55}}{n_2}\right) M_1 \operatorname{ch} \frac{\alpha_n h}{n_2} +$$
$$+ \left(\frac{d_{44} + k^2 d_{55}}{k}\right) F_1 \operatorname{th} \frac{\alpha_n h}{k} = 0\,.$$

Equations (26) determine $c_1(\alpha_n)$ and $M_1(\alpha_n)$.

3. TEMPERATURE DISTRIBUTION

i) Let us consider the particular case of constant temperature distribution characterised by

$$\begin{aligned} f(r) &= T_0\,, \quad 0 < r < b_1\,, \\ &= 0\,, \quad b_1 < r < a_1\,. \end{aligned}$$

By virtue of relations (16) and (17) we have

$$A_n = \frac{2T_0 b_1\, J_1(\alpha_n b_1)}{\alpha_n^2 a_1^2 J_0^2(\alpha_n a)}\,. \tag{27}$$

(ii) As the second case we consider the paraboloidal distribution of temperature characterised by

$$\begin{aligned} T &= T_0\left(1 - \frac{r^2}{b_1^2}\right), \quad 0 < r < b_1\,, \\ &= 0\,, \qquad b_1 < r < a_1\,. \end{aligned}$$

Applying Fourier Bessel integral and using relations (16) and (17) we have

$$A_n = \frac{4T_0}{\alpha_n^2 a_1^2\, J_0^2(\alpha_n a_1)} \left[\frac{2}{(\alpha_n b_1)}\, J_1(\alpha_n b_1) - J_0(\alpha_n b_1)\right]. \tag{28}$$

4. NUMERICAL RESULTS

For a Mooney type material, the Helmholtz's free energy function W is given be the relation

$$W = A_1 T_1 (I_1 - 3) + A_2 T_1 (I_2 - 3) ,$$

where A_1 and A_2 are known constants and T_1 is a constant temperature.

From relations (3) and (5), we have

$$\begin{aligned}
\Phi &= 2A_1 T_1 , \quad \Psi = 2A_2 T_1 , \quad A = B = F = 0 , \\
L &= 2A_1 , \quad M = 2A_2 , \\
a &= 2d_{55} = 4\lambda_1^2 (A_1 + A_2 \lambda_1^2)\, T_1 , \\
b &= 4A_2 T_1 \, \lambda_1^2 (\lambda_1^2 - \lambda_3^2) , \\
c &= 2d_{44} = 4\lambda_3^2 (A_1 + A_2 \lambda_1^2)\, T_1 , \\
\omega_1 &= 2A_1 \lambda_1^2 + 2A_2 \lambda_1^2 (\lambda_1^2 + \lambda_3^2) , \\
\omega_3 &= 2A_1 \lambda_3^2 + 4A_2 \lambda_3 .
\end{aligned}$$

By means of these relations, equation (11) reduces to

$$(n^2 - 1)(n^2 d_{55} - d_{44}) = 0$$

giving two roots n_1^2 and n_2^2 which are respectively

$$n_1^2 = 1 , \quad n_2^2 = \frac{d_{44}}{d_{55}} = \frac{\lambda_3^2}{\lambda_1^2} .$$

Again, from the graph (fig. 10·5) p. 298, in [4] the value of A_1 and A_2 may be assumed to be respectively

$$A_1 = 1{\cdot}9 \quad \text{and} \quad A_2 = {\cdot}2378 .$$

Let us assume $\lambda_1 = {\cdot}25$ so that $\lambda_3 = 16$ and the body was initially at a temperature of 300 °K so that $T_1 = 300$ °K.

Under these assumptions we have

$$\begin{aligned}
n_1 &= 1 , & n_2 &= 64 , \\
a &= 143{\cdot}6147 , & b &= -4554{\cdot}6131 , \\
c &= 588245{\cdot}76 , & d_{44} &= 294121{\cdot}88 , \\
d_{55} &= 71{\cdot}8073 , & & \\
\omega_1 &= 7{\cdot}8489 , & \omega_3 &= 988{\cdot}0192 , \\
L &= 3{\cdot}8 , & M &= {\cdot}4756 .
\end{aligned}$$

Further assuming $k^2 = 0{\cdot}5$ we get

$$E' = \cdot 00334\,, \quad \mu = - \quad 1968{\cdot}196\,,$$
$$l_1 = -15199{\cdot}7\,, \qquad l_2 = -577070{\cdot}1\,.$$

Taking $a_1 = 1$, $h = 1$ and $b_1 = \cdot 5$, the variation of the stress $[t_{\theta\theta}]_{r=1}$, has been tabulated. Table I gives $[t_{\theta\theta}]_{r=1}$ when the temperature distribution is constant on the plane ends. Table II gives $[t_{\theta\theta}]_{r=1}$ when the temperature distribution is paraboloidal on the plane ends.

Table 1

Z	0	·2	·4	·5	·6
$\dfrac{(t_{\theta\theta})_{r=1}}{T_0}$	139·1717	126·4025	89·2428	44·0817	−40·4093

Table 2

Z	0	·2	·4	·5	·6
$\dfrac{(t_{\theta\theta})_{r=1}}{T_0}$	600·6341	534·4405	290·7204	18·3066	−442·8808

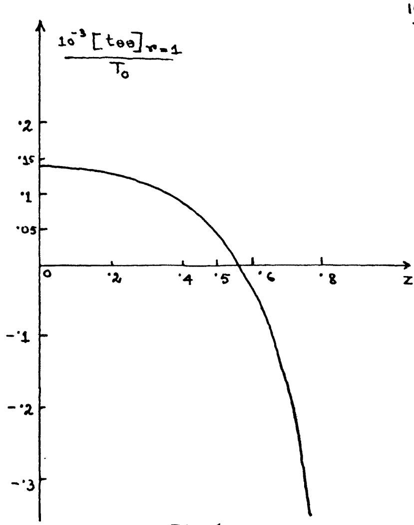

Fig. 1.

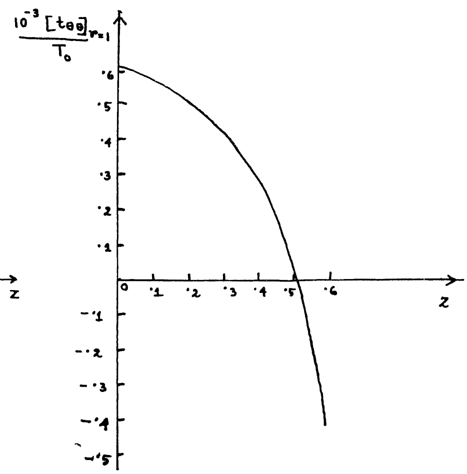

Fig. 2.

The variation of the stress $[t_{\theta\theta}]_{r=1} \cdot 10^{-3}/T_0$ is shown in Fig. 1 and Fig. 2 for different values of Z in the above two cases.

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Souhrn

TEPELNÉ NAPĚTÍ PŘEDPJATÉHO ROTAČNÍHO VÁLCE S HLADKÝM TUHÝM ISOLAČNÍM OBALEM PLÁŠTĚ

SUBHASH CHANDRA GHOSH

V článku jsou uvedeny vzorce pro tepelné napětí předpjatého isotropního konečného válce s hladkým tuhým isolačním obalem pláště, je-li dáno rozložení teploty na podstavcích. Jsou odvozeny numerické výsledky popisující průběh $[t_{\theta\theta}]_{r=1}$ pro speciální materiál známý jako materiál Mooneyova typu, jestliže rozložení teploty na podstavcích válce je buď konstantní nebo parabolické.

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