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AN ELEMENTARY CHARACTERIZATION OF THE CATEGORY OF RELATIONAL SYSTEMS

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Introduction

In [2] F. Lawvere characterized the category of sets by elementary axioms, using a language, with one sort of variable symbols (mappings) and two unary functions symbols (domain and codomain [1]) and one ternary relation, composition. $A \xrightarrow{f} B$ means f is a map with domain A and codomain B . D. Schlomiuk,[5], presented a method of characterizing the category of topological spaces by using the full subcategory of discrete spaces; the fact that the functor, inclusion, from sets to topological spaces has a left adjoint, together with additional axioms on a certain constant (the two-point space $(\{a, b\}, \{a\}, \{a, b\})$). Lawvere also characterized algebraic categories [1], using the special properties of a certain constant, the free object on one generator, and an adjointness condition. It is conjectured that what one needs is a previously characterized reflective subcategory C and finitely many

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1 An object is a map which is a domain and a codomain.

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objects $a_1 \dots a_m$ determined up to isomorphism, such that the full subcategory on $C \cup \bigcup_i a_i$, is in the transitive closure of "adequate". We shall adopt this technique in the present paper to the category of relational systems.

In [3],[4], Hedrlín, Pultr and Trnková defined morphisms to form the category of relational systems of type Δ, $R(\Delta)$, and showed the usefulness in representation of abstract concrete categories as full subcategories of a category of relational systems. The definition of the category of relational systems of Δ is the following:

Let $\Delta = \{n_i\}_{i \in I}$ where the n_i are cardinals, and I is a set. Then the objects are pairs $(X, \{R_i\}_{i \in I})$ where $R_i \subseteq X^{n_i}$.

$(X, \{R_i\}_{i \in I}) \xrightarrow{f} (Y, \{S_i\}_{i \in I})$ is a morphism iff f is a function from X to Y such that ${}^{n_i}f(R_i) \subseteq S_i$ for all $i \in I$; where if $n_i \xrightarrow{g} X \in X^{n_i}$, ${}^{n_i}f_i(g) = gf$, $n_i \xrightarrow{gf} Y$.

Although the present author was not able to find a complete characterization of $R(\Delta)$ such that every complete (categorically) model was naturally equivalent to the $R(\Delta)$ of Hedrlín and Pultr, he did find an axiom system denoted by R_I such that if C is a category satisfying the axioms of R_I, and C is complete then C is naturally equivalent to $R(\Delta)$ where

$\Delta = \{n_i\}_{i \in I}$ and one can determine the values of the n_i by a simple test in C, whenever I is a finite set. For I infinite one needs infinitely many a_i to form the category in the transitive closure of "adequate" for the generalization.

We shall start by stating the axioms for R_1 (where 1 is a one point set) and then develop from this the axioms for R_I.

<u>Characterization of</u> R_1.

<u>Axiom 1.</u> R_1 has an initial (0), and terminal (1) object, equalizers, coequalizers of pairs of maps, products and sums of pairs of objects.

<u>Axiom 2.</u> There is an object $1'$ such that if G is any object, $G \neq 0 \Rightarrow \exists f,\ 1' \xrightarrow{f} G$.

<u>Definition</u>: An object G is discrete $\Longleftrightarrow \exists v,\ G \xrightarrow{v} 1'$.

<u>Axiom 3.</u> $G \xrightarrow{v} 1'$ and $G \xrightarrow{u} 1' \Longrightarrow u = v$.

<u>Definition:</u> $X \in A$ (or X is an element of A) $\Longleftrightarrow 1' \xrightarrow{x} A$.

(Note the definition $1 \xrightarrow{x} A$ given by Lawvere,[1], and Schlomiuk [5] is not applicable here.)

<u>Axiom 4.</u> $1'$ is a projective generator.

It is clear that if $A \xrightarrow{f} B$ then f is *epi* $\Longleftrightarrow \forall x \in B, \exists y \in A\ \ \beta y f = x$; f is *mono* $\Longleftrightarrow$ for every pair of elements $x, y \in A$, $x \neq y$, $xf \neq yf$.

Definition: f is a bijection $\Longleftrightarrow f$ is *mono* and *epi*.

Axiom 5. Every non zero object has elements.

