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CONJUGATE POINTS OF SOLUTIONS OF A FOURTH-ORDER ITERATED LINEAR DIFFERENTIAL EQUATION

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The four-parameter space $\mathbf{Y} = y(t, C_1, \ldots, C_4) = \sum_{i=1}^{4} C_i \sin^{4-i} t \cos^{i-1} t$, $t \in (-\infty, +\infty)$, $C_i \in \boldsymbol{R}$, $i = 1, \ldots, 4$, of all solutions of the fourth-order iterated linear differential equation

$$y^{(IV)}(t) + 10y''(t) + 9y(t) = 0, \tag{1}$$

with base $[\sin^3 t, \sin^2 t \cos t, \sin t \cos^2 t, \cos^3 t]$ is broken up — with reference to triviality and nontriviality of the values of the parameters C_i ($i = 1, \ldots, 4$), respectively — into the following fifteen subspaces:

I. four one-parametric ($\mathbf{Y}^{I}_{1}, \mathbf{Y}^{I}_{2}, \mathbf{Y}^{I}_{3}, \mathbf{Y}^{I}_{4}$),
II. six two-parametric ($\mathbf{Y}^{II}_{12}, \mathbf{Y}^{II}_{13}, \mathbf{Y}^{II}_{14}, \mathbf{Y}^{II}_{23}, \mathbf{Y}^{II}_{24}, \mathbf{Y}^{II}_{34}$),
III. four three-parametric ($\mathbf{Y}^{III}_{123}, \mathbf{Y}^{III}_{124}, \mathbf{Y}^{III}_{134}, \mathbf{Y}^{III}_{234}$),
IV. one four-parametric ($\mathbf{Y}^{IV}_{1234}$).

This paper investigates the existence and the form of the conjugate points relating to the zeros of any (nontrivial) oscillatory solution of (1) of all subspaces cited in I.–IV.

Explanation to the Notation

Besides the standard logical symbols $\exists$ and $\forall$ (existential and universal quantifier), $\tilde{\exists}$ (non-existing), $\wedge$ ($\vee$, $\Rightarrow$, $\Leftrightarrow$) conjunction (disjunction, implication, equivalence), $\rightarrow$ (such that), : (holds), $\boldsymbol{R}(\boldsymbol{C})$ denotes the set of all real numbers (integers) and

$\nu(t)$ ... the multiplicity of the zero t, $N(t_1, \ldots, t_n)$ ... the number of the zeros $t_1, \ldots, t_n$ (including their multiplicities), $\rho\langle t_k, t_{k+n}\rangle$ ... the length of the closed interval $\langle t_k, t_{k+n}\rangle$, i.e. the distance of the zeros t_k, t_{k+n}, $\bar{\eta}_1(t_0)$ ... the first conjugate point of the point t_0, i.e. the smallest number $t_1 > t_0$ such that the nontrivial solution $y(t)$ of (1) with zeros t_0 and t_1 has exactly four zeros including their multiplicities on $\langle t_0, t_1\rangle$.

I. (1)

$$\forall[t\in(-\infty, +\infty)] \wedge \forall[C_1 \in \boldsymbol{R} - \{0\}] \exists[y(t, C_1) = C_1 \sin^3 t \in \mathbf{Y}_1^{\mathrm{I}} \to$$
$$\to y^{(\mathrm{IV})}(t, C_1) + 10y''(t, C_1) + 9y(t, C_1) = 0];$$
$$\{[y(t, C_1) = 0] \Leftrightarrow \forall(k \in C); [t_k = k\pi, \nu(t_k) = 3] \wedge$$
$$\wedge [t_k, t_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t_{k+1})] \wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}) = 6]\} \Rightarrow$$
$$\Rightarrow \tilde{\exists}\bar{\eta}_1(t_k).$$

I. (2)

$$\forall[t\in(-\infty, +\infty)] \wedge \forall[C_2 \in \boldsymbol{R} - \{0\}] \exists[y(t, C_2) = C_2 \sin^2 t \cos t \in \mathbf{Y}_2^{\mathrm{I}} \to$$
$$\to y^{(\mathrm{IV})}(t, C_2) + 10y''(t, C_2) + 9y(t, C_2) = 0];$$
$$\left\{[y(t, C_2) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}); \left[(t_k = k\pi, \nu(t_k) = 2) \wedge \left(t_k' = \frac{\pi}{2} + k\pi, \nu(t_k') = 1\right)\right] \wedge\right.$$
$$\wedge [t_k, t_{k+1}, t_k' \in \langle t_k, t_{k+1}\rangle \to (t_k < t_k' < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t_k') = 5] \wedge$$
$$\wedge [t_{k+1}, t_k', t_{k+1}' \in \langle t_k', t_{k+1}'\rangle \to (t_k' < t_{k+1} < t_{k+1}')] \wedge$$
$$\left.\wedge [\rho\langle t_k', t_{k+1}'\rangle = \pi, N(t_{k+1}, t_k', t_{k+1}') = 4]\right\} \Rightarrow$$
$$\Rightarrow \left\{\tilde{\exists}\bar{\eta}_1(t_k) \wedge \exists\left[\bar{\eta}_1(t_k') = \frac{\pi}{2} + (k+1)\pi\right]\right\}.$$

I. (3)

$$\forall[t\in(-\infty, +\infty)] \wedge \forall[C_3 \in \boldsymbol{R} - \{0\}] \exists[y(t, C_3) =$$
$$= C_3 \sin t \cos^2 t \in \mathbf{Y}_3^{\mathrm{I}} \to y^{(\mathrm{IV})}(t, C_3) + 10y''(t, C_3) + 9y(t, C_3) = 0];$$
$$\left\{[y(t, C_3) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \left[(t_k = k\pi, \nu(t_k) = 1) \wedge \left(t_k' = \frac{\pi}{2} + k\pi, \nu(t_k') = 2\right)\right] \wedge\right.$$
$$\wedge [t_k, t_{k+1}, t_k' \in \langle t_k, t_{k+1}\rangle \to (t_k < t_k' < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t_k') = 4] \wedge$$
$$\wedge [t_{k+1}, t_k', t_{k+1}' \in \langle t_k', t_{k+1}'\rangle \to (t_k' < t_{k+1} < t_{k+1}')] \wedge$$
$$\left.\wedge [\rho\langle t_k', t_{k+1}'\rangle = \pi, N(t_{k+1}, t_k', t_{k+1}') = 5]\right\} \Rightarrow \{\exists[\bar{\eta}_1(t_k) = (k+1)\pi] \wedge \tilde{\exists}\bar{\eta}_1(t_k')\}.$$

I. (4)

$$\forall[t\in(-\infty,+\infty)]\wedge\forall[C_4\in \boldsymbol{R}-\{0\}]\,\exists[y(t,C_4)=$$
$$=C_4\cos^3 t\in \mathbf{Y}_4^{\mathrm{I}}\to y^{(\mathrm{IV})}(t,C_4)+10y''(t,C_4)+9y(t,C_4)=0];$$
$$\Big\{[y(t,C_4)=0]\Leftrightarrow\forall(k\in \boldsymbol{C}):\Big[t_k=\frac{\pi}{2}+k\pi,\nu(t_k)=3\Big]\wedge$$
$$\wedge[t_k,t_{k+1}\in\langle t_k,t_{k+1}\rangle\to(t_k<t_{k+1})]\wedge[\rho\langle t_k,t_{k+1}\rangle=\pi,\ N(t_k,t_{k+1})=6]\Big\}\Rightarrow\tilde{\exists}\bar{\eta}_1(t_k).$$

II. (1.2)

$$\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_i\in \boldsymbol{R}-\{0\},\ i=1,2\wedge$$
$$\wedge\Bigg[\frac{C_1}{\sqrt{C_1^2+C_2^2}}=\sin\alpha\wedge\frac{C_2}{\sqrt{C_1^2+C_2^2}}=\cos\alpha,\ \text{i.e.}$$
$$\alpha=\operatorname{arctg}\frac{C_1}{C_2}\in\Big(-\frac{\pi}{2},0\Big)\cup\Big(0,\frac{\pi}{2}\Big)\Bigg]\Big\}\,\exists[y(t,C_1,C_2)=C_1\sin^3 t+C_2\sin^2 t\cos t=$$
$$=\sqrt{C_1^2+C_2^2}\sin^2 t\cos(t-\alpha)\in \mathbf{Y}_{12}^{\mathrm{II}}\to y^{(\mathrm{IV})}(t,C_1,C_2)+$$
$$+10y''(t,C_1,C_2)+9y(t,C_1,C_2)=0];$$
$$\Big\{[y(t,C_1,C_2)=0]\Leftrightarrow\forall(k\in \boldsymbol{C}):\Big[(t_k=k\pi,\nu(t_k)=2)\wedge$$
$$\wedge\Big(t_k'=\alpha+\frac{\pi}{2}+k\pi,\nu(t_k')=1\Big)\Big]\wedge$$

