Karel Rektorys

A proof of monotony of the Temple quotients in eigenvalue problems

Aplikace matematiky, Vol. 29 (1984), No. 2, 149–158

Persistent URL: http://dml.cz/dmlcz/104078

A PROOF OF MONOTONY OF THE TEMPLE QUOTIENTS IN EIGENVALUE PROBLEMS

KAREL REKTORYS

(Received September 20, 1983)

When applying the so-called Collatz method for twosided estimates of the first eigenvalue λ_1 (see, e.g., [2], [3]), two special sequences are constructed, that of the so-called Schwarz quotients (which are upper bounds for λ_1) and that of the so-called Temple ones (which are lower bounds). While the monotony of the first sequence was proved many years ago, the proof of monotony of the second one has been given only recently by F. Goerisch and J. Albrecht in their common paper [1], prepared for ZAMM, and announced on the Conference on Eigenvalue Problems in Oberwolfach this year. The proof is based on some properties of certain matrices. In the present paper, an other proof of this monotony is given — let us call it an elementary one.

Throughout the paper, the same notation is being used as in our common paper [3] with Z. Vospěl, or in my monography [2].

Thus let us investigate the eigenvalue problem

$$Au - \lambda Bu = 0 \quad \text{in } \Omega\,, \tag{1}$$

$$B_j u = 0 \quad \text{on } \Gamma\,, \quad j = 1, \ldots, \mu\,, \tag{2}$$

$$C_j u = 0 \quad \text{on } \Gamma\,, \quad j = 1, \ldots, k - \mu\,. \tag{3}$$

Here, Ω is a bounded domain in E_N with a Lipschitzian boundary Γ, A, or B is a linear differential operator of order $2k$, or $2l$, respectively,

$$A = \sum_{|i|,|j| \leq k} (-1)^{|i|}\, D^i(a_{ij} D^j)\,, \tag{4}$$

$$B = \sum_{|i|,|j| \leq l} (-1)^{|i|}\, D^i(b_{ij} D^j)\,, \tag{5}$$

$l < k$, with bounded measurable coefficients, (2), or (3) are linear boundary conditions stable (i.e. containing derivatives of orders $\leq k - 1$), or unstable for the operator A, respectively. Denote

$$V_A = \{v;\ v \in W_2^{(k)}(\Omega),\ B_j v = 0 \text{ on } \Gamma \text{ in the sense of traces}\}\,, \tag{6}$$

(7) $$V_B = \{v;\ v \in W_2^{(l)}(\Omega),\ D_j v = 0 \text{ on } \Gamma \text{ in the sense of traces}\}\,,$$

where $W_2^{(k)}(\Omega)$, $W_2^{(l)}(\Omega)$ are the well-known Sobolev spaces, $D_j v = 0$ on Γ are such of the boundary conditions (2) which are stable for the operator B (thus containing derivatives of orders $\leqq l - 1$).

Evidently, $V_A \subset V_B$.

In the weak formulation, the problem (1)–(3) consists in finding all values of λ such that to each of them there exists a nonzero function $u \in V_A$ satisfying the integral identity

(8) $$((v, u))_A - \lambda((v, u))_B = 0 \quad \forall v \in V_A\,,$$

where $((v, u))_A$, $((v, u))_B$ are bilinear forms corresponding, in the usual sense, to the operators A and B and to the given boundary conditions.[1])

In what follows, we assume that the forms $((v, u))_A$, $((v, u))_B$ are symmetric on V_A, V_B, i.e. that there holds

(9) $$((v, u))_A = ((u, v))_A \quad \forall u, v \in V_A\,,$$

(10) $$((v, u))_B = ((u, v))_B \quad \forall u, v \in V_B\,,$$

and that they are on V_A, V_B bounded and V_A- and V_B-elliptic, i.e. that such positive constants $K_1, K_2, \alpha_1, \alpha_2$ (not depending on u, v) exist that the inequalities

(11) $$|((v, u))_A| \leqq K_1 \|v\|_{V_A} \|u\|_{V_A} \quad \forall u, v \in V_A\,,$$

(12) $$|((v, u))_B| \leqq K_2 \|v\|_{V_B} \|u\|_{V_B} \quad \forall u, v \in V_B\,,$$