Axiom 6. Every element of a sum can be factored through exactly one of the injections.

Axiom 7. For each object G there exists a discrete object $|G|$ and a bijection t_G such that for each discrete object S and each map $S \xrightarrow{h} G$ there exists a unique f such that

$$\begin{tikzcd} |G| \arrow[rr, "t_G"] & & G \\ & S \arrow[ul, dashed, "f"] \arrow[ur, "h"'] & \end{tikzcd}$$

commutes.

It is clear that $||$ is a functor, and $|f|$ *mono* $\Rightarrow$ $\Rightarrow f$ *mono*, $|f|$ *epi* $\Rightarrow f$ *epi*.

Axiom 8. If C, A are discrete then there exists a discrete object B^C and a map $C \times B^C \xrightarrow{e} B$ such that for every discrete object X and mapping $C \times X \xrightarrow{f}$ $\xrightarrow{f} B$ there exists a unique mapping $X \xrightarrow{h} B^C$ such that

$$\begin{tikzcd} C \times X \arrow[d, "C \times h"'] \arrow[dr, "f"] & \\ C \times B^C \arrow[r, "e"] & B \end{tikzcd}$$

commutes.

It is clear that the elements of B^C are in one-one correspondence with the maps from B to C in the following way:

Let $y \in B^C$, define (y) to be the unique map for which

$$\begin{tikzcd} C\times 1' \arrow[r, "p_C"] \arrow[d, "C\times y"'] & C \arrow[d, "(y)"] \\ C\times B^C \arrow[r, "e"] & B \end{tikzcd}$$

commutes; if $C \xrightarrow{f} B$ let $[f]$ be the unique map such that

$$\begin{tikzcd} C\times 1' \arrow[r, "p_C"] \arrow[d, "C\times [f]"] & C \arrow[d, "f"] \\ C\times B^C \arrow[r, "e"] & B \end{tikzcd}$$

commutes.

It is clear B^C can be extended to a functor of two variable contravariant in the exponent.

<u>Axiom 9.</u> There exists a discrete object N and maps $1' \xrightarrow{0} N$, $N \xrightarrow{'} N$ such that for every discrete object X, $x_0 \in X$, and each map $X \xrightarrow{u} X$ there exists a unique map $N \xrightarrow{x} X$ such that

$$\begin{tikzcd} & N \arrow[r, "'"] \arrow[d, "x"] & N \arrow[d, "x"] \\ 1' \arrow[ur, "0"] \arrow[r, "x_0"'] & X \arrow[r, "u"] & X \end{tikzcd}$$

commutes.

<u>Axiom 10</u>. If C, B are discrete and C has elements then for every map $C \xrightarrow{f} B$ there exists $B \xrightarrow{g} C$ such that $fgf = f$.

We now have the following theorem schema: If Φ is a theorem of the elementary theory of the category of sets and Φ' is obtained from Φ by replacing "set" by discrete object then Φ' is a theorem of R_1 .

Axiom 11. There is an object A of R such that

(1) $A \xrightarrow{f} A \Rightarrow f = A$,

(2) $|A| \neq 0$,

(3) $\exists f,\ A \xrightarrow{f} G \Longleftrightarrow G$ is not discrete.

Definition: $|A| = n$.

We note that if $A \xrightarrow{f} G$ then $n \xrightarrow{|f|} |G|$ and thus there exists uniquely $1' \xrightarrow{[|f|]} |G|^n$.

Axiom 12. $\forall$ objects G there exists a monomorphism $R \xrightarrow{m} |G|^n$ such that for all f , $A \xrightarrow{f} G$ $\exists u$ such that

$$\begin{tikzcd} & {|G|^n} \\ 1' \arrow[ur, "{[|f|]}"] \arrow[r, "u"'] & R \arrow[u, "m"'] \end{tikzcd}$$

commutes

and if m' is any other monomorphism with this property there exists a unique monomorphism m'' such that

$$\begin{tikzcd} R \arrow[rr, "m"] \arrow[dr, "m''"'] & & {|G|^n} \\ & R' \arrow[ur, "m'"'] & \end{tikzcd}$$

commutes.