(1)
$$\alpha\in\Big(-\frac{\pi}{2},0\Big)\Rightarrow[t_k,t_{k+1},t_k'\in\langle t_k,t_{k+1}\rangle\to(t_k<t_k'<t_{k+1})]$$
$$\alpha\in\Big(0,\frac{\pi}{2}\Big)\Rightarrow[t_k,t_{k+1},t_k'\in\langle t_k,t_{k+1}\rangle\to(t_k<t_k'<t_{k+1})]$$

$$\wedge[\rho\langle t_k,t_{k+1}\rangle=\pi,\ N(t_k,t_{k+1},t_k')=5]$$

(2)
$$\alpha\in\Big(-\frac{\pi}{2},0\Big)\Rightarrow[t_{k+1},t_k',t_{k+1}'\in\langle t_k',t_{k+1}'\rangle\to(t_k'<t_{k+1}<t_{k+1}')]$$
$$\alpha\in\Big(0,\frac{\pi}{2}\Big)\Rightarrow[t_{k+1},t_k',t_{k+1}'\in\langle t_k',t_{k+1}'\rangle\to(t_k'<t_{k+1}<t_{k+1}')]$$

$$\wedge[\rho\langle t_k',t_{k+1}'\rangle=\pi,\ N(t_{k+1},t_k',t_{k+1}')=4]\Big\}\Rightarrow$$
$$\Big\{\tilde{\exists}\bar{\eta}_1(t_k)\wedge\exists\Big[\bar{\eta}_1(t_k')=\alpha+\frac{\pi}{2}+(k+1)\pi\Big]\Big\}$$

a) $\forall[t \in(-\infty,+\infty)] \wedge \forall[C_1, C_3 \in \boldsymbol{R}-\{0\} \wedge(\operatorname{sgn} C_1=\operatorname{sgn} C_3)]$

$\exists[y(t, C_1, C_3)=C_1 \sin^3 t+C_3 \sin t \cos^2 t=$

$=\sin t(C_1 \sin^2 t+C_2 \cos^2 t) \in \mathbf{Y}_{13}^{\mathrm{II}} \rightarrow y^{(\mathrm{IV})}(t, C_1, C_3)+10 y''(t, C_1, C_3)+$

$+9 y(t, C_1, C_3)=0]$;

$\{[y(t, C_1, C_3)=0] \Leftrightarrow \forall(k \in \boldsymbol{C}):[t_k=k \pi, v(t_k)=1] \wedge$

$\wedge[t_k, t_{k+1}, t_{k+2}, t_{k+3} \in\langle t_k, t_{k+3}\rangle \rightarrow(t_k<t_{k+1}<t_{k+2}<t_{k+3})] \wedge$

$\wedge[\rho\langle t_k, t_{k+3}\rangle=3 \pi, N(t_k, t_{k+1}, t_{k+2}, t_{k+3})=4]\} \Rightarrow \exists[\bar{\eta}_1(t_k)=(k+3) \pi]$

b) $\forall[t \in(-\infty,+\infty)] \wedge \forall\Big\{C_1, C_3 \in \boldsymbol{R}-\{0\} \wedge(\operatorname{sgn} C_1 \neq \operatorname{sgn} C_3) \wedge$

$\left(\sqrt{-\frac{C_3}{C_1}}=|p|>0\right) \wedge\left[\frac{1}{\sqrt{1+p^2}}=\sin \alpha \wedge \frac{p}{\sqrt{1+p^2}}=\cos \alpha\right.$, i.e. $\alpha=$

$\left.=\operatorname{arctg} \frac{1}{p} \in\left(-\frac{\pi}{2}, 0\right) \cup\left(0, \frac{\pi}{2}\right)\right]\Big\} \exists[y(t, C_1, C_3)=C_1 \sin^3 t+C_3 \sin t \cos^2 t=$

$=C_1 \sin t(\sin^2 t-p^2 \cos^2 t)=-C_1(1+p^2) \sin t \cos (t+\alpha) \cos (t-\alpha) \in \mathbf{Y}_{13}^{\mathrm{II}} \rightarrow$

$\rightarrow y^{(\mathrm{IV})}(t, C_1, C_3)+10 y''(t, C_1, C_3)+9 y(t, C_1, C_3)=0]$;

$\Big\{[y(t, C_1, C_3)=0] \Leftrightarrow \forall(k \in \boldsymbol{C}):\Big[(t_k=k \pi, v(t_k)=1) \wedge$

$\wedge\left(t_k'=-\alpha+\frac{\pi}{2}+k \pi, v(t_k')=1\right) \wedge\left(t_k''=\alpha+\frac{\pi}{2}+k \pi, v(t_k'')=1\right)\Big] \wedge$

$$(1)\begin{cases}\alpha \in\left(-\frac{\pi}{2}, 0\right) \Rightarrow[t_k, t_{k+1}, t_k', t_k'' \in\langle t_k, t_{k+1}\rangle \rightarrow(t_k<t_k''<t_k'<t_{k+1})] \\ \alpha \in\left(0, \frac{\pi}{2}\right) \Rightarrow[t_k, t_{k+1}, t_k', t_k'' \in\langle t_k, t_{k+1}\rangle \rightarrow(t_k<t_k'<t_k''<t_{k+1})]\end{cases}$$

$\wedge[\rho\langle t_k, t_{k+1}\rangle=\pi, N(t_k, t_{k+1}, t_k', t_k'')=4]$,

$$(2)\begin{cases}\alpha \in\left(-\frac{\pi}{2}, 0\right) \Rightarrow \\ \Rightarrow[t_{k+1}, t_k', t_{k+1}', t_{k+1}'' \in\langle t_k', t_{k+1}'\rangle \rightarrow(t_k'<t_{k+1}<t_{k+1}''<t_{k+1}')] \\ \alpha \in\left(0, \frac{\pi}{2}\right) \Rightarrow \\ \Rightarrow[t_{k+1}, t_k', t_{k+1}'', t_k'' \in\langle t_k', t_{k+1}'\rangle \rightarrow(t_k'<t_k''<t_{k+1}<t_{k+1}')]\end{cases}$$

$\wedge[\rho\langle t_k', t_{k+1}'\rangle=\pi, N(t_{k+1}, t_k', t_{k+1}', t_{k+1}'')=N(t_{k+1}, t_k', t_{k+1}', t_k'')=4]$

$$(3)\begin{cases}\alpha\in\left(-\frac{\pi}{2},0\right)\Rightarrow[t_{k+1},t'_k,t''_k,t''_{k+1}\in\langle t''_k,t''_{k+1}\rangle\rightarrow(t''_k<t'_k<t_{k+1}<t''_{k+1})]\\ \alpha\in\left(0,\frac{\pi}{2}\right)\Rightarrow[t_{k+1},t'_{k+1},t''_k,t''_{k+1}\in\langle t''_k,t''_{k+1}\rangle\rightarrow(t''_k<t_{k+1}<t'_{k+1}<t''_{k+1})]\end{cases}$$

$$\wedge[\rho\langle t''_k,t''_{k+1}\rangle=\pi,\ N(t_{k+1},t'_k,t''_k,t''_{k+1})=N(t_{k+1},t'_{k+1},t''_k,t''_{k+1})=4]\Big\}$$

$$\Big\{\exists[\bar{\eta}_1(t_k)=(k+1)\pi]\wedge\exists\left[\bar{\eta}_1(t'_k)=-\alpha+\frac{\pi}{2}+(k+1)\pi\right]\wedge$$

$$\wedge\exists\left[\bar{\eta}_1(t''_k)=\alpha+\frac{\pi}{2}+(k+1)\pi\right]\Big\}.$$

II.(1.4)

$$\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_1,C_4\in\boldsymbol{R}-\{0\}\wedge\left(\sqrt[3]{\frac{C_4}{C_1}}=p\right)\wedge$$

$$\wedge\left[\frac{1}{\sqrt{1+p^2}}=\sin\alpha\wedge\frac{p}{\sqrt{1+p^2}}=\cos\alpha,\text{ i.e. }\alpha=\operatorname{arctg}\frac{1}{p}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\wedge$$

$$\wedge\left[\frac{2}{\sqrt{4+p^2}}=\sin\beta\wedge\frac{p}{\sqrt{4+p^2}}=\cos\beta,\text{ i.e. }\beta=\operatorname{arctg}\frac{2}{p}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\Big\}$$

$$\exists\Big[y(t,C_1,C_4)=C_1\sin^3t+C_4\cos^3t=C_1(\sin t+p\cos t)\times$$

$$\times(\sin^2t-p\sin t\cos t+p^2\cos^2t)=\frac{C_1}{4}\sqrt{1+p^2}\cos(t-\alpha)\times$$

$$\times[(4+p^2)\cos^2(t+\beta)+3p^2\cos^2t]\in\mathbf{Y}^{II}_{14}\rightarrow y^{(IV)}(t,C_1,C_4)+$$

$$+10y''(t,C_1,C_4)+9y(t,C_1,C_4)=0\Big];$$

$$\Big\{[y(t,C_1,C_4)=0]\Leftrightarrow\forall(k\in\boldsymbol{C}):\left[t_k=\alpha+\frac{\pi}{2}+k\pi,\nu(t_k)=1\right]\wedge$$

$$\wedge[t_k,t_{k+1},t_{k+2},t_{k+3}\in\langle t_k,t_{k+3}\rangle\rightarrow(t_k<t_{k+1}<t_{k+2}<t_{k+3})]\wedge$$

$$\wedge[\varrho\langle t_k,t_{k+3}\rangle=3\pi,\ N(t_k,t_{k+1},t_{k+2},t_{k+3})=4]\Big\}\Rightarrow$$

$$\Rightarrow\exists\left[\bar{\eta}_1(t_k)=\alpha+\frac{\pi}{2}+(k+3)\pi\right].$$

