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EPIREFLECTIVE SUBCATEGORIES OF TOP NEED NOT BE COWELL-POWERED

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Abstract: It is shown that there exists an epireflective full subcategory $\underline{B}$ of TOP which is not cowellpowered, and a full subcategory of $\underline{B}$ which is strongly closed under the formation of limits in $\underline{B}$ and hence closed under the formation of limits in TOP but is not reflective in $\underline{B}$ or TOP .

Key words: (Epi)reflective subcategory, (strongly) closed under formation of limits, cowellpowered, TOP .

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Answering a problem in [3], V. Kannan and M. Rajagopalan [4, 5] , V. Trnková [7] and V. Koubek [6] proved that there exists a proper class K of Hausdorff spaces such that any continuous map between members of K is either constant or an identity. This result has several remarkable (unpleasant!) consequences. If A denotes the full subcategory of the category TOP of topological spaces and continuous maps whose objects are products of members of K , and if $\underline{B}$ denotes epireflective hull of K in TOP , i.e. the full subcategory of TOP whose members are subspaces (= extremal subobjects) of members of $\underline{A}$, then the following hold:

(1) $\underline{A}$ is a full subcategory of TOP which is closed under the formation of limits in TOP, but which is not reflective in TOP [5],

(2) $\underline{B}$ is an epireflective subcategory of TOP which is not cowellpowered,

(3) $\underline{A}$ is a full subcategory of $\underline{B}$ which is strongly closed under the formation of limits in $\underline{B}$, but which is not reflective in $\underline{B}$.

Using the fact that every continuous maps between $\underline{A}$-objects is a projection [1], proofs are straightforward.

These observations are especially interesting in view of the following propositions:

(a) A full, isomorphism-closed subcategory $\underline{A}$ of a complete, wellpowered, and cowellpowered category $\underline{B}$ is epireflective in $\underline{B}$ if and only if it is strongly closed under the formation of limits in $\underline{B}$ [2].

(b) If $\underline{C}$ is a complete, wellpowered and cowellpowered category, $\underline{A}$ is a full, isomorphism-closed subcategory of $\underline{C}$, and if the epireflective hull $\underline{B}$ of $\underline{A}$ in $\underline{C}$ is cowellpowered, then the following hold:

(1) $\underline{A}$ is reflective in $\underline{C}$ iff $\underline{A}$ is closed under the formation of limits in $\underline{C}$.

(2) $\underline{A}$ has a reflective hull $\underline{D}$ in $\underline{C}$, and $\underline{D}$ is simultaneously the epirefelctive hull of $\underline{A}$ in $\underline{B}$ and

the closure of $\underline{A}$ under the formation of limits in $\underline{C}$.

Hence, although the theory of full epireflective (resp. more general E-reflective for suitable classes E' of epimorphisms) subcategories of decent categories is well understood, we are not even able to characterize the full, reflective subcategories of TOP without using any smallness conditions.

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