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Existence and multiplicity results for nonlinear noncoercive equations

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ANNOUNCEMENTS OF NEW RESULTS

VARIETIES OF SUBREGULAR ALGEBRAS ARE DEFINABLE BY A MAL'CEV

CONDITION

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In [1], J. Timm introduced the concept of subregular algebra as follows: An algebra \mathcal{A} is called subregular if any congruence θ on \mathcal{A} is uniquely determined by its classes $[b]_\theta$, $b \in \mathcal{B}$, for every subalgebra \mathcal{B} of \mathcal{A} .

Theorem. For any variety V , the following conditions are equivalent:

- (1) Every algebra $\mathcal{A} \in V$ is subregular;
- (2) There exist unary polynomials u_1, \dots, u_n , ternary polynomials p_1, \dots, p_n and 4-ary polynomials s_1, \dots, s_n such that

$$x = s_1(x, y, z, u_1(z))$$

$$s_i(x, y, z, p_i(x, y, z)) = s_{i+1}(x, y, z, u_{i+1}(z)) \text{ for } 1 \leq i < n$$

$$y = s_n(x, y, z, p_n(x, y, z))$$

$$u_i(z) = p_i(x, x, z) \text{ for } 1 \leq i \leq n;$$

- (3) There exist unary polynomials u_1, \dots, u_n and ternary polynomials p_1, \dots, p_n such that

$$(u_i(z) = p_i(x, y, z), 1 \leq i \leq n) \Leftrightarrow x = y.$$

R e f e r e n c e s

- [1] J. TIMM: On regular algebras, in Contributions to universal algebra, Proceedings of the Colloquium held in Szeged, 1975. Coll. Math. Soc. J. Bolyai, Vol. 17. Norht-Holland, Amsterdam 1977, pp. 503-514.

EXISTENCE AND MULTIPLICITY RESULTS FOR NONLINEAR NONCOERCIVE

EQUATIONS

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We assume that $L: D(L) \subset L^2(\Omega) \rightarrow L^2(\Omega)$ is linear self-adjoint operator with dense domain $D(L)$ and closed range $R(L)$. Let 0 be an eigenvalue of L and let for the corresponding eigenspace $\dim N(L) < +\infty$; $L^2(\Omega) = N(L) \oplus R(L)$. We assume that the functions in $N(L)$ satisfy the "unique continuation property" (i.e. the only function $w \in N(L)$ which is vanishing on

the set of positive measure in Ω is $w \equiv 0$). Let $K:R(L) \rightarrow R(L)$ (the right inverse of L) be compact.

Let $G:L^2(\Omega) \rightarrow L^2(\Omega)$ be the Nemytskii operator associated with g (i.e. $G(u)(x) = g(u(x))$, $x \in \Omega$), where $g:R \rightarrow R$ is a continuous odd bounded function with continuous derivative g' on R , $c = \|K\| \sup_{z \in R} g'(z) < 1$ and $\int_0^{+\infty} |g(z)| dz < +\infty$.

Theorem. For $f_2 \in R(L)$ either

(i) for each $w \in N(L)$ there exists precisely one $v(w) \in R(L)$ such that $u = w + v(w)$ is solution of the equation $Lu + G(u) = f_2$ and there is no solution of $Lu + G(u) = f$ with $f = f_1 + f_2$, $f_1 \in N(L)$, $f_1 \neq 0$;

or (ii) the equation $Lu + G(u) = f_2$ has at least one solution and there is a real number $T(f_2) > 0$ such that the equation $Lu + G(u) = f_1 + f_2$ has at least two distinct solutions if $0 < \|f_1\| < T(f_2)$.

In distinction from the previous papers dealing with such a type of nonlinearity we assume nothing about the limits

$$\gamma(a)_+ = \lim_{x \rightarrow +\infty} \inf_{b \in \langle x, x+z \rangle} \chi_{\text{min}}^\infty(s).$$

The functions $g(s) = se^{-s^2}$ and $g(s) = \sin(s)e^{-s^2}$ can be given as an example.