Archivum Mathematicum

Jan Chvalina
Concerning a non-realizability of connected compact semi-separated closure operations by set-systems

Archivum Mathematicum, Vol. 12 (1976), No. 1, 45--51

Persistent URL: http://dml.cz/dmlcz/106926

Terms of use:

© Masaryk University, 1976

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.

This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://project.dml.cz

ARCH. MATH. 1, SCRIPTA FAC. SCI. NAT. UJEP BRUNENSIS
XII: 45—52, 1976

CONCERNING A NON-REALIZABILITY OF CONNECTED COMPACT SEMI-SEPARATED CLOSURE OPERATIONS BY SET-SYSTEMS

JAN CHVALINA, Brno
(Received August 25, 1975)

In this note there is given an elementary proof of a non-realizability of the category of connected compact semi-separated closure spaces and continuous mappings in the category of all set-systems and inversely compatible mappings. Further, there is formulated certain conclusion concerning a non-realizability of separated proximities by set-systems. Note that a closure space (in the sense of [1]) is said to be semi-separated (also it is called a closure T_1-space) if for every two different points there exists neighbourhood each of others which do not contain the other point. The category of all closure spaces with continuous mappings as morphisms will be denoted by Cl. Full subcategories of the last one, objects of which are all semi-separated closure spaces. connected compact semi-separated closure spaces we denote by Cl, Con_1 respectively. The symbol S^- will be used for the category of all pairs (P, S), where P is a set and $S \subset \exp P$ and inversely compatible mappings, i.e. $f \in [(P, S), (Q, T)]_S$ if $X \in T$ implies $f^{-1}(X) \in S$ (cf. e.g. [4], [8]). A realization of a concrete category (K_1, U_1) into a concrete category (K_2, U_2), (where U_1, U_2 are forgetful functors from K_i into the category Set of all sets and mappings), is a full functor F: $K_1 \to K_2$ one-to-one on objects and morphisms such that $U_2 \,.\, F = U_1$. Let X be a set, (K, U) a concrete category (cf. [2], [6]). A full subcategory of K formed by all objects $A \in \text{ob } K$ such that $U(A) = X$ will be denoted by $K(X)$. If $f \in \text{mor } K$, $U : K \to Set$ is a forgetful functor, we write f instead of $U(f)$. A closure space (P, u) is said to be compact if every proper filter of sets on (P, u) has a cluster point in (P, u) (def. 41A.3. in [1]).

1. We are going to prove a local non-realizability of the category Con_1 in the category S^-. Define certain closure operations u_1, v_1 on a three-element set $A = \{a_1, a_2, a_3\}$ as follows:

$$u_1\{a_1\} = \{a_1, a_2\}, \quad u_1\{a_2\} = \{a_2, a_3\}, \quad u_1\{a_3\} = \{a_1, a_3\},$$
$$v_1\{a_1\} = \{a_1, a_3\}, \quad v_1\{a_2\} = \{a_1, a_2\}, \quad v_1\{a_3\} = \{a_2, a_3\}.$$

Let P be an infinite set. Denote by u^* the coarsest T_1-topology on the set P, i.e. the topology of finite complements, which is also called a Fréchet topology. Let $\{P_i : i = 1, 2, 3\}$ be a decomposition of the set P such that card P_1 = card P_2 =

$= \text{card } P_3 = \text{card } P$. Further, denote by ξ a canonical mapping of P onto A such that $\xi^{-1}(a_i) = P_i$ for $i = 1, 2, 3$, by u a closure operation on the set P projectively generated by the system $\{\xi: P \to (A, u_1), \mathrm{id}_P: P \to (P, u^*)\}$ and by v a closure operation projectively generated by $\{\xi: P \to (A, v_1), \mathrm{id}_P: P \to (P, u^*)\}$ (see 32A.2. in [1]), i.e. u is the coarsest closure operation on P such that mappings $\xi: (P, u) \to (A, u_1)$, $\mathrm{id}_P: (P, u) \to (P, u^*)$ are continuous and similarly for v.