Thus if m, m' both satisfy the two above properties then we have

$$\begin{tikzcd} R \arrow[r, "m"] \arrow[d, shift left, "m''"] & {|G|^n} \\ R' \arrow[u, shift left, "m'''"] \arrow[ur, "m'"'] & \end{tikzcd}$$

commutes

thus $m'''m = m'$, $m''m' = m$ thus $m''m'''m''m' =$ $= m$, which by uniqueness gives $m''m'''m'' = m''$ and as m'' is mono, $m'''m'' = 1_{R'}$. Similarly $m''m''' = 1_R$.

Furthermore if h is an isomorphism $R \xrightarrow{h} R'$ and m satisfies the conditions of axiom 11 so does hm. Thus if we define $(|G|, m)$ represents G to mean that $(|G|, m)$ satisfies the properties of axiom 12 with respect to G, then if $(|G|, n)$ is another representation then $n = hm$ where h is an isomorphism.

<u>Axiom 13.</u> In every discrete object H and every monomorphism $R \xrightarrow{m} H^n$ there exists an object G such that $|G| = H$ and (H, m) represents G, furthermore if G' is represented by (H, m), G is isomorphic to G'.

<u>Axiom 14.</u> If $G \xrightarrow{f} H$ is any morphism and G is represented by $(|G|, m)$ and H by (H, m') then there exists a unique map f' such that

$$\begin{tikzcd} R \arrow[r, "m"] \arrow[d, "f'"] & {|G|^n} \arrow[d, "|f|^n"] \\ R' \arrow[r, "m'"] & {|H|^n} \end{tikzcd}$$

commutes.

It is clear that this is independant of the choice of representatives m, m' as can be seen from the commutative diagram

$$\begin{tikzcd}
\bar{R} \arrow[r, "hm"] \arrow[d, "h"'] & {|G|^n} \arrow[d, "1 = |1|^n"] \\
R \arrow[r, "m"] \arrow[d, "f'"'] & {|G|^n} \arrow[d, "|f|^n"] \\
R' \arrow[r, "m'"] \arrow[d, "k"'] & {|G|^n} \arrow[d, "1 = |1|^n"] \\
\bar{R}' \arrow[r, "k^{-1}m'"] & {|G|^n}
\end{tikzcd}$$

where h, k are isomorphisms.

Axiom 15. If G, H are discrete and m, m' are monomorphisms $R \xrightarrow{m} G^n$, $R' \xrightarrow{m'} H^n$ and f is a map $G \xrightarrow{f} H$ such that there exists a unique map f' for which

$$\begin{tikzcd}
R \arrow[r, "m"] \arrow[d, "f'"] & G^n \arrow[d, "f^n"] \\
R \arrow[r, "m'"] & H^n
\end{tikzcd}$$

commutes then if G^*, H^* are represented by (G, m), and (H, m') respectively, there exists uniquely a map $G^* \xrightarrow{f^*} H^*$ such that $|f^*| = f$.

Definition: The pair category P_1 is the category whose objects are pairs (G, m) where G is discrete and $R \xrightarrow{m} G^n$ is a monomorphism; a morphism $(G, m) \xrightarrow{f} (H, m')$ is a map $G \xrightarrow{f} H$ for which there exists a unique map f' such that

$$\begin{tikzcd} R \arrow[r, "m"] \arrow[d, "f'"] & G^n \arrow[d, "f^n"] \\ R' \arrow[r, "m'"] & H^n \end{tikzcd}$$

commutes.

<u>Meta-theorem I:</u> Let $\mathcal{A}$ be any model of R_1 and B be the pair category constructed from $\mathcal{A}$. Then there exists a natural equivalence $\mathcal{A} \xrightarrow{T} B$ with the property that H is discrete $\Longleftrightarrow T(H) = (H, o_H)$ where $0 \xrightarrow{o_H} H^n$; and $T(A)$ can be chosen to be $(n, [|1_A|])$.

<u>Proof:</u> Axioms 12 - 15 insure the existence of a natural equivalence. We need only show that the equivalence can be chosen with the two given properties.