(13) $$((v, v))_A \geqq \alpha_1 \|v\|^2_{V_A} \quad \forall v \in V_A$$

(14) $$((v, v))_B \geqq \alpha_2 \|v\|^2_{V_B} \quad \forall v \in V_B$$

hold. (Here $\|v\|_{V_A}$, or $\|v\|_{V_B}$ means $\|v\|_{W_2^{(k)}(\Omega)}$, or $\|v\|_{W_2^{(l)}(\Omega)}$ for $v \in V_A$, or $v \in V_B$, respectively.)[2])

Under the assumptions (9)–(14), the eigenvalue problem (8) has a countable set of (positive) eigenvalues

(15) $$\lambda_1 \leqq \lambda_2 \leqq \lambda_3 \leqq \ldots, \quad \lim_{n \to \infty} \lambda_n = +\infty\,.$$

---

[1]) Multiplying (1) by an arbitrary function $v \in V_A$ and using the Green theorem (with (2) and (3)) in the usual way, one comes to (8). For details see [2], Chap. 32, or 39.

[2]) In [1] a slightly different approach to the problematics considered has been chosen: Instead of imposing certain requirements on the bilinear forms $((v, u))_A$, $((v, u))_B$, some properties of symmetry and positive definiteness of the operators A and B on their domains of definition are required. Each of these two aproaches has its preferences. However, they are in a very closed connection together. Let us note that the way of our proof of monotony of the Temple quotients, which we are going to give in the following text, is well applicable in both the cases. (In essential, only (31) is needed, and this holds under very general assumptions.)

It is not necessary to say that the priority in proving the monotony belongs to F. Goerisch and J. Albrecht. Only the idea of our proof is different.

The corresponding system

(16) $$v_1, v_2, v_3, \dots$$ [1])

of eigenfunctions, orthonormalized in the sense of the form $((v, u))_A$, i.e.

$$((v_i, v_j))_A = \delta_{ik},$$

is complete in V_A. The system of functions

(17) $$\varphi_n = v_n \sqrt{\lambda_n}, \quad n = 1, 2, 3, \dots,$$

is then orthonormalized in the sense of the form $((v, u))_B$ and is complete in V_B.

Let the last eigenvalue λ_1 be simple. (This assumption can be weakened.) The well-known Collatz method how to obtain two-sided estimates for this λ_1 consists in the following:

Choose a nonzero function $f_0 \in V_B$ and construct, subsequently, the functions

(18) $$f_j \in V_A, \quad j = 1, 2, 3, \dots,$$

satisfying

(19) $$((v, f_j))_A = ((v, f_{j-1}))_B \quad \forall v \in V.$$

Let us construct, further, the so-called *Schwarz coefficients*

(20) $$a_j = ((f_0, f_j))_B > 0, \quad j = 0, 1, 2, \dots,$$

Schwarz quotients

(21) $$\varkappa_j = \frac{a_{j-1}}{a_j}, \quad j = 1, 2, 3$$

and *Temple quotients*

(22) $$\tau_j(L) = \frac{La_j - a_{j-1}}{La_{j+1} - a_j}, \quad j = 1, 2, 3, \dots,$$

defined for

(23) $$\varkappa_1 < L < \lambda_2$$

(provided

(24) $$\varkappa_1 < \lambda_2).$$

Then

(25) $$\tau_j \leqq \lambda_1 \leqq \varkappa_j, \quad j = 1, 2, 3, \dots$$

(see e.g. [2], Chap. 40). At the same time,

(26) $$\varkappa_1 \geqq \varkappa_2 \geqq \varkappa_3 \geqq \dots$$

[1]) The usual convention is chosen for ordering of eigenvalues in order that the correspondance between (15) and (16) be one-to-one.

([2], [3]; for the case of ordinary differential equations this result was derived by L. Collatz many years ago). If, moreover, the functions f_0, f_1 are linearly independent, then the sequence $\{\varkappa_j\}$ is even strictly decreasing, i.e. we have

$$\varkappa_1 > \varkappa_2 > \varkappa_3 > \ldots > \lambda_1 . \tag{27}$$

In what follows, linear independency of f_0, f_1 is everywhere assumed, so that (27) *holds.*

Under the given assumptions, the following theorem is valid:

Theorem 1. *The sequence of the Temple quotients is strongly increasing, i.e. we have*

$$\tau_1(L) < \tau_2(L) < \tau_3(L) < \ldots \quad \textit{for every} \quad L \in (\varkappa_1, \lambda_2) . \tag{28}$$