1.1. Lemma. *Let P be an infinite set, u, v, be the above defined closure operations on P. It holds*: $(P, u), (P, v) \in \text{ob } \mathsf{Con}_1$ *and* $u^2 \neq u \neq v \neq v^2$, (*where u^2 is the second iteration of u*).

Proof. Consider the space (P, u). Since $id_p: (P, u) \to (P, u^*)$ is a continuous mapping, (P, u) is semi-separated. Let $X \subseteqq P$ be a non-void subset. From above and the definition of (P, u) it follows that finite sets, $\emptyset$ and P are the only closed sets in the space (P, u). Hence (P, u) does not contain any proper non-void clopen subset thus according to 20 B.2. in [1] it is connected. Let R be an interior cover of (P, u), i.e. $R \subset \exp P$ and $P = \bigcup_{X \in R} \text{int}_u X$. According to the definition of (P, u), for $X \subset P$ it holds $\text{int}_u X \neq \emptyset$ if and only if there exist at least two indexes $i, j \in \{1, 2, 3\}$, $i \neq j$ with $P_i \cup P_j \subseteqq X$. Thus R contains a finite subcover of (P, u) hence in regard with theorem 41 A.9. [1] the space (P, u) is compact. We have $(P, u) \in \text{ob } \mathsf{Con}_1$. The same holds evidently for the space (P, v). Further, we have $uP_1 = P_1 \cup P_2$, $uP_2 = P_2 \cup P_3$, $vP_1 = P_1 \cup P_3$, $vP_3 = P_2 \cup P_3$, $u^2P_1 = u(P_1 \cup P_2) = P$ and $v^2P_1 = v(P_1 \cup P_3) = P$, q.e.d.

In what follows we suppose that P is a fixed infinite set and for $X \subseteqq P$ put $X_i = X \cap P_i$, $i = 1, 2, 3$. Let $B = \{b_1, b_2\}$ be a set, τ a topology on B defined by the rules: $\tau\{b_1\} = B$, $\tau\{b_2\} = \{b_2\}$. Consider a two-element decomposition $\{Q_i : i = 1, 2\}$ of the set P such that $\text{card } Q_1 = \text{card } Q_2 = \text{card } P$ and denote by t a T_1-topology on P projectively generated by the system $\{\gamma : P \to (B, \tau), \mathrm{id}_P : P \to (P, u^*)\}$, where $\gamma(x) = b_i$ for $x \in Q_i$, $i = 1, 2$. Evidently (P, t) is a connected compact semi-separated topological space.

1.2. Theorem. *Let K be a full subcategory of Con_1 such that* $\{(P, u), (P, u^*), (P, t)\} \subseteqq \text{ob } \mathsf{K}$. *There is no realization of the category K into the category $\mathsf{S}^-(P)$.*

Proof. Let $F: \mathsf{K} \to \mathsf{S}^-$ be a realization. Let $F(P, u^*) = (P, S(u^*))$. According to proposition 5. in [8] it holds that $S(u^*)$ is either the system of all closed sets or the system of all open sets in (P, u^*). Without loss of generality it can be supposed that the first case occurs. (A functor $G : \mathsf{S}^- \to \mathsf{S}^-$ for which $G(P, S) = \{P - X : X \in S\}$ is a realization). Every bijection $f: P \to P$ belongs to $[(P, u), (P, u^*)]_{\mathsf{Con}_1}$ thus $X \subseteqq P$, $\text{card } X < \aleph_0$ implies $X \in S(u)$, where $(P, S(u)) = F(P, u)$. Since every bijection $g_i : P \to P_i$, $i \in \{1, 2, 3\}$, is an element from $[(P, u^*), (P, u)]_{\mathsf{Con}_1}$ we have that for $X \in S(u)$ the set $X_i = X \cap P_i$ is either finite or $X_i = P_i$. Let $X \in S(u)$, $X_{i_1} = P$, card

