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THREE CLASSES OF DIAMETER EDGE-INVARIANT GRAPHS

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Abstract: Three classes of diameter edge-invariant graphs are presented.

Key word and phrases: Diameter, diameter-edge-invariant (d.e.i), Young tableau graph with diagonal crossing, 2-dimensional polyominoes , communication network.

Classification: 05C99

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1. Introduction. In this paper, a graph G=(V,E) is always undirected, without multiple edges and loops. A graph G=(V,E) is said to be connected, if for each u, v in G there exists a path connecting u and v. For a connected graph G, we can define a distance function d_G on V(G) as follows: For each u and v in G, $d_G(u,v)$ is the length of the shortest path between u and v. The diameter D(G) of the graph G is defined as the $\max(d_G(u,v)$: for all $u,v \in V(G))$ [6].

A connected graph G is said to be diameter-edge-invariant (d.e.i), if $D(G\setminus e)=D(G)$ for all $e \in E(G)$.

In a communication network, the diameter of the network graph is a deciding factor in choosing the system topology which defines the interprocessor communication architecture. The authors of the survey articles [1] and [2] showed the importance of the diameter in a computer communication system. In fact, it is essential not to increase the maximum time delay of communication of messages when there is a failure in one of the communication links between any two nodes in the system. Thus, we need to consider the design of diameter-edge-invariant networks.

In [5], several constructions of d.e.i. graphs are proposed by the first author. In this paper, we are interested in the design of three families of planar networks which are d.e.i. In the first section, we consider special types of 2-dimensional polyominoes [3] which are called Young Tableau graphs. We will show that almost all Young Tableau graphs are d.e.i.

2. Young Tableau Graphs which are diameter-edge-invariant

In [3], the first author introduced the concept of a Young Tableau graph. Given $\beta=(n_1,n_2,\ldots n_k)$ where $k\geq 1$, and $1\leq n_1\leq n_2\leq \leq\ldots\leq n_k$, the Young Tableau graph $Y(\beta)$ is a graph of the form (Fig. 1):

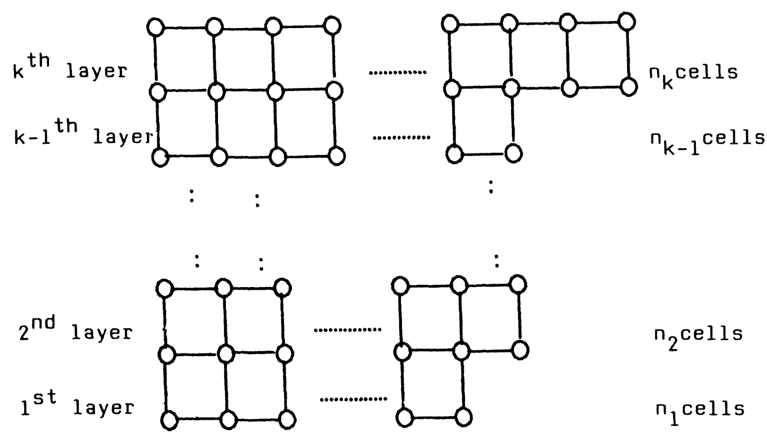


Figure 1. Young Tableau Graph $Y(\beta)$

In [3], it is shown that $Y(\beta)$ is d-graceful.

Theorem 2.1. For any $\beta=(n_1,n_2,\ldots,n_k)$ with the property $k\geq 1$, and $1\leq n_1\leq n_2\leq\ldots\leq n_k$, and not of the form (1,1,...,1) or. (n), a Young Tableau graph $Y(\beta)$ is d.e.i. with the diameter equal to n_k+k.

Proof. If β is of the form (1,1,...,1) with k 1's, then $Y(\beta)$ is isomorphic to the graph $G=P_2\times P_{k+1}$ with the diameter k+1. (Figure 2)

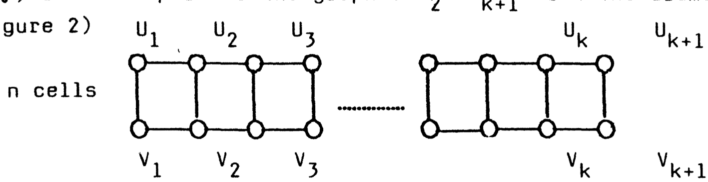


Figure 2. Young Tableau Graph Y(1,1,...,1) which is not d.e.i.