Proof. In the proof, we utilize the following relation, proved in our work [3] (eq. (2.26), p. 221): Let

$$f_0 = \sum_{i=1}^{\infty} \alpha_i \varphi_i \quad \text{in } V_B . \tag{29}$$

(Thus

$$\alpha_i = ((f_0, \varphi_i))_B , \quad i = 1, 2, 3, \ldots .) \tag{30}$$

Then

$$a_j = \sum_{i=1}^{\infty} \frac{\alpha_i^2}{\lambda_i^j} , \quad j = 0, 1, 2, \ldots . \tag{31}$$

To prove (28), we have to show that

$$\tau_{j+1}(L) - \tau_j(L) > 0 \quad \text{for every} \quad L \in (\varkappa_1, \lambda_2) \quad \text{and for every} \quad j \geqq 1 . \tag{32}$$

Thus let $j \geqq 1$ be fixed, otherwise arbitrary. We have

$$\tau_{j+1}(L) - \tau_j(L) = \tag{33}$$

$$= \frac{La_{j+1} - a_j}{La_{j+2} - a_{j+1}} - \frac{La_j - a_{j-1}}{La_{j+1} - a_j} = \frac{y_j(L)}{(La_{j+2} - a_{j+1})(La_{j+1} - a_j)} ,$$

where

$$y_j(L) = (a_{j+1}^2 - a_j a_{j+2}) L^2 + (a_{j-1} a_{j+2} - a_j a_{j+1}) L + + (a_j^2 - a_{j-1} a_{j+1}) . \tag{34}$$

Now,

$$La_{j+1} - a_j = a_{j+1}(L - \varkappa_{j+1}) > 0 ,$$

because of (27) and (23), and, by the same reasoning,

$$La_{j+2} - a_{j+1} > 0 .$$

Thus to prove (32) (for the given j) we have to prove that

$$y_j(L) > 0 \quad \text{for all} \quad L \in (\varkappa_1, \lambda_2) . \tag{35}$$

The function y_j is defined (by (34)) for *all* (real) L. Because

$$a_{j+1}^2 - a_j a_{j+2} = a_{j+1}^2 \left(1 - \frac{\varkappa_{j+1}}{\varkappa_{j+2}}\right) < 0 \tag{36}$$

in virtue of (27), this function is strictly concave elsewhere. Thus to prove (35), it is sufficient to find two points L_1, L_2 such that

$$L_1 \leqq \varkappa_1\,, \quad L_2 \geqq \lambda_2 \tag{37}$$

and that, at the same time, we have

$$y_j(L_1) \geqq 0\,, \quad y_j(L_2) \geqq 0\,, \tag{38}$$

(see Fig. 1).

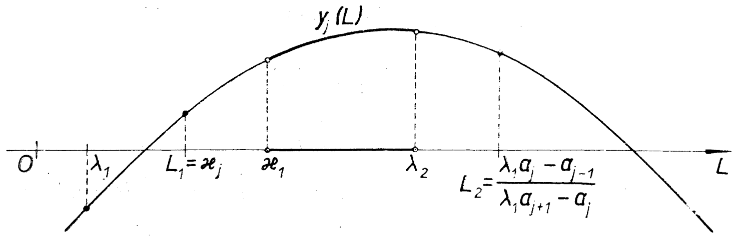

Fig 1.

j being fixed, let us choose

$$L_1 = \varkappa_j\,, \quad L_2 = \frac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j}\,. \tag{39}$$

Both the requirements (37) are satisfied. The first one because of (27) (even for $j = 1$). For the second inequality,

$$\frac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j} \geqq \lambda_2\,,$$

see [3] (ineqs. (2.38), (2.39), p. 223). Now, by (34),

$$y_j(\varkappa_j) = y_j\left(\frac{a_{j-1}}{a_j}\right) = (a_{j+1}^2 - a_j a_{j+2})\frac{a_{j-1}^2}{a_j^2} + \tag{40}$$

$$(a_{j-1}a_{j+2} - a_j a_{j+1})\frac{a_{j-1}}{a_j} + (a_j^2 - a_{j-1}a_{j+1}) =$$

$$= \frac{1}{a_j^2}(a_{j-1}^2 a_{j+1}^2 - a_{j-1}^2 a_j a_{j+2} + a_{j-1}^2 a_j a_{j+2} - a_{j-1} a_j^2 a_{j+1} +$$

$$+ a_j^4 - a_{j-1} a_j^2 a_{j+1}) = \frac{1}{a_j^2}(a_{j-1}a_{j+1} - a_j^2)^2 > 0\,.$$