$X_{i_2} < \aleph_0$. Suppose that $i_1 = 1$, $i_2 = 2$. Let $f_1 : P \to P$ be such a mapping that restrictions $f_1 |_{Q_1} : Q_1 \to X_1$, $f_1 |_{Q_2} : Q_2 \to (P - X_2)$ are bijective. Since $f_1 \in \operatorname{mor} Cl$, $f_1 \in [(P, t), (P, u^*)]_{Con_1}$ we have according to theorem 32 A. 10. in [1] that $f_1 \in [(P, t), (P, u)]_{Con_1}$ and thus $Q_1 \in S(t)$. If $X_3 = P$, we consider a mapping $f_2 : P \to P$ such that $f_2 |_{Q_1} : Q_1 \to (P_2 - X_2)$ and $f_2 |_{Q_2} : Q_2 \to X_3$ are bijections. As above, there is $f_2 \in [(P, t), (P, u)]_{Con_1}$ hence $Q_2 \in S(t)$. If card $X_3 < \aleph_0$, we take a mapping $f_3 : P \to P$ with properties: $f_3 |_{Q_1} : Q_1 \to (P_3 - X_3)$ and $f_3 |_{Q_2} : Q_2 \to X$ are bijections. From the continuity of the mapping $f_3 : (P, t) \to (P, u)$ we get that $Q_2 \in S(t)$. Therefore it holds $Q_1, Q_2 \in S(t)$ which is a contradiction with lemma 2.b. in [8]. From here we have the equality

$$S(u) = \{X \subseteq P : \operatorname{card} X < \aleph_0\} \cup \{P\} = S(u^*),$$

which contradits $[(P, u), (P, u)]_{Con_1} \neq [(P, u^*), (P, u^*)]_{Con_1}$. The proof is complete.

Remark. It is evident that we can consider the closure space (P, v) instead of (P, u) and after the corresponding changes we get the result of the same type as above. Further, from the just proved theorem it follows especially the corresponding part of proposition 3. in [4], which is also a corollary of the theorem from paper [7] p. 537 obtained in another way.

It is to be noted that the closure operation u is realizable by a set-system with respect to the semigroup of all continuous transformations of the space (P, u). More precisely, it holds:

1.3. Proposition. *Let P be an infinite set. There exists a set-system $T \subset \exp P$ such that* $[(P, u), (P, u)]_{Cl} = [(P, T), (P, T)]_{S-}$.

Proof. Denote by λ the cyclic permutation $(1, 2, 3)$ of the set $J = \{1, 2, 3\}$ and put $T = \bigcup_{i=1}^{3} \{P_i \cup X : \emptyset \neq X \subset P_{\lambda(i)}, \operatorname{card} X < \aleph_0\} \cup \{X : X \subset P, \operatorname{card} X < \aleph_0\} \cup \cup \{P\}$. Evidently, the mapping $f : P \to P$ belongs to $[(P, u), (P, u)]_{Cl}$ if and only if it is either a constant mapping or there exists a morphism $\varphi \in [(A, u_1), (A, u_1)]_{Cl}$ such that $\xi f = \varphi \xi$, where u_1 and ξ are defined in the beginning of this paragraph and $f^{-1}(x)$ is a finite set for each $x \in P$. It is easy to verify that $[(P, u), (P, u)]_{Cl} \subset \subset [(P, T), (P, T)]_{S-}$. We are going to show that a mapping $f \in P^P$ such that $f \notin \notin [(P, T), (P, T)]_{S-}$ is not a continuous transformation of the space (P, u). It is sufficient to examine three following cases:

1° It does not exist a mapping $\varphi \in A^A$ such that $\xi f = \varphi \xi$.

2° There exists a mapping $\varphi \in A^A$ satisfying the condition $\xi f = \varphi \xi$, however $\varphi \notin [(A, u_1), (A, u_1)]_{Cl}$.

