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24 SELF-INSCRIBED DECAGONS IN DESARGUES CONFIGURATION 10_3

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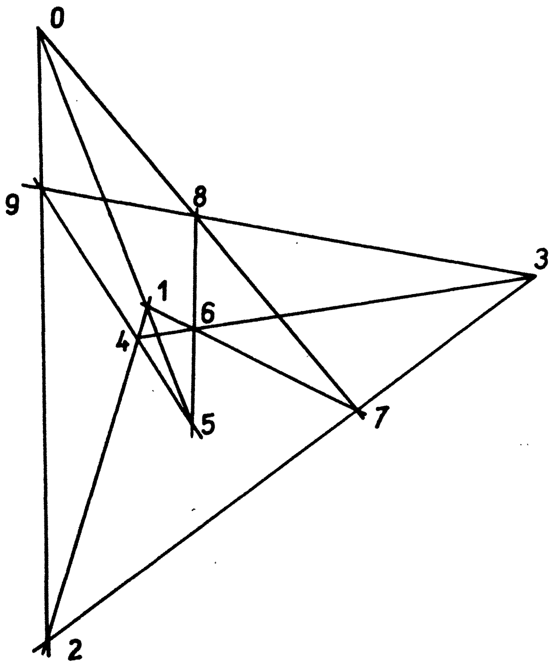
In this note we shall show that the Desargues configuration 10_3 can be regarded as a set of 24 self-inscribed decagons having a certain group structure.

The Desargues configuration 10_3 is a figure of 10 lines and 10 points, 3 points on each line and 3 lines through each point.



The existence of this configuration is a consequence of the Desargues theorem which states: *If two triangles are perspective from a point then they are perspective from a line.*

This configuration can be regarded in 10 ways as a pair of triangles in perspective, since every pair of triangles formed by the points ik, il, im and jk, jl, jm are perspective from the vertex ij and the side klm.

The Desargues configuration reveals five pairs of quadrangles and quadrilaterals, the latter inscribed into the former. The six sides, *ijk*, *ilm*, *ikl*, *imj*, *ijl*, *ikm*, of every quadrangle pass, respectively, through the six vertices *jk*, *lm*, *kl*, *jm*, *jl*, *km*, of the quadrilateral.

The same Desargues configuration reveals six pairs of mutually inscribed pentagons. The five vertices *ij*, *jk*, *kl*, *lm*, *mi*, of one lie on the five sides *jli*, *lik*, *ikm*, *kmj*, *mjl* of the other.

By placing the vertices of a pair of pentagons appropriately we get a self-inscribed decagon and by the use of 24 permutations of A, B, C, D (where $A, B, C, D =$ $= i, j, k, l, m$), we get 24 selfinscribed decagons as given below.

Writing 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 for *ij*, *jl*, *jk*, *km*, *kl*, *il*, *lm*, *jm*, *im*, *ik*, respectively we get the decagons

(1). 0 1 2 3 4 5 6 7 8 9,	(2). 0 7 2 9 4 1 6 3 8 5,
(3). 0 1 4 5 8 9 2 3 6 7,	(4). 0 9 4 3 8 7 2 1 6 5,
(5). 0 2 1 6 4 9 3 7 8 5,	(6). 0 7 1 5 4 2 3 6 8 9,
(7). 0 2 4 9 8 5 1 6 3 7,	(8). 0 5 4 6 8 7 1 2 3 9,
(9). 0 2 7 6 3 9 4 1 5 8,	(10). 0 1 7 8 3 2 4 6 5 9,
(11). 0 2 3 9 5 8 7 6 4 1,	(12). 0 8 3 6 5 1 7 2 4 9,
(13). 0 5 9 3 4 1 6 8 7 2.	(14). 0 8 9 2 4 5 6 3 7 1,
(15). 0 5 4 1 7 2 9 3 6 8,	(16). 0 2 4 3 7 8 9 5 6 1,
(17). 0 7 2 4 3 8 6 1 5 9,	(18). 0 1 2 9 3 7 6 4 5 8,
(19). 0 7 3 8 5 9 2 4 6 1,	(20). 0 9 3 4 5 1 2 7 6 8,
(21). 0 7 1 4 6 8 3 2 9 5,	(22). 0 2 1 5 6 7 3 4 9 8,
(23). 0 7 6 8 9 5 1 4 3 2,	(24). 0 5 6 4 9 2 1 7 3 8.

The group G of symmetries of the Desargues figure which leaves both the set P of 10 points and the set L of 10 lines invariant is the group of permutations of the 10 points, which preserve the set L of lines. If we also allow the reciprocity which interchanges P and L, we obtain the group G'. It has been shown by Coxeter (1950) that G has order 120 while G' has order 240. It is clear from this paper that G contains a subgroup of order 24, the symmetric group of degree four on A, B, C, D, (where $A, B, C, D = i, j, k, l, m$).

There are several ways in which G can be calculated. G is transitive over 10 points, 10 lines, the 24 self-inscribed decagons.

The stabilizers G_P (of a point), G_1 (of a line), G_d (of a self-inscribed decagon) have orders 12, 12, and 5 respectively. Hence G has order $10 \times 12 = 10 \times 12 = 24 \times$ $\times 5 = 120$.

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Reference

Coxeter, H. S. M., Self-dual configurations and graphs, Bulletin of the American Mathematical Society, *56*, 413—455; (1950).

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