In this way, validity of the first inequality in (38) is established. It remains to prove validity of the second one. However, we have

$$(41) \qquad y_j\left(\frac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j}\right) =$$

$$= \frac{1}{(\lambda_1 a_{j+1} - a_j)^2}\left[(a_{j+1}^2 - a_j a_{j+2})(\lambda_1^2 a_j^2 - 2\lambda_1 a_{j-1} a_j + a_{j-1}^2) + \right.$$

$$+ (a_{j-1}a_{j+2} - a_j a_{j+1})(\lambda_1^2 a_j a_{j+1} - \lambda_1 a_j^2 - \lambda_1 a_{j-1}a_{j+1} + a_{j-1}a_j) +$$

$$\left. + (a_j^2 - a_{j-1}a_{j+1})(\lambda_1^2 a_{j+1}^2 - 2\lambda_1 a_j a_{j+1} + a_j^2)\right] =$$

$$= \frac{1}{(\lambda_1 a_{j+1} - a_j)^2}\left[\lambda_1^2(a_j^2 a_{j+1}^2 - a_j^3 a_{j+2} + \right.$$

$$+ a_{j-1}a_j a_{j+1}a_{j+2} - a_j^2 a_{j+1}^2 + a_j^2 a_{j+1}^2 - a_{j-1}a_{j+1}^3) +$$

$$+ \lambda_1(-2a_{j-1}a_j a_{j+1}^2 + 2a_{j-1}a_j^2 a_{j+2} - a_{j-1}a_j^2 a_{j+2} +$$

$$+ a_j^3 a_{j+1} - a_{j-1}^2 a_{j+1}a_{j+2} + a_{j-1}a_j a_{j+1}^2 - 2a_j^3 a_{j+1} + 2a_{j-1}a_j a_{j+1}^2) +$$

$$+ a_{j-1}^2 a_{j+1}^2 - a_{j-1}^2 a_j a_{j+2} + a_{j-1}^2 a_j a_{j+2} - a_{j-1}a_j^2 a_{j+1} +$$

$$\left. + a_j^4 - a_{j-1}a_j^2 a_{j+1})\right] =$$

$$= \frac{1}{(\lambda_1 a_{j+1} - a_j)^2}\left[\lambda_1^2(a_{j-1}a_j a_{j+1}a_{j+2} - a_j^3 a_{j+2} - a_{j-1}a_{j+1}^3 + a_j^2 a_{j+1}^2) + \right.$$

$$+ \lambda_1(a_{j-1}a_j a_{j+1}^2 - a_j^3 a_{j+1} - a_{j-1}^2 a_{j+1}a_{j+2} + a_{j-1}a_j^2 a_{j+2}) +$$

$$\left. + (a_{j-1}^2 a_{j+1}^2 - 2a_{j-1}a_j^2 a_{j+1} + a_j^4)\right] =$$

$$= \frac{1}{(\lambda_1 a_{j+1} - a_j)^2}(a_j^2 - a_{j-1}a_{j+1})\left[(a_{j+1}^2 - a_j a_{j+2})\lambda_1^2 + \right.$$

$$\left. + (a_{j-1}a_{j+2} - a_j a_{j+1})\lambda_1 + (a_j^2 - a_{j-1}a_{j+1})\right] =$$

$$= \frac{a_j^2 - a_{j-1}a_{j+1}}{(\lambda_1 a_{j+1} - a_j)^2}\, y_j(\lambda_1)\,.$$

In the same way as in (36) we obtain $a_j^2 - a_{j-1}a_{j+1} < 0$. Consequently, to prove validity of the second inequality in (38), it is sufficient to prove that

$$(42) \qquad y_j(\lambda_1) \leqq 0\,.$$

Having proved (42), the proof of Theorem 1 will be completed.