3° $\xi f = \varphi \xi$ for some $\varphi \in [(A, u_1), (A, u_1)]_{Cl}$ but there exists an infinite set $X \subset P$ with the property card $f(X) < \aleph_0$ and f is not a constant mapping.

In the case 1° there exists such an index $i_0 \in J$ that it holds: $f(P_{i_0}) \cap P_i \neq \emptyset \neq$ $\neq f(P_{i_0}) \cap P_j$, where $i, j \in J$, $i \neq j$. Consider $j \in J$ such that card $(P_j \cap f(P_{i_0})) =$ $=$ card P, that is card $f^{-1}(P_j) \geqq \aleph_0$. There exists $x_0 \in P_{\lambda(j)}$ with the property $f^{-1}(x_0) \cup f^{-1}(P_j) \subsetneqq P_{i_0}$. Then $\{x_0\} \cup P_j \in T$ but $f^{-1}(\{x_0\} \cup P_j) \notin T$ for card $f^{-1}(P_j) \geqq \aleph_0$.

In the second case we suppose that $\varphi(a_1) = a_2$, $\varphi(a_2) = a_1$, $\varphi(a_3) = a_3$. Since $\lambda(1) = 2$, $\lambda(2) = 3$, $\lambda(3) = 1$ and $\xi(P_i) = \{a_i\}$, for $i \in \{1, 2, 3\}$ we have $P_3 \cup \{x_0\} \in T$, where $x_0 \in P_1$ and $f^{-1}(P_3 \cup \{x_0\}) = P_3 \cup f^{-1}(x_0) \notin T$ for $f^{-1}(x_0) \subseteqq P_2$. If $\varphi(a_1) =$ $= \varphi(a_2) = a_1$ and $\varphi(a_3) = a_3$ or $\varphi(a_3) = a_2$ then for $x_0 \in P_2$ we have $P_1 \cup \{x_0\} \in T$, $P_1 \cup P_2 \subseteqq f^{-1}(P_1 \cup \{x_0\})$ hence $f^{-1}(P_1 \cup \{x_0\}) \notin T$. Similarly, we get in the remaining cases which are permutations of above introduced that f is not an inversely compatible mapping.

Let us consider case 3°. Choose a point $x_0 \in P_{i_0}$, $(i_0 \in J)$ for which card $f^{-1}(x_0) \geqq$ $\geqq \aleph_0$. There exists $j_0 \in J$ with $f^{-1}(x_0) \subseteqq P_{j_0}$. Let $k \in J$ be an index with the property $\lambda(k) = i_0$. It holds $P_k \cup \{x_0\} \in T$ and $f^{-1}(P_k) \cup f^{-1}(x_0) \notin T$.

Therefore we have got that $f \notin [(P, T), (P, T)]_{S-}$ hence $[(P, u), (P, u)]_{Cl} =$ $= [(P, T), (P, T)]_{S-}$, q.e.d.

2. We turn our attention to certain proximity induced by a closure operation. Some notions, first. By a proximity space it will be understood a proximity space in the sense of Čech's book [1]. Moreover we deal with general separated proximity spaces which are pairs (P, p), where P is a set and p a binary relation on $\exp P$ satisfying following conditions:

1° $\emptyset$ non p P,

2° p is symetric,

3° $X_1, X_2 \subset P$ then $(X_1 \cup X_2)\, pY$ if and if X_1pY or X_2pY,

4° if $x, y \in P$ then xpy if and only if $x = y$.

A proximity space (P, p) is called discrete if for X, $Y \subset P$ it holds XpY if and only if $X \cap Y \neq \emptyset$ and it is called p-connected if for each $X \subset P$, $X \neq \emptyset$ it holds $Xp(P - X)$. Notice that a p-connected space is defined in [5] (def. 2.1.) as a proximity space which cannot be mapped by a proximally continuous mapping onto a two-element discrete proximity space. Here, a proximity space in the sense of Jefremovič–Smirnov is ment but it is easy to verify that the above introduced definition is equivalent to the last one also in the case of general proximity spaces (cf. theorem 2.1. in [5]).