Since we observe $D(G \setminus (u_1,u_2))$ is k+2, it is not d.e.i.

Now suppose $\beta=(n_1,n_2,\ldots,n_k)$ and not of the form $(1,1,\ldots,1)$ or (n), it is clear that $D(Y(\beta))=n_k+k \geq 4$. For each $e=(x,y)$ in $E(Y(\beta))$, we see that $d_{Y(\beta)\setminus e}(x,y)=3$, and for each pair of nodes u and v in $Y(\beta)$, there are more than two paths from u to v. Thus we have $D(Y(\beta)\setminus e)=D(Y(\beta))$. □

<u>Example 1</u>. For k=2, and $\beta=(3,4)$, $Y(\beta)$ is d.e.i. with diameter = 2+4=6. We observe that there are several paths of length 6 connecting nodes 1 & 14.

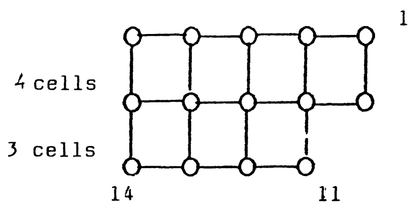


Figure 3. Young Tableau Graph Y(3,4) which is d.e.i. of diameter 6.

<u>Corollary 2.2</u>. For any $n,m > 2$, the grid graph $P_n \times P_m$ is diameter-edge-invariant.

<u>Proof</u>. For $P_n \times P_m = Y(\underbrace{m-1,m-1,\ldots,m-1}_{n-1 \text{ terms}})$. □

3. <u>Young Tableau Graphs with Diagonal Crossing</u>

For any $k \geq 1$, and $\beta=(n_1,n_2,n_3,\ldots,n_k)$ with $1 \leq n_1 \leq n_2 \leq \leq \ldots \leq n_k$, a graph $N(\beta)$ is constructed from the Young Tableau Graph $Y(\beta)$ such that each cell has an additional diagonal from upper left corner to lower right corner. (See Fig. 3.)

<u>Theorem 3.3</u>. For any $k \geq 1$, and $\beta=(n_1,n_2,n_3,\ldots,n_k)$ with integers $1 \leq n_1 \leq n_2 \leq \ldots \leq n_k$, the graph $N(\beta)$ is d.e.i. with diameter n_k+k.

<u>Example 2</u>: If k=3, and $\beta=(5,6,7)$, then $N(\beta)$ has a diameter of 12.

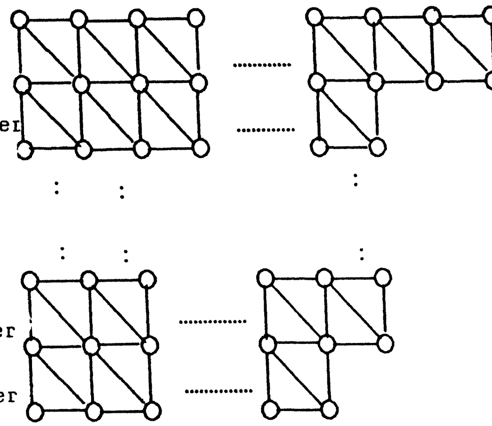


Figure 3: Young Tableau Graph with Diagonal Crossing, $N(n_1, n_2, \ldots, n_k)$.