By definition (cf. (33), (34)), we have

(43)
$$y_j(\lambda_1) =$$
$$= (\lambda_1 a_{j+2} - a_{j+1})(\lambda_1 a_{j+1} - a_j)\left(\frac{\lambda_1 a_{j+1} - a_j}{(\lambda_1 a_{j+2} - a_{j+1}} - \frac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j}\right).$$

However,

$$\lambda_1 a_{j+2} - a_{j+1} = a_{j+2}(\lambda_1 - \varkappa_{j+2}) < 0\,,$$

and, in the same way,

$$\lambda_1 a_{j+1} - a_j < 0\,, \quad \lambda_1 a_j - a_{j-1} < 0\,.$$

Thus, to prove (42), we have to prove that

$$\frac{\lambda_1 a_{j+1} - a_j}{\lambda_1 a_{j+2} - a_{j+1}} - \frac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j} \leqq 0\,,$$

or, because each of the two fractions is positive, that

$$Q = \frac{\dfrac{\lambda_1 a_{j+1} - a_j}{\lambda_1 a_{j+2} - a_{j+1}}}{\dfrac{\lambda_1 a_j - a_{j-1}}{\lambda_1 a_{j+1} - a_j}} \leqq 1\,,$$

or, what is the same, that

(44)
$$Q = \frac{\dfrac{a_j - \lambda_1 a_{j+1}}{a_{j+1} - \lambda_1 a_{j+2}}}{\dfrac{a_{j-1} - \lambda_1 a_j}{a_j - \lambda_1 a_{j+1}}} \leqq 1\,.$$

To this purpose, (31), i.e. the relation

(45)
$$a_j = \sum_{i=1}^{\infty} \frac{\alpha_i^2}{\lambda_i^j}\,,$$

will be applied. Because

$$\sum_{i=1}^{\infty} \alpha_i^2 < \infty$$

and λ_i tend to infinity for $i \to \infty$, we have

(46)
$$Q = \lim_{p \to \infty} Q_p\,,$$

where Q_p is obtained from Q if in (45) only the finite number p of summands is taken, i.e.

$$(47)\qquad Q_p = \frac{\dfrac{\sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^j} - \lambda_1 \sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^{j+1}}}{\sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^{j+1}} - \lambda_1 \sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^{j+2}}}}{\dfrac{\sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^{j-1}} - \lambda_1 \sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^j}}{\sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^j} - \lambda_1 \sum_{i=1}^{p} \dfrac{\alpha_i^2}{\lambda_i^{j+1}}}} .$$

Let us note that, making the differences, the first summands drop out.[1]) Thus, after an obvious rearranging, we obtain

$$(48)\qquad Q_p = \frac{\dfrac{\frac{\alpha_2^2}{\lambda_2^j}\left(1 - \frac{\lambda_1}{\lambda_2}\right) + \frac{\alpha_3^2}{\lambda_3^j}\left(1 - \frac{\lambda_1}{\lambda_3}\right) + \ldots + \frac{\alpha_p^2}{\lambda_p^j}\left(1 - \frac{\lambda_1}{\lambda_p}\right)}{\frac{\alpha_2^2}{\lambda_2^{j+1}}\left(1 - \frac{\lambda_1}{\lambda_2}\right) + \frac{\alpha_3^2}{\lambda_3^{j+1}}\left(1 - \frac{\lambda_1}{\lambda_3}\right) + \ldots + \frac{\alpha_p^2}{\lambda_p^{j+1}}\left(1 - \frac{\lambda_1}{\lambda_p}\right)}}{\dfrac{\frac{\alpha_2^2}{\lambda_2^{j-1}}\left(1 - \frac{\lambda_1}{\lambda_2}\right) + \frac{\alpha_3^2}{\lambda_3^{j-1}}\left(1 - \frac{\lambda_1}{\lambda_3}\right) + \ldots + \frac{\alpha_p^2}{\lambda_p^{j-1}}\left(1 - \frac{\lambda_1}{\lambda_p}\right)}{\frac{\alpha_2^2}{\lambda_2^j}\left(1 - \frac{\lambda_1}{\lambda_2}\right) + \frac{\alpha_3^2}{\lambda_3^j}\left(1 - \frac{\lambda_1}{\lambda_3}\right) + \ldots + \frac{\alpha_p^2}{\lambda_p^j}\left(1 - \frac{\lambda_1}{\lambda_p}\right)}} ,$$

or, denoting, for simplicity,

$$\alpha_i^2\left(1 - \frac{\lambda_1}{\lambda_i}\right) = \beta_i \geqq 0 , \quad v_i = \frac{1}{\lambda_i} > 0 ,$$

$$(49)\qquad Q_p = \frac{\dfrac{\beta_2 v_2^j + \beta_3 v_3^j + \ldots + \beta_p v_p^j}{\beta_2 v_2^{j+1} + \beta_3 v_3^{j+1} + \ldots + \beta_p v_p^{j+1}}}{\dfrac{\beta_2 v_2^{j-1} + \beta_3 v_3^{j-1} + \ldots + \beta_p v_p^{j-1}}{\beta_2 v_2^j + \beta_3 v_3^j + \ldots + \beta_p v_p^j}} .$$