By P will be denoted the category of all proximity spaces (satisfying conditions 1°, 2°, 3° and (prox 3) from [1] p. 439[1] and proximally continuous mappings, by Pc, P_1 full subcategories of P of all p-connected, proximity spaces, separated proximity

[1] Axiom (prox3) in [1], 25, p. 439 says that $X \subset P$, $Y \subset P$, $X \cap Y \neq \emptyset$ implies XpY. It follows from 3 and 4°.

spaces respectively. Pc_1 means $Pc \cap P_1$. A proximity p for closure operation u on P defined by $X, Y \subset P$, XpY if and only if $uX \cap uY \neq \emptyset$, is called a Wallman proximity of (P, u), (cf. 25 A.18. in [1]). We shall denote it by p_u. Further, we denote by W a covariant functor from the category Cl into the category P such that for $(P, u) \in \text{ob } Cl$ it is $W(P, u) = (P, p_u)$ and which preserves actual forms of morphisms. Evidently such a functor exists since the continuity of the mapping $f : (P, u) \to (Q, v)$ implies the proximal continuity of $f : (P, p_u) \to (Q, p_v)$. If $(P, u) \in \text{ob } Cl_1$ then $W(P, u) \in \text{ob } P_1$. By W_1, W_2, W_T are denoted restrictions of W onto Cl_1, Con_1, B_1 respectively, where Cl_1 means the full subcategory of Cl of all semi-separated closure spaces and B_1 the full subcategory of the last one of all topological T_1-spaces.

2.1. Lemma. *The functor $W_1 : Cl_1 \to P_1$ is a realization.*

Proof. It is sufficient to show that the functor W_1 is one-to-one on objects and full. Let u, v be arbitrary various semi-separated closure operations on a set P, $W_1(P, u) = (P, p_u)$, $W_1(P, v) = (P, p_v)$. Let $X \subset P$ be a non-void set such that $uX \neq vX$; suppose that $uX - vX \neq \emptyset$. For $x \in uX - vX$ it holds $\{x\} p_u X$ and $\{x\}$ non $p_v X$, thus $p_u \neq p_v$. Let $(P, p), (Q, q) \in \text{ob } P_1$, $f \in [(P, p), (Q, q)]_{P_1}$ and $(P, u), (Q, v)$ be such objects of Cl_1 that $W_1(P, u) = (P, p)$, $W_1(Q, v) = (Q, q)$. Let X be a non-void subset of P, $x \in uX$ be a point. Since $\{x\} pX$, $\{f(x)\} qf(X)$ i.e. $f(x) \in vf(X)$, we have $f(uX) \subset vf(X)$. Hence $f \in [(P, u), (Q, v)]_{Cl_1}$ and we get that W_1 is full.

Remark. The functor $W : Cl \to P$ is not a realization because of e.g. closure operations τ, σ on the set $B = \{b_1, b_2\}$ defined by the rules: $\tau\{b_1\} = \sigma\{b_2\} = B$, $\tau\{b_2\} = \{b_2\}$, $\sigma\{b_1\} = \{b_1\}$ are different i.e. $\sigma \neq \tau$ and $W(B, \sigma) = (B, p^*) = W(B, \tau)$, where p^* is the coarsest proximity on B, i.e. $X, Y \subset B$, Xp^*Y if and only if $X \neq \emptyset \neq Y$.

Considering lemma 2.1. or example 1. from [3] we get from theorem 1.2.:

2.2. Theorem. *Let P be an infinite set. There is no realization of the category $Pc_1(P)$ into the category $S^-(P)$.*

Proof. Assuming that there exists a realization $G : Pc_1(P) \to S^-(P)$, we get according to lemma 2.1. that the functor $F = G \circ W_2 : Con_1(P) \to S^-(P)$ is also a realization, which is a contradiction with theorem 1.2.