II.(2.3)

$$\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_2,C_3\in\boldsymbol{R}-\{0\}\wedge\left[\frac{C_2}{\sqrt{C_2^2+C_3^2}}=\sin\alpha\wedge\frac{C_3}{\sqrt{C_2^2+C_3^2}}=\right.$$

$$= \cos\alpha, \text{ i.e. } \alpha = \operatorname{arctg}\frac{C_2}{C_3} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\Big]\Big\}$$

$$\exists[y(t, C_2, C_3) = C_2 \sin^2 t \cos t + C_3 \sin t \cos^2 t = \sqrt{C_2^2 + C_3^2} \sin t \cos(t - \alpha) \in$$
$$\in \mathbf{Y}_{23}^{II} \to y^{(IV)}(t, C_2, C_3) + 10y''(t, C_2, C_3) + 9y(t, C_2, C_3) = 0];$$

$$\Big\{[y(t, C_2, C_3) = 0] \Leftrightarrow \forall(k \in C) : \left[(t_k + k\pi, \nu(t_k) = 1) \wedge \left(t'_k = \frac{\pi}{2} + k\pi, \nu(t'_k) = 1\right) \wedge\right.$$
$$\left.\wedge \left(t''_k = \alpha + \frac{\pi}{2} + k\pi,\ \nu(t''_k) = 1\right)\right] \wedge$$

$$(1) \begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_k < t_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t''_k < t_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_k, t''_k) = 4],$$

$$(2) \begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t''_{k+1} < t'_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_{k+1}, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_{k+1} < t'_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_{k+1}, t'_k, t'_{k+1}, t''_k) = N(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$

$$(3) \begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_{k+1}, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_k < t_{k+1} < t''_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_{k+1}, t'_k, t''_k, t''_{k+1}) = N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4]\Big\} \Rightarrow$$

$$\Rightarrow \Big\{\exists[\bar{\eta}_1(t_k) = (k + 1)\pi] \wedge \exists\left[\bar{\eta}_1(t'_k) = \frac{\pi}{2} + (k + 1)\pi\right] \wedge$$
$$\wedge \exists\left[\bar{\eta}_1(t''_k) = \alpha + \frac{\pi}{2} + (k + 1)\pi\right]\Big\}.$$

II. (2.4)

a) $\forall[t \in (-\infty, +\infty)] \wedge \forall[C_2, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_2 = \operatorname{sgn} C_4)]$

$$\exists[y(t, C_2, C_4) = C_2 \sin^2 t \cos t + C_4 \cos^3 t = \cos t(C_2 \sin^2 t + C_4 \cos^2 t) \in \mathbf{Y}_{24}^{II} \to$$
$$\to y^{(IV)}(t, C_2, C_4) + 10y''(t, C_2, C_4) + 9y(t, C_2, C_4) = 0];$$

$$\left\{[y(t, C_2, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}): \left[t_k = \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right] \wedge\right.$$

$$[t_k, t_{k+1}, t_{k+2}, t_{k+3} \in \langle t_k, t_{k+3}\rangle \to (t_k < t_{k+1} < t_{k+2} < t_{k+3})] \wedge$$

$$\left.\wedge [\rho\langle t_k, t_{k+3}\rangle = 3\pi, N(t_k, t_{k+1}, t_{k+2}, t_{k+3}) = 4]\right\} \Rightarrow$$

$$\Rightarrow \exists\left[\bar{\eta}_1(t_k) = \frac{\pi}{2} + (k+3)\pi\right]$$

b) $$\forall[t \in (-\infty, +\infty)] \wedge \forall\left\{C_2, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_2 \neq \operatorname{sgn} C_4) \wedge\right.$$

$$\wedge\left(\sqrt{-\frac{C_4}{C_2}} = |p| > 0\right) \wedge \left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha, \text{ i.e.}\right.$$

$$\left.\left.\alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\right\} \exists[y(t, C_2, C_4) =$$

$$= C_2 \sin^2 t \cos t + C_4 \cos^3 t = C_3 \cos t(\sin^2 t - p^2 \cos^2 t) =$$

$$= -C_2(1+p^2)\cos t \cos(t+\alpha)\cos(t-\alpha) \in \mathbf{Y}^{II}_{24} \to$$

$$\to y^{(IV)}(t, C_2, C_4) + 10y''(t, C_2, C_4) + 9y(t, C_2, C_4) = 0];$$

$$\left\{[y(t, C_2, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}): \left[\left(t_k = \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right) \wedge\right.\right.$$

$$\left.\wedge\left(t'_k = -\alpha + \frac{\pi}{2} + k\pi, \nu(t'_k) = 1\right) \wedge \left(t''_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t''_k) = 1\right)\right] \wedge$$

$$(1)\begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_k, t_{k+1}, t'_k, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t''_{k+1} < t_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_k, t_{k+1}, t'_{k+1}, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_{k+1} < t_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k, t''_{k+1}) = N(t_k, t_{k+1}, t'_{k+1}, t''_k) = 4]$$

$$(2)\begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_{k+1} < t_{k+1} < t'_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t''_k < t'_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = N(t_k, t'_k, t'_{k+1}, t''_k) = 4]$$

$$(3)\begin{cases} \alpha \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow [t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_k < t'_k < t''_{k+1})] \\ \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow [t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \end{cases}$$

$$\wedge\,[\rho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_k, t'_k, t''_k, t''_{k+1}) = N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4]\Big\} \Rightarrow$$

$$\Rightarrow \Big\{\exists\Big[\bar{\eta}_1(t_k) = \frac{\pi}{2} + (k+1)\pi\Big] \wedge \exists\Big[\bar{\eta}_1(t'_k) = -\alpha + \frac{\pi}{2} + (k+1)\pi\Big] \wedge$$

$$\wedge\,\exists\Big[\bar{\eta}_1(t''_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\Big]\Big\}.$$

II. (3.4)

$$\forall[t \in (-\infty, +\infty)] \wedge \forall\Big\{C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge \Big[\frac{C_3}{\sqrt{C_2^2 + C_4^2}} = \sin\alpha \wedge$$

$$\wedge\,\frac{C_4}{\sqrt{C_2^2 + C_4^2}} = \cos\alpha,\ \text{i.e. } \alpha = \operatorname{arctg}\frac{C_3}{C_4} \in \Big(-\frac{\pi}{2}, 0\Big) \cup \Big(0, \frac{\pi}{2}\Big)\Big]\Big\}\ \exists[y(t, C_2, C_4) =$$

$$= C_3 \sin t \cos^2 t + C_4 \cos^3 t = \sqrt{C_3^2 + C_4^2}\cos^2 t \cos(t - \alpha) \in \mathbf{Y}^{II}_{34} \rightarrow$$

$$\rightarrow y^{(IV)}(t, C_3, C_4) + 10y''(t, C_3, C_4) + 9y(t, C_3, C_4) = 0];$$

$$\Big\{[y(t, C_3, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \Big[\Big(t_k = \frac{\pi}{2} + k\pi,\ \nu(t_k) = 2\Big) \wedge$$

$$\wedge\Big(t'_k = \alpha + \frac{\pi}{2} + k\pi,\ \nu(t'_k) = 1\Big)\Big] \wedge$$

$$(1)\ \begin{cases} \alpha \in \Big(-\frac{\pi}{2}, 0\Big) \Rightarrow [t_k, t_{k+1}, t'_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_{k+1} < t_{k+1})] \\ \alpha \in \Big(0, \frac{\pi}{2}\Big) \Rightarrow [t_k, t_{k+1}, t'_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_k < t_{k+1})] \end{cases}$$

$$\wedge\,[\varrho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_{k+1}) = N(t_k, t_{k+1}, t'_k) = 5],$$

$$(2)\ \begin{cases} \alpha \in \Big(-\frac{\pi}{2}, 0\Big) \Rightarrow [t_k, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t'_{k+1})] \\ \alpha \in \Big(0, \frac{\pi}{2}\Big) \Rightarrow [t_{k+1}, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_{k+1} < t'_{k+1})] \end{cases}$$

$$\wedge\,[\varrho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_k, t'_k, t'_{k+1}) = N(t_{k+1}, t'_k, t'_{k+1}) = 4]\Big\} \Rightarrow$$

$$\Rightarrow \Big\{\tilde{\exists}\bar{\eta}_1(t_k) \wedge \exists\Big[\bar{\eta}_1(t'_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\Big]\Big\}.$$

III. (1.2.3)

a) $\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_i\in \boldsymbol{R}-\{0\}, i=1,2,3\wedge(\operatorname{sgn}C_1=\operatorname{sgn}C_3)\wedge$

$\wedge(4C_1C_3-C_2^2>0)\wedge\left[\frac{2C_1}{\sqrt{4C_1^2+C_2^2}}=\sin\alpha\wedge\frac{C_2}{\sqrt{4C_1^2+C_2^2}}=\cos\alpha,\right.$

$\left.\text{i.e. } \alpha=\operatorname{arctg}\frac{2C_1}{C_2}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\Big\}$

$\exists\Big[y(t,C_1,C_2,C_3)=C_1\sin^3 t+C_2\sin^2 t\cos t+C_3\sin t\cos^2 t=$

$=\frac{1}{4C_1}\sin t[(4C_1^2+C_2^2)\cos^2(t-\alpha)+(4C_1C_3-C_2^2)\cos^2 t]\in \mathbf{Y}^{\mathrm{III}}_{123}\to$

