Jiří Rosický

Model theoretic approach to concrete categories

Commentationes Mathematicae Universitatis Carolinae, Vol. 20 (1979), No. 1, 192--193

Persistent URL: `http://dml.cz/dmlcz/105917`

INCLUSION ORDERING OF CLASSES OF E-COMPACTNESS

Jan Pelant, Alexander Šostak (Českosl. Akad. Věd, Praha 1, Československo), received 23.11. 1978

We answered negatively Mrówka's question [M] of whether classes between $\mathcal{K}(T(\omega_\alpha))$ and $\mathcal{K}(\mathcal{D})$ are linearly ordered by inclusion $\subset$. ($\mathcal{D}$ denotes the two point discrete space, $T(\omega_\alpha)$ denotes the ordered space of ordinals less than ω_α). Our results can be divided into two parts:

1. More general results: using the Solovay theorem on stationary sets we proved:

<u>Theorem</u>: Let ω_α be an uncountable regular initial ordinal. Then there are $2^{2^{\omega_\alpha}}$ classes of E-compactness which are contained in $\mathcal{K}(T(\omega_\alpha))$, contain $\mathcal{K}(\mathcal{D})$ and are not comparable by inclusion.

2. More concrete results: we constructed several particular examples which solve Mrówka's question as well. In one of these constructions we used the compactification cN of a countable discrete space N satisfying: 1) no subsequence of N converges in cN, 2) there is no $M \subset N$ such that

$\overline{M}^{cN} = \beta N$ (= the Čech-Stone compactification)

These results were achieved mainly during the second author's visit to Prague in December 1973.
P. Simon has recently constructed a very similar compactification b(N) of N for which b(N) $-$ N is sequentially compact.

<u>Reference</u>:

[M] S. Mrówka: Further results on E-compact spaces, Acta math. (120)(1968), 161-185.

ORDERABILITY OF SPACES WITH LINEARLY ORDERED UNIFORM BASE

M. Hušek (Karlova Universita, 18600 Praha, Československo), received 30.11. 1978

<u>Theorem:</u> Let X be a nonmetrizable topological T_1-space induced by a uniformity with a linearly ordered base. Then the topology of X is an order-topology.

This result generalizes that for topological groups proved by P.J. Nyikos, H.-C. Reichel (Gen. Top. Appl. 5(1975), 195-204) and that for spaces without isolated points proved by R. Frankiewicz, W. Kulpa (this issue). The result for metrizable 0-dimensional spaces was proved by H. Herrlich (Math. Ann. 159(1965), 77-80).

MODEL THEORETIC APPROACH TO CONCRETE CATEGORIES

Jiří Rosický (Universita J.E. Purkyně,Brno, Československo), received 24.11. 1978

An infinitary first-order language $L_{\infty,\infty}$ has a class of function symbols, a class of relation symbols and a class of variables. Arities of function and relation symbols are arbitrary cardinals. Infinitary conjunctions and quantifiers are admitted. This language is suitable for the study of concrete categories as the following results indicate.

Any concrete category is equivalent to a category of models of $L_{\infty,\infty}$. Any concrete category is equivalent to the category of all models of some theory of $L_{\infty^+,\infty}$ (class-indexed conjunctions are admitted). A theory of $L_{\infty,\infty}$ will be called normal if it has (up to the equivalence) only a set of n-ary atomic formulas for each cardinal n. A concrete category is strongly fibre-small (in the sense of Adámek, Herrlich and Strecker) iff it is equivalent to the category of all models of some normal theory of $L_{\infty,\infty}$. The Beck's theorem gives the characterization of categories of models of normal equational theories of $L_{\infty,\infty}$. Categories of models of normal universal Horn theories of $L_{\infty,\infty}$ was described in the foregoing author's paper (Arch. Math. (Brno) 4(1978), 219-226). Further, universal-uniquely existencial Horn theories correspond to the existence of a left adjoint to the underlying functor of the category of models and Horn theories to the existence of a stable left adjoint. The subsequent author's paper will also contain the model-theoretic treatment of Mac Neille completions and of (cartesian closed) initially complete categories.

SEQUENCE SOLUTIONS OF THE DIRICHLET PROBLEM

Jiří Veselý (Karlova Universita, 18600 Praha, Československo),
received 22.1. 1979

Let X be a $\mathfrak{P}$-harmonic space with the countable base in the sense of Constantinescu and Cornea in which constants are harmonic functions. A sequence solution of the Dirichlet problem is the solution obtained as the limit of a properly defined sequence of functions. Suppose that $D \subset X$ is an open relatively compact set and $f \in C(\partial D)$ a boundary condition ($C(M)$ denotes the system ot continuous functions on M). A generalization of Lebesgue's procedure can be obtained in the following way: denote by ρ a metric compatible with the topology of X and by ε_x^t the Dirac measure ε_x swept-out on the complement of the open sphere with centre x and radius t. For an $r \in C(D)$, $0 < r(x) \leq \operatorname{dist}(x, X \setminus D)$, an increasing $g \in C(\,[0,\infty[\,)$, $g(0) = 0$, $g > 0$ on $]0,\infty[$ and $G \in C(\bar{D})$ put

$$AG(x) = [g(r(x))]^{-1} \int_0^{r(x)} \varepsilon_x^t(G)\,dg(t), \quad x \in D.$$

Now choose an $F \in C(\bar{D})$, $F = f$ on ∂D and define $F_n = A^n F$, $n \in N$. Then $\{F_n\}$ is convergent (uniformly on compact subsets of D) to the solution which coincides with the PWB-solution H_f^D corresponding to f and D. Remark that in this context there is eventually more than one reasonable generalized solution. Some other related results will appear under the same title in Časopis Pěst. Mat.