Let G be discrete and (G, m) represent G. If $m \neq 0$ then $R \neq 0$ which implies R has elements; thus there exists x such that $1' \xrightarrow{x} R \xrightarrow{m} G^n$. Now as G is discrete there are no maps from $A \xrightarrow{f} H$ thus the following diagram commutes vacuously:

$$\begin{tikzcd} & {|H|^n} \\ 1' \arrow[ur, "{[|f|]}"] \arrow[dr, "u"'] & \\ & 0 \arrow[uu, "o"'] \end{tikzcd}$$

By axiom 12 there is a monomorphism m' from R to 0; thus we have $1' \xrightarrow{x} R \xrightarrow{m'} 0$, i.e. 0 has elements. Thus $R = 0$

and m must be the unique map $0 \xrightarrow{\circ_N} H^n$. The converse is similar.

Since A is determined by its definition up to isomorphism, and $|A| = n$ by definition, we need only that $(n, [|1_A|])$ satisfies the three defining properties of A.

(1) If $(n,[|1_A|]) \xrightarrow{f} (n,[|1_A|]) \Rightarrow f = 1_n$.

Let f be such a morphism, then we have the following commutative diagram:

$$\begin{array}{ccc} 1' & \xrightarrow{[|1_A|]} & n^n \\ \downarrow{\scriptstyle 1'} & & \downarrow{\scriptstyle f^n} \\ 1' & \longrightarrow & n^n \end{array}$$

As $1'$ is the only map $1' \xrightarrow{1'}$ $'$. This implies that

$$\begin{array}{ccc} n & \xrightarrow{n} & n \\ \downarrow{\scriptstyle n} & & \downarrow{\scriptstyle f} \\ n & \xrightarrow{n} & n \end{array}$$

commutes i.e.

$f = 1_n = |1_A|$.

(2) $|(n,[|1_A|])| \neq 0$ is obvious as $n \neq 0$.

(3) $\exists f,\ |(n,[|1_A|])| \xrightarrow{f} (G,m) \Longleftrightarrow m \neq 0$.

Let (G,m) be an object in B . If $m = 0$, and $(n,[|1_n|]) \xrightarrow{f} (G,m)$, there exists f' such that

$$\begin{tikzcd} 1' \arrow[r, "{[\,|\,1_A|\,]}"] \arrow[d, "f'"] & n^n \arrow[d, "f"] \\ 0 \arrow[r] & G^n \end{tikzcd}$$

commutes, but there are no maps from $1'$ to 0. Thus (G, m) discrete $\Rightarrow$ there are no maps from $(n, [\,|\,1_A|\,])$ to (G, m). If $m \neq 0$ and, $R \xrightarrow{m} G^n$, then R has elements. Let $1 \xrightarrow{x'} R \xrightarrow{m} G^n$. Then $n \xrightarrow{(x'm)} G$. It is claimed that $(x'm)$ is a morphism in B from $(n, [\,|\,1_A|\,])$ to (G, m). This will be true if the following diagram commutes:

$$\begin{tikzcd} 1' \arrow[r, "{[\,|\,1_A|\,]}"] \arrow[d, "x'"] & n^n \arrow[d, "(xm)^n"] \\ R \arrow[r, "m"] & G^n \end{tikzcd}$$

This is equivalent to the commutativity of

$$\begin{tikzcd} & n \arrow[d, "x'm"] \\ n \arrow[ur, "1_n"] \arrow[r, "x'm"] & G \end{tikzcd}$$

which is trivial.

<u>Meta-lemma I</u>: In B (as in meta-theorem I), $|(G, m)|_B = (G, o)$.

Proof: It has been shown that the discrete objects of B are those of the form (G, o). It is clear that $(G, o) \xrightarrow{G} (G, m)$ is a bijection and the following diagram commutes

$$\begin{tikzcd}
(G,o) \arrow[r, "G"] & (G,m) \\
(H,o) \arrow[u, "f'=f"] \arrow[ur, "f"'] &
\end{tikzcd}$$

; and f' is uniquely determined by f.

Meta-lemma 2: Let B, C be complete models of P_1. Then there are functors $B^d \xrightarrow{Law} C^d$, and $C^d \xrightarrow{vere} B^d$ such that $Lawvere \simeq 1$ and $vereLaw \cong 1$ where B^d and C^d are the full subcategories of discrete objects of B and C respectively.

Proof: This is a restatement of the principal result of [2].