$\to y^{(IV)}(t,C_1,C_2,C_3)+10y''(t,C_1,C_2,C_3)+9y(t,C_1,C_2,C_3)=0\Big];$

$\{[y(t,C_1,C_2,C_3)=0]\Leftrightarrow\forall(k\in \boldsymbol{C}):[t_k=k\pi, \nu(t_k)=1]\wedge$

$\wedge[t_k,t_{k+1},t_{k+2},t_{k+3}\in\langle t_k,t_{k+3}\rangle\to(t_k<t_{k+1}<t_{k+2}<t_{k+3})]\wedge$

$\wedge[\varrho\langle t_k,t_{k+3}\rangle=3\pi, N(t_k,t_{k+1},t_{k+2},t_{k+3})=4]\}\Rightarrow\exists[\bar{\eta}_1(t_k)=(k+3)\pi].$

b) $\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_i\in \boldsymbol{R}-\{0\}, i=1,2,3\wedge(\operatorname{sgn}C_1=\operatorname{sgn}C_3)\wedge$

$\wedge(4C_1C_3=C_2^2)\wedge\left[\frac{2C_1}{\sqrt{4C_1^2+C_2^2}}=\sin\alpha\wedge\frac{C_2}{\sqrt{4C_1^2+C_2^2}}=\cos\alpha,\right.$

$\left.\text{i.e. } \alpha=\operatorname{arctg}\frac{2C_1}{C_2}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\Big\}\exists[y(t,C_1,C_2,C_3)=$

$=C_1\sin^3 t+C_2\sin^2 t\cos t+C_3\sin t\cos^2 t=(C_1+C_3)\sin t\cos^2(t-\alpha)\in$

$\in \mathbf{Y}^{\mathrm{III}}_{123}\to y^{(IV)}(t,C_1,C_2,C_3)+10y''(t,C_1,C_2,C_3)+9y(t,C_1,C_2,C_3)=0];$

$\Big\{[y(t,C_1,C_2,C_3)=0]\Leftrightarrow\forall(k\in \boldsymbol{C}):\Big[\ t_k=k\pi, \nu(t_k)=1)\wedge$

$\wedge\left(t'_k=\alpha+\frac{\pi}{2}+k\pi, \nu(t'_k)=2\right)\Big]\wedge$

(1) $\left[\alpha\in\left(-\frac{\pi}{2},0\right)\vee\alpha\in\left(0,\frac{\pi}{2}\right)\right]\Rightarrow\{[t_k,t_{k+1},t'_k\in\langle t_k,t_{k+1}\rangle\to$

$\to(t_k<t'_k<t_{k+1})]\wedge[\rho<t_k,t_{k+1}\rangle=\pi, N(t_k,t_{k+1}t'_k)=4]\},$

(2) $\left[\alpha\in\left(-\frac{\pi}{2},0\right)\vee\alpha\in\left(0,\frac{\pi}{2}\right)\right]\Rightarrow\{[t_{k+1},t'_k,t'_{k+1}\ \varepsilon\ \langle t'_k,t'_{k+1}\rangle\to$

$\to(t'_k<t_{k+1}<t'_{k+1})]\wedge[\varrho\langle t'_k,t'_{k+1}\rangle=\pi, N(t_{k+1},t'_k,t'_{k+1})=5]\}\Big\}$

$\Rightarrow\{\exists[\bar{\eta}_1(t_k)=(k+1)\pi]\wedge\bar{\exists}\bar{\eta}_1(t'_k)\},$

c) $\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_i\in\boldsymbol{R}-\{0\}, i=1,2,3\wedge(C_2^2-4C_1C_3>0)\wedge$

$\wedge\left(\dfrac{C_2+\sqrt{C_2^2-4C_1C_3}}{2C_1}=p\wedge\dfrac{C_2-\sqrt{C_2^2-4C_1C_3}}{2C_1}=q, p\neq 0, q\neq 0,\right.$

$\left. p\pm q\neq 0\right)\wedge\left[\dfrac{1}{\sqrt{1+p^2}}=\sin\alpha\wedge\dfrac{p}{\sqrt{1+p^2}}=\cos\alpha, \text{ i.e. } \alpha=\operatorname{arctg}\dfrac{1}{p}\in\right.$

$\left.\in\left(-\dfrac{\pi}{2},0\right)\cup\left(0,\dfrac{\pi}{2}\right)\right]\wedge\left[\dfrac{1}{\sqrt{1+q^2}}=\sin\beta\wedge\dfrac{q}{\sqrt{1+q^2}}=\cos\beta,\right.$

$\left.\text{i.e. } \beta=\operatorname{arctg}\dfrac{1}{q}\in\left(-\dfrac{\pi}{2},0\right)\cup\left(0,\dfrac{\pi}{2}\right)\right]\Big\}$

$\exists[y(t,C_1,C_2,C_3)=C_1\sin^3 t+C_2\sin^2 t\cos t+C_3\sin t\cos^2 t=$
$=C_1\sin t(\sin t+p\cos t)(\sin t+q\cos t)=C_1\sqrt{(1+p^2)(1+q^2)}\times$
$\times\sin t\cos(t-\alpha)\cos(t-\beta)\in\mathbf{Y}_{123}^{\mathrm{III}}\to y^{(\mathrm{IV})}(t,C_1,C_2,C_3)+$
$+10y''(t,C_1,C_2,C_3)+9y(t,C_1,C_2,C_3)=0];$

$\Big\{[y(t,C_1,C_2,C_3)=0]\Leftrightarrow\forall(k\in\boldsymbol{C}):\Big[(t_k=k\pi, \nu(t_k)=1)\wedge$

$\wedge\left(t'_k=\alpha+\dfrac{\pi}{2}+k\pi, \nu(t'_k)=1\right)\wedge\left(t''_k=\beta+\dfrac{\pi}{2}+k\pi, \nu(t''_k)=1\right)\Big]\wedge$

$(1_1)\ \left[\left(-\dfrac{\pi}{2}<\alpha<\beta<0\right)\vee\left(-\dfrac{\pi}{2}<\alpha<0<\beta<\dfrac{\pi}{2}\right)\vee\left(0<\alpha<\beta<\dfrac{\pi}{2}\right)\right]\Rightarrow$
$\Rightarrow[t_k, t_{k+1}, t'_k, t''_k\in\langle t_k, t_{k+1}\rangle\to(t_k<t'_k<t''_k<t_{k+1})]$

$(1_2)\ \left[\left(-\dfrac{\pi}{2}<\beta<\alpha<0\right)\vee\left(-\dfrac{\pi}{2}<\beta<0<\alpha<\dfrac{\pi}{2}\right)\vee\left(0<\beta<\alpha<\dfrac{\pi}{2}\right)\right]\Rightarrow$
$\Rightarrow[t_k, t_{k+1}, t'_k, t''_k\in\langle t_k, t_{k+1}\rangle\to(t_k<t''_k<t'_k<t_{k+1})]$

$\wedge[\varrho\langle t_k, t_{k+1}\rangle=\pi, N(t_k, t_{k+1}, t'_k, t''_k)=4]$

$(2_1)\ \left[\left(-\dfrac{\pi}{2}<\alpha<\beta<0\right)\vee\left(-\dfrac{\pi}{2}<\alpha<0<\beta<\dfrac{\pi}{2}\right)\vee\left(0<\alpha<\beta<\dfrac{\pi}{2}\right)\right]\Rightarrow$
$\Rightarrow[t_{k+1}, t'_k, t'_{k+1}, t''_k\in\langle t'_k, t'_{k+1}\rangle\to(t'_k<t''_k<t_{k+1}<t'_{k+1})]$

$(2_2)\ \left[\left(-\dfrac{\pi}{2}<\beta<\alpha<0\right)\vee\left(-\dfrac{\pi}{2}<\beta<0<\alpha<\dfrac{\pi}{2}\right)\vee\left(0<\beta<\alpha<\dfrac{\pi}{2}\right)\right]\Rightarrow$
$\Rightarrow[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}\in\langle t'_k, t'_{k+1}\rangle\to(t'_k<t_{k+1}<t''_{k+1}<t'_{k+1})]$

$\wedge[\varrho\langle t'_k, t'_{k+1}\rangle=\pi, N(t_{k+1}, t'_k, t'_{k+1}, t''_k)=N(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1})=4]$

$(3_1)\ \left[\left(-\dfrac{\pi}{2}<\alpha<\beta<0\right)\vee\left(-\dfrac{\pi}{2}<\alpha<0<\beta<\dfrac{\pi}{2}\right)\vee\left(0<\alpha<\beta<\dfrac{\pi}{2}\right)\right]\Rightarrow$
$\Rightarrow[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}\in\langle t''_k, t''_{k+1}\rangle\to(t''_k<t_{k+1}<t'_{k+1}<t''_{k+1})]$

$$(3_2)\left[\left(-\frac{\pi}{2}<\beta<\alpha<0\right)\vee\left(-\frac{\pi}{2}<\beta<0<\alpha<\frac{\pi}{2}\right)\vee\left(0<\beta<\alpha<\frac{\pi}{2}\right)\right]\Rightarrow$$

$$\Rightarrow[t_{k+1}, t'_k, t''_k, t''_{k+1}\in\langle t''_k, t''_{k+1}\rangle\to(t''_k<t'_k<t_{k+1}<t''_{k+1})]$$

$$\wedge\,[\rho\langle t''_k, t''_{k+1}\rangle=\pi,\ N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1})=N(t_{k+1}, t'_k, t''_k, t''_{k+1})=4]\Big\}\Rightarrow$$

$$\Rightarrow\left\{\exists[\bar{\eta}_1(t_k)=(k+1)\pi]\wedge\exists\left[\bar{\eta}_1(t'_k)=\alpha+\frac{\pi}{2}+(k+1)\pi\right]\wedge\right.$$

$$\left.\wedge\,\exists\left[\bar{\eta}_1(t''_k)=\beta+\frac{\pi}{2}+(k+1)\pi\right]\right\}.$$