Remark. The obtained result can be specified as in theorem 1.2. with the use of proximity spaces $W_1(P, u)$, $W_2(P, t)$ and $W_1(P, u)$, where corresponding closure spaces are those considered in the preceeding paragraph.

In paper [3] is contained a general method for the construction of special functors between considered concrete categories as full embeddings, especially realizations. There is also given a schema which makes possible to obtain the following proposition which we prove directly in another way.

2.3. Proposition. *The functor $W_T : B_1 \to P_1$ is the only realization of B_1 into P_1.*

Proof. The functor W_T is a realization according to lemma 2.1. Let $F: \mathcal{B}_1 \to \mathcal{P}_1$ be a realization, $(P, u) \in \operatorname{ob} \mathcal{B}_1$, $F(P, u) = (P, p)$. Let X, Y be arbitrary non-void subsets of P such that X non pX. Put $Q = u(X \cup Y)$ and consider the relativization u_Q of u onto Q. If we put $(Q, q) = F(Q, u_Q)$ we have X non qY for (uX) non $p(uY)$ in (P, p). Let $a \in X$, $b \in Y$. A mapping φ: $Q \to Q$ defined

$$\varphi(x) = \begin{cases} a \text{ for } x \in uX \\ b \text{ for } x \in uY \end{cases}$$

is a proximally continuous mapping of (Q, q) into itself. Indeed, if for $X_1, X_2 \subseteqq Q$ it holds $X_1 q X_2$ then either $X_1 \cap uX \neq \emptyset$ and $X_2 \cap uX \neq \emptyset$ or $X_1 \cap uY \neq \emptyset$ and $X_2 \cap uY \neq \emptyset$, hence $\varphi(X_1)\, q \varphi(X_2)$. From the fullness of F it follows that $\varphi \in$ $\in [(Q, u_Q), (Q, u_Q)]_{Cl_1}$ which implies $uX \cap uY = \emptyset$, hence X non $p_u Y$. Thus we have $p_u \subseteqq p$.

We shall show that $p \subseteqq p_u$. Admit that there exists a pair X, Y of non-void subsets of P such that XpY and $uX \cap uY = \emptyset$. Suppose that card $[P - u(X \cup Y)] \geqq \aleph_0$. Let v_0 be a topology for the set $A = \{a_1, a_2, a_3\}$ such that $v_0\{a_1\} = \{a_1\}$, $v_0\{a_2\} = \{a_2\}$, $v_0\{a_3\} = A$. Denote by ξ_1 a mapping of the set P onto A and by ξ_2 a mapping of the set $R = \{x_1, x_2\} \cup [P - u(X \cup Y)]$, (where $\{x_1, x_2\} \in X \times Y$ is an arbitrary but fixed pair), onto the set A defined as follows:

$$\xi_1(x) = \begin{cases} a_1 \text{ for } x \in uX, \\ a_2 \text{ for } x \in uY, \\ a_3 \text{ for } x \in P - u(X \cup Y), \end{cases} \qquad \xi_2(x) = \begin{cases} a_1 \text{ for } x = x_1, \\ a_2 \text{ for } x = x_2, \\ a_3 \text{ for } x \in P - u(X \cup Y). \end{cases}$$

Denote by v a T_1-topology on P projectively generated by the system $\{\xi_1 : P \to (A, v_0)$, $\mathrm{id}_P : P \to (P, u^*)\}$ and by w a T_1-topology on R, which is projectively generated by the system $\{\xi_2 : R \to (A, v_0)$, $\mathrm{id}_R : R \to (P, u^*)\}$, where u denotes as above the topology of finite complements. Finally, by f will be denoted a mapping of P onto R such that $f(x) = x_1$ for $x \in uX$, $f(x) = x_2$ for $x \in uY$ and $f(x) = x$ for $x \in [P -$ $- u(X \cup Y)]$. The considered diagram is following:

$$\begin{tikzcd}
(P, u) \arrow[r, "\mathrm{id}_P"] & (P, v) \arrow[dl, "\mathrm{id}_P"'] \arrow[d, "f"] \arrow[dr, "\xi_1"] & \\
(P, u^*) & (R, w) \arrow[l, "\mathrm{id}_R"'] \arrow[r, "\xi_2"] & (A, v_0)
\end{tikzcd}$$