Meta-theorem 2: Let B, C be complete models of R, let B', C' be pair categories constructed from B, C respectively. Let $|A|_B = n_1$ and $|A|_C = n_2$ then B' and C' are naturally equivalent iff $Law(n_1) \cong n_2$. If B' and C' are naturally equivalent the equivalence can be given by $Law'(G, m) = (Law(G), Law(m))$. Hence B and C are naturally equivalent $\Longleftrightarrow n_1 \cong n_2$.

Proof: If $\Phi : B \longrightarrow C$ is a natural equivalence then $\Phi \mid B_d$ is a natural equivalence from B^d to C^d

and thus there is a natural equivalence $\Phi' : B' \longrightarrow C'$ and $\Phi'(H, o) = |\phi(H)|$, $\Phi'(|A|, o) \cong \Phi'|A| \cong n_1$, but $|A| = n_2$ thus $n_1 \simeq n_2$.

Conversely, we have $Law : B'^d \longrightarrow C'^d$ and define $Law'(G, m) = (Law(G), Law(m))$. The fact that Law' is an equivalence can be seen from the commutativity of the following diagram and its inverse. ($vere'$ is defined similarly to Law' .)

$$\begin{tikzcd}
R \arrow[r, "m"] \arrow[d, shift right, "\theta_R"'] & G^n \arrow[r, "\theta_G^n"] \arrow[dr, shift left, "\theta^{-1}_{G_n}"] & Lawvere(G)^n \arrow[r, "\theta_n"] & Lawvere(G)^{Lawvere(n)} \arrow[dd, no head] \\
& & & \\
Lawvere(R) \arrow[u, shift right, "\theta^{-1}_R"'] \arrow[r, "Lawvere(m)"] & Lawvere\, G^n \arrow[ul, shift left, "\theta_{G_m}"] \arrow[rr, "{\left[\theta^{-1}_{G_n} \;\; \theta_{G_n}\right]^{\theta_n}}"] & & Lawvere(G)^{Lawvere(n)}
\end{tikzcd}$$

where θ is the natural isomorphism given by the equivalence. If B' and C' are naturally equivalent then so are B and C .

Remark 1. If n is finite one can characterize $R(n)$ completely by changing the axiom 11 - (2) to $|A| = n \cdot {}' = {}' + {}' + {}' \ldots {}'$ (*n times*) .

Remark 2. If $R(n)$ is the usual category of relational systems of type $\{n\}$, and B is any complete model of R in which $|A| = n$,

$Law'(G, m) = (\{x \mid {}' \xrightarrow{x} |G|\}, m')$ where m' is defined by $m'(x) = xm'$, $x \in G^n$.

<u>The Characterization of</u> R_I, I <u>finite</u>

Let $I = \{0, 1, \dots m-1\}$. Then we have the following axiom system for R_I.

<u>Axioms 1 - 10</u> are the same as for R.

<u>Axiom 11</u>. For each object G, there exists objects $|G|_0, |G|_1 \dots |G|_n$, bijections $t_{|G|_i}$, θ_G^i such that for every discrete object S, and any map $S \xrightarrow{f} G$ there exists unique maps f_i such that

$$\begin{tikzcd}
{|G|} \arrow[r, "t_{|G|_i}"] \arrow[rr, bend left, "t_G"] & {|G|_i} \arrow[r, "\theta_G^i"] & G \\
& S \arrow[ul, "f'"] \arrow[u, dashed, "f_i"'] \arrow[ur, "f"'] &
\end{tikzcd}$$

commutes where $t, ||$ are as in axiom 7.

<u>Definition:</u> G is an i-object $\Longleftrightarrow$ $0_i = 1$

<u>Axiom 12.</u> If $\theta_i = \theta_j$ $i \neq j$ then $\theta_i = \theta_j$ for all i, j and furthermore $t_G = 1_G$ i.e. G is discrete.

<u>Axiom 14_i</u>. For any i object H and any map $H \xrightarrow{f} G$ there exists uniquely a map f^* such that

$$\begin{tikzcd} {|G|} \arrow[rr, dashed, "\theta^i_G"] & & G \\ & H \arrow[ul, "f_i^*"] \arrow[ur, "f"'] & \end{tikzcd}$$

commutes; moreover $|f|' = |f_i^*|$.