III. (1.2.4)

a) $\forall[t\in(-\infty, +\infty)]\wedge\forall\Big\{C_1, C_2, C_4\in\boldsymbol{R}-\{0\}\wedge(\operatorname{sgn}C_2\neq\operatorname{sgn}C_4)\wedge$

$$\wedge\,(27C_1^2C_4+4C_2^3=0)\wedge\left(\frac{C_2}{C_1}=-3p\wedge\frac{C_4}{C_1}=4p^3,\ \text{i.e. } p=-\frac{C_4}{4C_1}=\sqrt[3]{\frac{C_2}{4C_1}}\right)\wedge$$

$$\wedge\left[\frac{1}{\sqrt{1+p^2}}=\sin\alpha\wedge\frac{p}{\sqrt{1+p^2}}=\cos\alpha,\ \text{i.e. } \alpha=\operatorname{arctg}\frac{1}{p}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\wedge$$

$$\wedge\left[\frac{1}{\sqrt{1+4p^2}}=\sin\beta\wedge\frac{2p}{\sqrt{1+4p^2}}=\cos\beta,\right.$$

$$\left.\text{i.e. } \beta=\operatorname{arctg}\frac{1}{2p}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\Big\}$$

$$\exists[y(t, C_1, C_2, C_4)=C_1\sin^3 t+C_2\sin^2 t\cos t+C_4\cos^3 t=$$

$$=C_1(\sin t+p\cos t)(\sin t-2p\cos t)^2=C_1\sqrt{1+p^2}\,(1+4p^2).$$

$$\cos(t-\alpha)\cos^2(t+\beta)\in\mathbf{Y}^{\mathrm{III}}_{124}\to y^{(\mathrm{IV})}(t, C_1, C_2, C_4)+$$

$$+\,10y''(t, C_1, C_2, C_4)+9y(t, C_1, C_2, C_4)=0];$$

$$\left\{[y(t, C_1, C_2, C_4)=0]\Leftrightarrow\forall(k\in\boldsymbol{C}):\left[\left(t_k=\alpha+\frac{\pi}{2}+k\pi, \nu(t_k)=1\right)\wedge\right.\right.$$

$$\left.\wedge\left(t'_k=-\beta+\frac{\pi}{2}+k\pi, \nu(t'_k)=2\right)\right]\wedge$$

$$(1_1)\left(p>0,\ \text{i.e. } 0<\beta<\alpha<\frac{\pi}{2}\right)\Rightarrow$$

$$\Rightarrow[t_k, t_{k+1}, t'_{k+1}\in\langle t_k, t_{k+1}\rangle\to(t_k<t'_{k+1}<t_{k+1})]$$

$$(1_2)\left(p<0,\ \text{i.e. } -\frac{\pi}{2}<\alpha<\beta<0\right)\Rightarrow$$

$$\Rightarrow[t_k, t_{k+1}, t'_k\in\langle t_k, t_{k+1}\rangle\to(t_k<t'_k<t_{k+1})]$$

$$\wedge\,[\rho\langle t_k, t_{k+1}\rangle=\pi,\ N(t_k, t_{k+1}, t'_{k+1})=N(t_k, t_{k+1}, t'_k)=4]$$

$$(2_1)\ \left(p > 0, \text{ i.e. } 0 < \beta < \alpha < \frac{\pi}{2}\right) \Rightarrow$$

$$\Rightarrow [t_k, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t'_{k+1})]$$

$$(2_2)\ \left(p < 0, \text{ i.e. } -\frac{\pi}{2} < \alpha < \beta < 0\right) \Rightarrow$$

$$\Rightarrow [t_{k+1}, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_{k+1} < t'_{k+1})]$$

$$\left.\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_k, t'_k, t'_{k+1}) = N(t_{k+1}, t'_k, t'_{k+1}) = 5]\right\} \Rightarrow$$

$$\Rightarrow \left\{\exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge \tilde{\exists}\bar{\eta}_1(t'_k)\right\}.$$

b) $\forall[t \in (-\infty, +\infty)] \wedge \forall\Big\{C_1, C_2, C_4 \in \boldsymbol{R} - \{0\} \wedge$

$$\wedge [(p > 0 \wedge -4p < q < 0) \vee (p < 0 \wedge 0 < q < -4p)] \wedge (p + q \neq 0) \wedge$$

$$(\operatorname{sgn} C_1C_4 \neq \operatorname{sgn} q) \wedge [\operatorname{sgn}(C_1p - C_2) = \operatorname{sgn} C_4] \wedge$$

$$\wedge \left[\left(\frac{C_2}{C_1} = p + q\right) \wedge \left(\frac{C_4}{C_1} = -p^2q\right),\right.$$

$$\left.\text{i.e. } p^2(C_1p - C_2) = C_4 \wedge C_1C_4 + q(C_2 - C_1q)^2 = 0\right] \wedge$$

$$\wedge \left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha,\right.$$

$$\left.\text{i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right] \wedge \left[\frac{2}{\sqrt{4+q^2}} = \sin\beta \wedge\right.$$

$$\left.\left.\wedge \frac{q}{\sqrt{4+q^2}} = \cos\beta, \text{ i.e. } \beta = \operatorname{arctg}\frac{2}{q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\right\}$$

$$\exists\Big[y(t, C_1, C_2, C_4) = C_1 \sin^3 t + C_2 \sin^2 t \cos t + C_4 \cos^3 t =$$

$$= C_1(\sin t + p\cos t)(\sin^2 t + q \sin t \cos t - pq \cos^2 t) =$$

$$= \frac{C_1}{4}\sqrt{1+p^2}\cos(t-\alpha)[(4+q^2)\cos^2(t-\beta) - q(4p+q)\cos^2 t] \in Y^{III}_{124} \rightarrow$$

$$\left.\rightarrow y^{(IV)}(t, C_1, C_2, C_4) + 10y''(t, C_1, C_2, C_4) + 9y(t, C_1, C_2, C_4) = 0\right];$$

$$\Big\{[y(t, C_1, C_2, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}): \left[t_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right] \wedge$$

$$\wedge [t_k, t_{k+1}, t_{k+2}, t_{k+3} \in \langle t_k, t_{k+3}\rangle \rightarrow (t_k < t_{k+1} < t_{k+2} < t_{k+3})] \wedge$$

$$\left.\wedge [\rho\langle t_k, t_{k+3}\rangle = 3\pi, N(t_k, t_{k+1}, t_{k+2}, t_{k+3}) = 4]\right\} \Rightarrow$$

$$\Rightarrow \exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+3)\pi\right].$$

c) $\forall[t\in(-\infty,+\infty)]\wedge\forall\Big\{C_1, C_2, C_4\in \boldsymbol{R}-\{0\}\wedge(\operatorname{sgn} C_2 \neq \operatorname{sgn} C_4)\wedge$

$$\wedge\left[\left(C_1>0\wedge\frac{4C_2^3+27C_1^2C_4}{C_1^3}>0\wedge\frac{C_4}{C_1}<0\right)\vee\right.$$

$$\left.\vee\left(C_1<0\wedge\frac{4C_2^3+27C_1^2C_4}{C_1^3}<0\wedge\frac{C_4}{C_1}>0\right)\right]\wedge$$

$$\wedge\left[\frac{C_2}{C_1}=\frac{p^2+pq+q^2}{p+q}\wedge\frac{C_4}{C_1}=-\frac{p^2q^2}{p+q},\ p\neq 0,\ q\neq 0,\ p\pm q\neq 0,\right.$$

$$\left.p+2q\neq 0,\ q+2p\neq 0\right]\wedge\left[\frac{1}{\sqrt{1+p^2}}=\sin\alpha\wedge\frac{p}{\sqrt{1+p^2}}=\cos\alpha,\right.$$

$$\left.\text{i.e. }\alpha=\operatorname{arctg}\frac{1}{p}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\wedge\left[\frac{1}{\sqrt{1+q^2}}=\sin\beta\wedge\frac{q}{\sqrt{1+q^2}}=\cos\beta,\right.$$

$$\left.\text{i.e. }\beta=\operatorname{arctg}\frac{1}{q}\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\wedge\left[\frac{p+q}{\sqrt{(p+q)^2+p^2q^2}}=\right.$$

$$=\sin\gamma\wedge-\frac{pq}{\sqrt{(p+q)^2+p^2q^2}}=\cos\gamma,\ \text{i.e. }\gamma=\operatorname{arctg}\left(-\frac{p+q}{pq}\right)\in$$

$$\left.\left.\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)\right]\right\}\exists\Big[y(t,C_1,C_2,C_4)=C_1\sin^3 t+C_2\sin^2 t\cos t+$$

$$+C_4\cos^3 t=C_1\left(\sin^3 t+\frac{p^2+pq+q^2}{p+q}\sin^2 t\cos t-\frac{p^2q^2}{p+q}\cos^3 t\right)=$$

$$=C_1(\sin t+p\cos t)(\sin t+q\cos t)\left(\sin t-\frac{pq}{p+q}\cos t\right)=$$

$$=\frac{C_1}{p+q}\sqrt{(1+p^2)(1+q^2)[(p+q)^2+p^2q^2]}\cos(t-\alpha)\cos(t-\beta)\cos(t-\gamma)\in$$

$$\in\mathbf{Y}^{\mathrm{III}}_{124}\rightarrow y^{(\mathrm{IV})}(t,C_1,C_2,C_4)+10y''(t,C_1,C_2,C_4)+9y(t,C_1,C_2,C_4)=0\Big];$$

$$\Big\{[y(t,C_1,C_2,C_4)=0]\Leftrightarrow\forall(k\in\boldsymbol{C}):\left[\left(t_k=\alpha+\frac{\pi}{2}+k\pi,\ \nu(t_k)=1\right)\wedge\right.$$

$$\left.\wedge\left(t'_k=\beta+\frac{\pi}{2}+k\pi,\ \nu(t'_k)=1\right)\wedge\left(t''_k=\gamma+\frac{\pi}{2}+k\pi,\ \nu(t''_k)=1\right)\right]\wedge$$