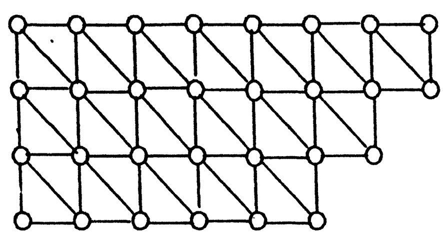


Figure 4: Young Tableau Graph with Diagonal Crossing, N(5,6,7)

4. Reverse Young Tableau Graph with Diagonal Crossing

For any $k \geq 1$, and $\beta = (a_1, a_2, a_3, \ldots, a_k)$ with $1 \leq a_1 \leq a_2 \ldots \ldots \leq a_k$, a graph $T(\beta)$ is of the following form (see Fig. 5):

Theorem 4.4. For any $k \geq 1$, and $\beta = (a_1, a_2, a_3, \ldots, a_k)$ with $1 \leq a_1 \leq a_2 \leq \ldots \leq a_k$, a planar network $T(\beta)$ is d.e.i.

Computing the diameter of $T(\beta)$ is much more difficult than that of $N(\beta)$. However, we have found
$D(T(\beta))$=max (the diameter of the maximal subrectangles of $T(\beta)$).

Example 3: For i=6, the graph T(5,5,8,8,8,9) is d.e.i. and its diameter is equal to the diameter of the second subrectangle (see Fig. 6)

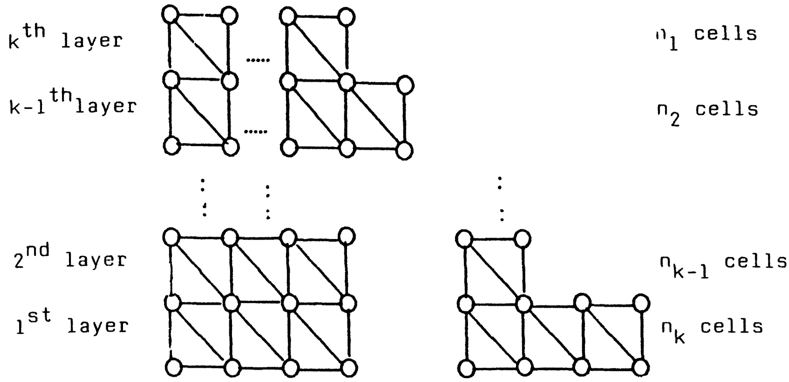


Figure 5: Reverse Young Tableau Graph with Diagonal Crossing

$D(T(\beta)) = \max(6+5,\ 4+8,\ 1+9) = 12.$

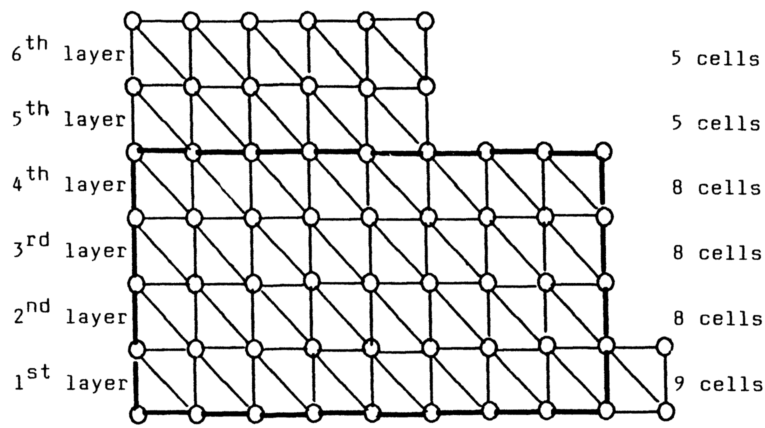
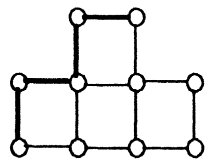


Figure 6: Reverse Young Tableau Graph with Diagonal Crossing, $T(\beta)$.

Note: Not all 2-dimensional polyominoes are d.e.i. For example, in the following graph (see Figure 7a) the diameter is 4, and if one edge is removed (see Figure 7b), diameter becomes 5.

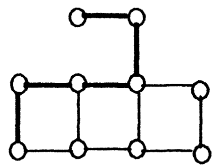
We propose the following

Problem: Characterize 2-dimensional polyominoes which are d.e.i.



diameter = 4

Figure 7 a



diameter = 5

Figure 7 b

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