In accordance with theorem 32 A.8. in [1] (as in the preceeding paragraph) we have that $f \in \operatorname{mor} Cl_1$. Then $F(f) \in [(P, s), (R, r)]_{\mathcal{P}_1}$ where $((P, s) = F(P, v)$, $(R, r) = F(R, w)$ and $\mathrm{id}_P \in [(P, p), (P, s)]_{\mathcal{P}_1}$, hence $\{x_1\} = f(X)\, r f(Y) = \{x_2\}$, thus $x_1 = x_2$, which contradicts $X \cap Y = \emptyset$. The same contradiction we get also in the case when $P \neq$

$\neq u(X \cup Y)$ and card $[P - u(X \cup Y)] < \aleph_0$. Indeed, considering a mapping ψ of (P, u) into itself such that $\psi(x) = x_1$ for $x \in uX$ and $\psi(x) = x_2$ for $x \in uY \cup \cup [P - u(X \cup Y)]$, where $x_1 \in X$, $x_2 \in Y$, we get $\{x_1\} = \psi(X) p \psi(Y) = \{x_2\}$ for $\psi \in [(P, u), (P, u)]_{Cl_1}$. Thus $x_1 = x_2$. In a similar way we obtain this contradiction also in the case when $X \cup Y$ is a dense set in (P, u), i.e. $P = u(X \cup Y)$, with the use of the restriction of ψ onto $u(X \cup Y)$; it is the simplest case. Therefore it holds $uX \cap uY \neq \emptyset$ and we have that $p \subseteqq p_u$, hence $p = p_u$. Consequently $F(P, u) = W_T(P, u)$ for each object $(P, u) \in \operatorname{ob} B_1$. The proof is complete.

In the first part of the proof it is not required that considered closure spaces are topological. We have either from here or using methods described in the second paragraph of [8] that the Wallman proximity for $(P, u) \in \operatorname{ob} Cl_1$ is not coarser then a proximity which is assigned to u by a realization of Cl_1 into P_1.

The author is indebted to Jiří Rosický for his valuable remarks and the critical perusal of the whole manuscript.

REFERENCES

[1] E. Čech, *Topological spaces,* Academia, Prague 1966.

[2] Z. Hedrlín, A., Putr and V. Trnková, *Concerning a categorial approach to topological and algebraic theories,* Proceedings of the Second Prague Topological Symposium 1966, Academia Prague 1967, 176—181.

[3] M. Hušek, *Constructions of special functors and its applications,* Comment. Math. Univ Carolinae 8, 3 (1967), 555—566.

[4] J. Chvalina and M. Sekanina, *Realizations of closure spaces by set-systems,* Proceedings of the Third Prague Topological Symposium 1971, Academia Prague 1972, 85—87.

[5] Z. P. Mamuzič, *Koneksni prostori,* Mat. institut, Posebna izdanja 11, Beograd 1974.

[6] A. Pultr, *On selecting of morphisms among all mappings between underlying sets of objects in concrete categories and realizations of these,* Comment. Math. Univ. Carolinae 8, (1967), 53—83.

[7] J. Rosický, *Full embeddings with a given restriction,* Comment. Math. Univ. Carolinae 14, 3 (1973), 519—540.

[8] J. Rosický and M. Sekanina, *Realizations of topologies by set-systems,* Proceedings of Colloquium on Topology, Keszthely 1972, 535—555.

J. Chvalina
662 95 Brno, Janáčkovo nám. 2a
Czechoslovakia