(1) $$\left[\left(\frac{1}{p}<0<\frac{1}{q}<-\frac{p+q}{pq},\ \text{i.e. }-\frac{\pi}{2}<\alpha<0<\beta<\gamma<\frac{\pi}{2}\right)\vee\right.$$

$$\left.\vee\left(\frac{1}{p}<\frac{1}{q}<0<-\frac{p+q}{pq},\ \text{i.e. }-\frac{\pi}{2}<\alpha<\beta<0<\gamma<\frac{\pi}{2}\right)\right]$$

$$\Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_k\in\langle t_k, t_{k+1}\rangle\rightarrow(t_k<t'_k<t''_k<t_{k+1})]\wedge$$

$$\wedge[\varrho\langle t_k, t_{k+1}\rangle=\pi,\ N(t_k, t_{k+1}, t'_k, t''_k)=4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_k) = 4],$$
$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(2) $$\left[\left(\frac{1}{p} < 0 < -\frac{p+q}{pq} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < 0 < \gamma < \beta < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee\left(\frac{1}{p} < -\frac{p+q}{pq} < 0 < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < \gamma < 0 < \beta < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_k < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_k) = 4],$$
$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_{k+1}, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_k < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t''_k, t''_{k+1}) = 4],$$

(3) $$\left[\left(\frac{1}{q} < 0 < \frac{1}{p} < -\frac{p+q}{pq}, \text{ i.e. } -\frac{\pi}{2} < \beta < 0 < \alpha < \gamma < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee\left(\frac{1}{q} < \frac{1}{p} < 0 < -\frac{p+q}{pq}, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < 0 < \gamma < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_{k+1}, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_k) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t''_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$
$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_{k+1} < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(4) $$\left[\left(\frac{1}{q} < 0 < -\frac{p+q}{pq} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < 0 < \gamma < \alpha < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee\left(\frac{1}{q} < -\frac{p+q}{pq} < 0 < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < \gamma < 0 < \alpha < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_{k+1} < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_k, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t_k < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, N(t_k, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

$$(5) \left[\left(-\frac{p+q}{pq} < 0 < \frac{1}{p} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \gamma < 0 < \alpha < \beta < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee \left(-\frac{p+q}{pq} < \frac{1}{p} < 0 < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \alpha < 0 < \beta < \frac{\pi}{2}\right)\right]$$

$$\Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_k < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k, t''_{k+1}) = 4],$$
$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t''_{k+1} < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t_k < t'_k < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, N(t_k, t'_k, t''_k, t''_{k+1}) = 4],$$

$$(6) \left[\left(-\frac{p+q}{pq} < 0 < \frac{1}{q} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \gamma < 0 < \beta < \alpha < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee \left(-\frac{p+q}{pq} < \frac{1}{q} < 0 < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \beta < 0 < \alpha < \frac{\pi}{2}\right)\right]$$

$$\Rightarrow$$

$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t''_{k+1} < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_k, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t'_k < t_k < t'_{k+1})] \wedge$$
$$\left. \wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, N(t_k, t'_k, t''_k, t''_{k+1}) = 4]\right\} \Rightarrow$$

$$\left\{\exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge \exists\left[\bar{\eta}_1(t'_k) = \beta + \frac{\pi}{2} + (k+1)\pi\right] \wedge \right.$$
$$\left. \wedge \exists\left[\bar{\eta}_1(t''_k) = \gamma + \frac{\pi}{2} + (k+1)\pi\right]\right\}.$$

III. (*1.3.4*)

$$\text{a) } \forall[t \in (-\infty, +\infty)] \wedge \forall\left\{C_1, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_1 \neq \operatorname{sgn} C_3) \wedge \right.$$
$$\wedge (27C_1C_4^2 + 4C_3^3 = 0) \wedge \left(\frac{C_3}{C_1} = -3q^2 \wedge \frac{C_4}{C_1} = -2q^3,\right.$$

$$\text{i.e. } q = \pm\sqrt{-\frac{C_3}{3C_1}} = \sqrt[3]{-\frac{C_4}{2C_1}}\Bigg) \wedge \left[\frac{1}{\sqrt{1+4q^2}} = \sin\alpha \wedge \frac{2q}{\sqrt{1+4q^2}} = \cos\alpha,\right.$$

$$\text{i.e. } \alpha = \operatorname{arctg}\frac{1}{2q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\Bigg] \wedge \left[\frac{1}{\sqrt{1+q^2}} = \sin\beta \wedge\right.$$

$$\left. \wedge \frac{q}{\sqrt{1+q^2}} = \cos\beta, \text{ i.e. } \beta = \operatorname{arctg}\frac{1}{q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\Bigg\}$$

$$\exists[y(t, C_1, C_3, C_4) = C_1 \sin^3 t + C_3 \sin t \cos^2 t + C_4 \cos^3 t =$$
$$= C_1(\sin t - 2q \cos t)(\sin t + q \cos t)^2 =$$
$$= -C_1(1+q^2)\sqrt{1+4q^2}\cos(t+\alpha)\cos(t-\beta) \in \mathbf{Y}^{\mathrm{III}}_{134} \rightarrow$$
$$\rightarrow y^{(\mathrm{IV})}(t, C_1, C_3, C_4) + 10y''(t, C_1, C_3, C_4) + 9y(t, C_1, C_3, C_4) = 0];$$

$$\Big\{[y(t, C_1, C_3, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \left[\left(t_k = -\alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right) \wedge\right.$$
$$\left.\wedge \left(t'_k = \beta + \frac{\pi}{2} + k\pi, \nu(t'_k) = 2\right)\right] \wedge$$

$$(1)\ \begin{cases} \left(q > 0, \text{ i.e. } 0 < \alpha < \beta < \frac{\pi}{2}\right) \Rightarrow \\ \Rightarrow [t_k, t_{k+1}, t'_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_k < t_{k+1})] \\ \left(q < 0, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < 0\right) \Rightarrow \\ \Rightarrow [t_k, t_{k+1}, t'_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_{k+1} < t_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k) = N(t_k, t_{k+1}, t'_{k+1}) = 4]$$

$$(2)\ \begin{cases} \left(q > 0, \text{ i.e. } 0 < \alpha < \beta < \frac{\pi}{2}\right) \Rightarrow \\ \Rightarrow [t_{k+1}, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_{k+1} < t'_{k+1})] \\ \left(q < 0, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < 0\right) \Rightarrow \\ \Rightarrow [t_k, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t'_{k+1})] \end{cases}$$

$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}) = N(t_k, t'_k, t'_{k+1}) = 5]\Big\} \Rightarrow$$

$$\Rightarrow \left\{\exists\left[\bar{\eta}_1(t_k) = -\alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge \tilde{\exists}\bar{\eta}_1(t'_k)\right\}$$

b) $\forall[t \in (-\infty, +\infty)] \wedge \forall\Big\{C_1, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (p \neq 0.\ r > 0 \wedge 4r - p^2 > 0) \wedge$

$$\wedge \left(\operatorname{sgn}\frac{C_4}{C_1} = \operatorname{sgn} p\right) \wedge \left[\left(\frac{C_3}{C_1} = r - p^2\right) \wedge \left(\frac{C_4}{C_1} = pr\right),\right.$$

i.e. $p(C_3 + C_1 p^2) = C_4 \wedge C_1 r^2(rC_1 - C_3) = C_4^2\Big] \wedge [\operatorname{sgn}(rC_1 - C_3) = \operatorname{sgn} C_1] \wedge$

$$\wedge\left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha,\ \text{i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right] \wedge$$

$$\wedge\left[\frac{2}{\sqrt{4+p^2}} = \sin\beta \wedge \frac{p}{\sqrt{4+p^2}} = \cos\beta,\ \text{i.e. } \beta = \operatorname{arctg}\frac{2}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup\right.$$

$$\left.\cup\left(0, \frac{\pi}{2}\right)\right]\Bigg\} \exists\Big[y(t, C_1, C_3, C_4) = C_1 \sin^3 t + C_3 \sin t \cos^2 t + C_4 \cos^3 t =$$

$$= C_1(\sin t + p\cos t)(\sin^2 t - p \sin t \cos t + r\cos^2 t) = \frac{C_1}{4}\sqrt{1+p^2}\cos(t-\alpha) \times$$

$$\times [(4+p^2)\cos^2(t+\beta) + (4r - p^2)\cos^2 t] \in \mathbf{Y}^{\mathrm{III}}_{134} \to y^{(\mathrm{IV})}(t, C_1, C_3, C_4) +$$