Multiplication of the corresponding sums in the composite fraction yields

$$(50)\qquad Q_p = \frac{\sum_{i=2}^{p} \beta_i^2 v_i^{2j} + 2 \sum_{\substack{i,k=2 \\ i<k}}^{p} \beta_i \beta_k v_i^j v_k^j}{\sum_{i=2}^{p} \beta_i^2 v_i^{2j} + \sum_{\substack{i,k=2 \\ i<k}}^{p} \beta_i \beta_k (v_i^{j+1} v_k^{j-1} + v_i^{j-1} v_k^{j+1})} \leqq 1 ,$$

---

[1])
$$\frac{\alpha_1^2}{\lambda_1^j} - \lambda_1 \frac{\alpha_1^2}{\lambda_1^{j+1}} = 0 , \quad \text{etc.}$$

because

$$2v_i^j v_k^j = v_i^{j-1} v_k^{j-1} \cdot 2v_i v_k\,,$$

$$v_i^{j+1} v_k^{j-1} + v_i^{j-1} v_k^{j+1} = v_i^{j-1} v_k^{j-1} (v_i^2 + v_k^2)$$

and

$$2v_i v_k \leqq v_i^2 + v_k^2\,.$$

(50) and (46) imply (44), or (42) which simultaneously with (41) yields the second of the inequalities (38).

This completes the proof of Theorem 1.

Remark. If, moreover, the initial function f_0 in the Collatz process is not orthogonal, in the sense of the bilinear form $((v, u))_B$, to the first eigenfunction v_1, i.e. if

(51) $$((f_0, v_1))_B \neq 0\,,$$

then (see [3], (2.43), p. 224)

(52) $$\lim_{j\to\infty} \varkappa_j = \lambda_1\,.$$

This fact implies, by an easy computation, that also

(53) $$\lim_{j\to\infty} \tau_j(L) = \lambda_1 \quad \text{for every } L \in (\varkappa_1, \lambda_2)\,.$$

Thus, in this case, λ_1 is the limit of two strictly monotonic sequences, the decreasing sequence of the Schwarz quotients and the increasing sequence of the Temple quotients.

In the case that λ_1 is not simple, the condition (51) is to be replaced by the condition that f_0 is not orthogonal (in the sense of the bilinear form $((v, u))_B$) simultaneously to all eigenfunctions corresponding to λ_1.

References

[1] *F. Goerisch, J. Albrecht:* Die Mononie der Templeschen Quotienten. ZAMM (in print).

[2] *K. Rektorys:* Variational Methods in Mathematics, Science and Engineering. 2nd Ed. Dordrecht—Boston—London, J. Reidel 1979. (Czech: Praha, SNTL 1974.)

[3] *K. Rektorys, Z. Vospěl:* On a method of twosided eigenvalue estimates for elliptic equations of the form $Au - \lambda Bu = 0$. Aplikace matematiky 26 (1981), 211—240.

JINÝ DŮKAZ MONOTÓNNOSTI TEMPLEOVÝCH KVOCIENTŮ V PROBLÉMECH VLASTNÍCH ČÍSEL

KAREL REKTORYS

Aplikujeme-li tzv. Collatzovu metodu k sestrojení dvoustranných odhadů prvního vlastního čísla λ_1 (viz např. [2], [3]), konstruujeme dvě posloupnosti, posloupnost tzv. Schwarzových kvocientů (kterými odhadujeme číslo λ_1 shora) a posloupnost tzv. Templeových kvocientů (kterými odhadujeme λ_1 zdola). Zatímco monotónnost první z těchto posloupností je známá řadu let, monotónnost posloupnosti Templeových kvocientů byla dokázána (za velmi přirozených předpokladů) teprve nedávno F. Goerischem a J. Albrechtem v jejich společné práci [1], připravené pro ZAMM. O této práci bylo referováno letos na konferenci o vlastních číslech v Oberwolfachu. Důkaz, uvedený v citované práci, je založen na určitých vlastnostech některých matic.

V předložené práci je uveden jiný – nazvěme jej elementární – důkaz monotónnosti posloupnosti Templeových kvocientů.

Author's address: Prof. RNDr. *Karel Rektorys*, DrSc., Stavební fakulta ČVUT, Thákurova 7, 166 29 Praha 6.