<u>Axioms</u> 15_i - 19_i . The i-objects satisfies axiom 11 - 14 of R .

Let G be any object, let $(|G|, m_i)$ represent $|G|_i$, $(||G|_i| = |G|)$ as in axiom 14_i . Thus we may say $(|G|, m_i)_{i<n}$ represents G. If $G \xrightarrow{f} H$ then we have $|G|_i \xrightarrow{|f|_i} |H|_i$, and $||f|_i| = |f|$, thus we have the following commutative diagrams

$$\begin{tikzcd} R_i \arrow[r, "m_i"] \arrow[d, "f'"'] & {|G|^{n_i}} \arrow[d, "|f|^{n_i}"] \\ R_i \arrow[r, "m_i"] & {|H|} \end{tikzcd}$$

where $n_i = |A_i|$ where A_i is the object A in the copy of R associated with i .

<u>Axiom 20</u>. $(|G|, m_i)_{i<n}$ represents G , and $(|G|, m'_i)_{i<n}$ represents G, then $m'_i = h_i m_i$ where the h_i are isomorphisms.

__Axiom 21.__ If G is discrete and m_i are monomorphisms $R_i \xrightarrow{m_i} |G|^{n_i}$ then there exists uniquely up to isomorphism on object H, $|H| = G$ and $(G, m_i)_{i<n}$ represents H.

__Axiom 22.__ If $(G\, m_i)_{i<n}$, $(H\, m'_i)_{i<n}$ represent~~s~~ G^*, H^* respectively, and furthermore if $G \xrightarrow{f} H$ such that $\exists !\, f'_i$ for which

$$\begin{tikzcd} R_i \arrow[r, "m_i"] \arrow[d, "f'_i"'] & G^{n_i} \arrow[d, "f^{n_i}"] \\ R'_i \arrow[r, "m'_i"] & H^{n_i} \end{tikzcd}$$

commutes then there exists f^*, $G^* \xrightarrow{f^*} H^*$, $|f^*| = f$; furthermore $(G, k^j_i)_{i<n}$, $(H, h^j_i)_{i<n}$ represent $|G^*|_j$ and $|H^*|_j$ respectively then f represents $|f^*|_i$.

__Definition:__ The n-tuple category N_I is the category where objects are n-tuples $(G, m_i)_{i<n}$ where $R \xrightarrow{m_i} |G|^{n_i}$, $|A_i| = n_i$ and G is discrete and $(G, m_i)_{i<n} \xrightarrow{f} (H, k_i)_{i<n} \Longleftrightarrow G \xrightarrow{f} H$ and for all i, $\exists f'$

$$\begin{tikzcd}
R_i \arrow[r, "m_i"] \arrow[d, "f'"'] & G^{n_i} \arrow[d, "f^{n_i}"] \\
R'_i \arrow[r, "k_i"] & H^{n_i}
\end{tikzcd}$$

commutes.

<u>Definition</u>: $\delta_i^j(G, m_i)_{i<n} = (G\ m'_i)_{i<n}$ where $m'_i = 0 \quad i \neq j \quad m'_j = m_j$.

<u>Meta-theorem I'</u>: Let $\mathcal{A}$ be any model of R_I and let B be the n-tuple category constructed from $\mathcal{A}$. Then there exists a natural equivalence $T : \mathcal{A} \longrightarrow B$ such that $T(|G|_i) = \delta_k^i(T(G))$,

$T(A_i) = (n_i, m_i)$ where $m_i = 0 \quad i \neq j$ $m_j = [|_{A_j}|]$.

<u>Meta-theorem 2'</u>: Let B, C be complete models of R_I . Let B', C' be the pair categories constructed from them. Let $|A_i|_B = n_i$ and $|A_i|_C = m_i$ then B' and C' are naturally equivalent iff there exists a function $f : I \longrightarrow I$ one-one, onto such that $Law(n_i) \cong m_{f(i)}$. If B' and C' are naturally equivalent then the equivalence can be given by $Law'(G, n_i) \cong (Law(G), Law\ n_{f(i)})$.

Hence $\mathcal{B}$ and $\mathcal{C}$ are equivalent iff an f with the above property exists.

Remark. One can make the same remarks modulo the appropriate changes about R_I that one can make about R .

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