$$+ 10y''(t, C_1, C_3, C_4) + 9y(t, C_1, C_3, C_4) = 0\Big];$$

$$\Big\{[y(t, C_1, C_3, C_4) = 0] \Leftrightarrow \forall(k \in C) : \left[t_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right] \wedge$$

$$\wedge [t_k, t_{k+1}, t_{k+2}, t_{k+3} \in \langle t_k, t_{k+3}\rangle \to (t_k < t_{k+1} < t_{k+2} < t_{k+3})] \wedge$$

$$\wedge [\rho\langle t_k, t_{k+1}\rangle = 3\pi, N(t_k, t_{k+1}, t_{k+2}, t_{k+3}) = 4]\Big\} \Rightarrow$$

$$\Rightarrow \exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+3)\pi\right]$$

c) $\forall[t \in (-\infty, +\infty)] \wedge \forall\Big\{C_1, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_1 \neq \operatorname{sgn} C_3) \wedge$

$$\wedge\left[\left(C_1 > 0 \wedge 2\left(-\frac{C_3}{C_1}\right)^{3/2} + 3\sqrt{3}\frac{C_4}{C_1} > 0 \wedge -2\left(-\frac{C_3}{C_1}\right)^{3/2} + 3\sqrt{3}\frac{C_4}{C_1} < 0\right) \vee\right.$$

$$\left.\vee\left(C_1 < 0 \wedge 2\left(-\frac{C_3}{C_1}\right)^{3/2} + 3\sqrt{3}\frac{C_4}{C_1} < 0 \wedge -2\left(-\frac{C_3}{C_1}\right)^{3/2} + 3\sqrt{3}\frac{C_4}{C_1} > 0\right)\right] \wedge$$

$$\wedge\left[\frac{C_3}{C_1} = pq - (p+q)^2 \wedge \frac{C_4}{C_1} = -pq(p+q), p \neq 0, q \neq 0, p \pm q \neq 0,\right.$$

$$\left. p + 2q \neq 0, q + 2p \neq 0\right] \wedge \left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha,\right.$$

$$\left.\text{i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right] \wedge \left[\frac{1}{\sqrt{1+q^2}} = \sin\beta \wedge \frac{q}{\sqrt{1+q^2}} = \cos\beta,\right.$$

$$\left.\text{i.e. } \beta = \operatorname{arctg}\frac{1}{q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right] \wedge \left[\frac{1}{\sqrt{1+(p+q)^2}} = \sin\gamma \wedge\right.$$

$$-\frac{p+q}{\sqrt{1+(p+q)^2}} = \cos\gamma,\ \text{i.e. } \gamma = \operatorname{arctg}\left(-\frac{1}{p+q}\right) \in \left(-\frac{\pi}{2}, 0\right) \cup$$

$$\left.\cup\left(0, \frac{\pi}{2}\right)\right]\Bigg\} \exists[y(t, C_1, C_3, C_4) = C_1 \sin^3 t + C_3 \sin t \cos^2 t + C_4 \cos^3 t =$$

$$= C_1\{\sin^3 t + [pq - (p+q)^2]\sin t\cos^2 t - pq(p+q)\cos^3 t\} =$$
$$= C_1(\sin t + p\cos t)(\sin t + q\cos t)[\sin t - (p+q)\cos t] =$$
$$= C_1\sqrt{(1+p^2)(1+q^2)[1+(p+q)^2]}\cos(t-\alpha)\cos(t-\beta)\cos(t-\gamma) \in \mathbf{Y}^{III}_{134} \rightarrow$$
$$\rightarrow y^{(IV)}(t, C_1, C_3, C_4) + 10y''(t, C_1, C_3, C_4) + 9y(t, C_1, C_3, C_4) = 0];$$

$$\Big\{[y(t, C_1, C_3, C_4) = 0] \Leftrightarrow \forall(k \in C): \left[\left(t_k = \alpha + \frac{\pi}{2} + k\pi, v(t_k) = 1\right) \wedge\right.$$
$$\left.\wedge\left(t'_k = \beta + \frac{\pi}{2} + k\pi, v(t'_k) = 1\right) \wedge \left(t''_k = \gamma + \frac{\pi}{2} + k\pi, v(t'') = 1\right)\right] \wedge$$

$$(1)\ \left[\left(\frac{1}{p} < 0 < \frac{1}{q} < -\frac{1}{p+q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < 0 < \beta < \gamma < \frac{\pi}{2}\right) \vee\right.$$
$$\left.\vee\left(\frac{1}{p} < \frac{1}{q} < 0 < -\frac{1}{p+q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < \beta < 0 < \gamma < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_k < t''_k < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k, t''_k) = 4],$$
$$[t_{k+1}, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t''_k < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}, t''_k) = 4],$$
$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

$$(2)\ \left[\left(\frac{1}{p} < 0 < -\frac{1}{p+q} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < 0 < \gamma < \beta < \frac{\pi}{2}\right) \vee\right.$$
$$\left.\vee\left(\frac{1}{p} < -\frac{1}{p+q} < 0 < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < \gamma < 0 < \beta < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t''_k < t'_k < t_{k+1})] \wedge$$
$$\wedge [\varrho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k, t''_k) = 4],$$
$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_{k+1} < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_{k+1}, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t'_k < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t''_k, t''_{k+1}) = 4],$$

$$(3)\ \left[\left(\frac{1}{q} < 0 < \frac{1}{p} < -\frac{1}{p+q}, \text{ i.e. } -\frac{\pi}{2} < \beta < 0 < \alpha < \gamma < \frac{\pi}{2}\right) \vee\right.$$
$$\left.\vee\left(\frac{1}{q} < \frac{1}{p} < 0 < -\frac{1}{p+q}, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < 0 < \gamma < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$

$$[t_k, t_{k+1}, t'_{k+1}, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_k) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t''_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$
$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_{k+1} < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(4) $$\left[\left(\frac{1}{q} < 0 < -\frac{1}{p+q} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < 0 < \gamma < \alpha < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee \left(\frac{1}{q} < -\frac{1}{p+q} < 0 < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < \gamma < 0 < \alpha < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_{k+1} < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\varrho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$
$$[t_k, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_k < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(5) $$\left[\left(-\frac{1}{p+q} < 0 < \frac{1}{p} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \gamma < 0 < \alpha < \beta < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee \left(-\frac{1}{p+q} < \frac{1}{p} < 0 < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \alpha < 0 < \beta < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_k, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\varrho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_{k+1}) = 4],$$
$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_{k+1} < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_k < t'_k < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t''_k, t''_{k+1}) = 4],$$

(6) $$\left[\left(-\frac{1}{p+q} < 0 < \frac{1}{q} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \gamma < 0 < \beta < \alpha < \frac{\pi}{2}\right) \vee \right.$$
$$\left. \vee \left(-\frac{1}{p+q} < \frac{1}{q} < 0 < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \beta < 0 < \alpha < \frac{\pi}{2}\right)\right]$$
$$\Rightarrow$$
$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_{k+1} < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\varrho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t_k', t_{k+1}', t_{k+1}'' \in \langle t_k', t_{k+1}'\rangle \to (t_k' < t_k < t_{k+1}'' < t_{k+1}')] \wedge$$
$$\wedge [\varrho\langle t_k', t_{k+1}'\rangle = \pi, \mathrm{N}(t_k, t_k', t_{k+1}', t_{k+1}'') = 4],$$
$$[t_k, t_k', t_k'', t_{k+1}'' \in \langle t_k'', t_{k+1}''\rangle \to (t_k'' < t_k' < t_k < t_{k+1}'')] \wedge$$
$$\wedge [\varrho\langle t_k'', t_{k+1}''\rangle = \pi, \mathrm{N}(t_k, t_k', t_k'', t_{k+1}'') = 4]\} \Rightarrow$$
$$\left\{\exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge \exists\left[\bar{\eta}_1(t_k') = \beta + \frac{\pi}{2} + (k+1)\pi\right] \wedge\right.$$
$$\left.\wedge \exists\left[\bar{\eta}_1(t_k'') = \gamma + \frac{\pi}{2} + (k+1)\pi\right]\right\}.$$

III. (2. 3. 4)

a) $$\forall[t \in (-\infty, +\infty)] \wedge \forall\left\{C_2, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_2 = \operatorname{sgn} C_4) \wedge\right.$$
$$\wedge (4C_2C_4 - C_3^2 > 0) \wedge \left[\frac{2C_2}{\sqrt{4C_2^2 + C_3^2}} = \sin\alpha \wedge \frac{C_3}{\sqrt{4C_2^2 + C_3^2}} =\right.$$
$$\left.\left.= \cos\alpha, \text{ i.e. } \alpha = \operatorname{arctg}\frac{2C_2}{C_3} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\right\}$$
$$\exists\left[y(t, C_2, C_3, C_4) = C_2 \sin^2 t \cos t + C_3 \sin t \cos^2 t + C_4 \cos^3 t =\right.$$
$$= \frac{1}{4C_2} \cos t[(4C_2^2 + C_3^2)\cos^2(t - \alpha) + (4C_2C_4 - C_3^2)\cos^2 t] \in \mathbf{Y}_{234}^{\mathrm{III}} \to$$
$$\left.\to y^{(\mathrm{IV})}(t, C_2, C_3, C_4) + 10y''(t, C_2, C_3, C_4) + 9y(t, C_2, C_3, C_4) = 0\right];$$
$$\left\{[y(t, C_2, C_3, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \left[t_k = \frac{\pi}{2} + k\pi, v(t_k) = 1\right] \wedge\right.$$
$$\wedge [t_k, t_{k+1}, t_{k+2}, t_{k+3} \in \langle t_k, t_{k+3}\rangle \to (t_k < t_{k+1} < t_{k+2} < {}_{k+3})] \wedge$$
$$\left.\wedge [\rho\langle t_k, t_{k+3}\rangle = 3\pi, \mathrm{N}(t_k, t_{k+1}, t_{k+2}, t_{k+3}) = 4]\right\} \Rightarrow \exists\left[\bar{\eta}_1(t_k) = \frac{\pi}{2} + (k+3)\pi\right]$$

b) $$\forall[t \in (-\infty, +\infty)] \wedge \forall\left\{C_2, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (\operatorname{sgn} C_2 = \operatorname{sgn} C_4) \wedge\right.$$
$$\wedge (4C_2C_4 = C_3^2) \wedge \left[\frac{2C_2}{\sqrt{4C_2^2 + C_3^2}} = \sin\alpha \wedge \frac{C_3}{\sqrt{4C_2^2 + C_3^2}} = \cos\alpha,\right.$$
$$\left.\left.\text{i.e. } \alpha = \operatorname{arctg}\frac{2C_2}{C_3} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\right\} \exists[y(t, C_2, C_3, C_4) =$$
$$= C_2 \sin^2 t \cos t + C_3 \sin t \cos^2 t + C_4 \cos^3 t = (C_2 + C_4)\cos t \cos^2(t - \alpha) \in$$
$$\in \mathbf{Y}_{234}^{\mathrm{III}} \to y^{(\mathrm{IV})}(t, C_2, C_3, C_4) + 10y''(t, C_2, C_3, C_4) + 9y(t, C_2, C_3, C_4) = 0];$$

$$\Big\{[y(t, C_2, C_3, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}): \Big[\Big(t_k = \frac{\pi}{2} + k\pi, \nu(t_k) = 1\Big) \wedge$$

$$\wedge \Big(t'_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t'_k) = 2\Big)\Big] \wedge$$

$$(1_1) \ \Big[\alpha \in \Big(-\frac{\pi}{2}, 0\Big)\Big] \Rightarrow \{[t_k, t_{k+1}, t'_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_{k+1} < t_{k+1})] \wedge$$

$$\wedge [\varrho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_{k+1}) = 4]\},$$

$$(1_2) \ \Big[\alpha \in \Big(0, \frac{\pi}{2}\Big)\Big] \Rightarrow \{[t_k, t_{k+1}, t'_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t_{k+1})] \wedge$$

$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k) = 4]\},$$

$$(2_1) \ \Big[\alpha \in \Big(-\frac{\pi}{2}, 0\Big)\Big] \Rightarrow \{[t_k, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t'_{k+1})] \wedge$$

$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_k, t'_k, t'_{k+1}) = 5]\},$$

$$(2_2) \ \Big[\alpha \in \Big(0, \frac{\pi}{2}\Big)\Big] \Rightarrow \{[t_{k+1}, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t'_{k+1})] \wedge$$

$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, N(t_{k+1}, t'_k, t'_{k+1}) = 5]\}\Big\} \Rightarrow$$

$$\Rightarrow \Big\{\exists \Big[\bar{\eta}_1(t_k) = \frac{\pi}{2} + (k+1)\pi\Big] \wedge \bar{\exists}\bar{\eta}_1(t'_k)\Big\},$$

c) $\forall[t \in (-\infty, +\infty)] \wedge \forall \Big\{C_2, C_3, C_4 \in \boldsymbol{R} - \{0\} \wedge (C_3^2 - 4C_2C_4 > 0) \wedge$

$$\wedge \Big(\frac{C_3 + \sqrt{C_3^2 - 4C_2C_4}}{2C_2} = p \wedge \frac{C_3 - \sqrt{C_3^2 - 4C_2C_4}}{2C_2} = q, \ p \neq 0,$$

$$q \neq 0, \ p \pm q \neq 0\Big) \wedge \Big[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha,$$

$$\text{i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \Big(-\frac{\pi}{2}, 0\Big) \cup \Big(0, \frac{\pi}{2}\Big)\Big] \wedge \Big[\frac{1}{\sqrt{1+q^2}} = \sin\beta \wedge \frac{q}{\sqrt{1+q^2}} =$$

$$= \cos\beta, \ \text{i.e. } \beta = \operatorname{arctg}\frac{1}{q} \in \Big(-\frac{\pi}{2}, 0\Big) \cup \Big(0, \frac{\pi}{2}\Big)\Big]\Big\}$$

$$\exists[y(t, C_2, C_3, C_4) = C_2 \sin^2 t \cos t + C_3 \sin t \cos^2 t + C_4 \cos^3 t =$$

$$= C_2 \cos t (\sin t + p \cos t)(\sin t + q \cos t) = C_2 \sqrt{(1+p^2)(1+q^2)} \times$$

$$\times \cos t \cos(t - \alpha) \cos(t - \beta) \in \mathbf{Y}_{234}^{\mathrm{III}} \to y^{(\mathrm{IV})}(t, C_2, C_3, C_4) +$$

$$+ 10y''(t, C_2, C_3, C_4) + 9y(t, C_2, C_3, C_4) = 0];$$

$$\Big\{[y(t, C_2, C_3, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}): \Big[\Big(t_k = \frac{\pi}{2} + k\pi, \nu(t_k) = 1\Big) \wedge$$

$$\wedge \Big(t'_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t'_k) = 1\Big) \wedge \Big(t''_k = \beta + \frac{\pi}{2} + k\pi, \nu(t''_k) = 1\Big)\Big] \wedge$$

$$(1)\quad \left[\frac{1}{p} < \frac{1}{q} < 0,\ \text{i.e.}\ -\frac{\pi}{2} < \alpha < \beta < 0\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_{k+1} < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t''_k < t_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_k, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t_k < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_k, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

$$(2)\quad \left[\frac{1}{p} < 0 < \frac{1}{q},\ \text{i.e.}\ -\frac{\pi}{2} < \alpha < 0 < \beta < \frac{\pi}{2}\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_{k+1}, t''_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t''_k < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_{k+1}, t''_k) = 4],$$

$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t''_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t'_{k+1} < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

$$(3)\quad \left[0 < \frac{1}{p} < \frac{1}{q},\ \text{i.e.}\ 0 < \alpha < \beta < \frac{\pi}{2}\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t'_k < t''_k < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_k, t''_k) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t''_k < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_{k+1}, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

$$(4)\quad \left[\frac{1}{q} < \frac{1}{p} < 0,\ \text{i.e.}\ -\frac{\pi}{2} < \beta < \alpha < 0\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t''_{k+1} < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi,\ N(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi,\ N(t_k, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t'_k < t_k < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi,\ N(t_k, t'_k, t''_k, t''_{k+1}) = 4],$$

$$(5)\quad \left[\frac{1}{q} < 0 < \frac{1}{p},\ \text{i.e.}\ -\frac{\pi}{2} < \beta < 0 < \alpha < \frac{\pi}{2}\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_{k+1}) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_{k+1} < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_k < t'_k < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t''_k, t''_{k+1}) = 4],$$

$$(6)\quad \left[0 < \frac{1}{q} < \frac{1}{p},\ \text{i.e.}\ 0 < \beta < \alpha < \frac{\pi}{2}\right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_k < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_k) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_{k+1}, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_k < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t''_k, t''_{k+1}) = 4]\Big\} \Rightarrow$$

$$\left\{\exists\left[\bar{\eta}_1(t_k) = \frac{\pi}{2} + (k+1)\pi\right] \wedge \exists\left[\bar{\eta}_1(t'_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge\right.$$
$$\left.\wedge \exists\left[\bar{\eta}_1(t''_k) = \beta + \frac{\pi}{2} + (k+1)\pi\right]\right\}.$$

IV. (1. 2. 3. 4)

$$\text{a)}\ \forall[t \in (-\infty, +\infty)] \wedge \forall\Big\{C_i \in \boldsymbol{R} - \{0\}, i = 1, \ldots, 4 \wedge (\operatorname{sgn} C_1 = \operatorname{sgn} C_3) \wedge$$
$$\wedge (\operatorname{sgn} C_2 = \operatorname{sgn} C_4) \wedge (C_2C_3 = 9C_1C_4) \wedge (3C_1C_3 - C_2^2 = 0) \wedge$$
$$\wedge (2C_2^3 - 9C_1C_2C_3 + 27C_1^2C_4 = 0) \wedge \left(\frac{C_2}{3C_1} = p,\ \frac{C_3}{3C_1} = p^2,\ \frac{C_4}{C_1} = p^3\right) \wedge$$
$$\wedge\left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha,\ \text{i.e.}\ \alpha = \operatorname{arctg}\frac{1}{p} \in\right.$$
$$\left.\in\left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\Big\}\ \exists[y(t, C_1, C_2, C_3, C_4) = \sum_{i=1}^{4} C_i \sin^{4-i} t \cos^{i-1} t =$$
$$= C_1 \sum_{i=0}^{3}\binom{3}{i}\sin^{3-i} t (p\cos t)^i = C_1(\sin t + p\cos t)^3 = C_1(1+p^2)^{3/2}\cos^3(t-\alpha) \in$$
$$\in \mathbf{Y}^{IV}_{1234} \to y^{(IV)}(t, C_1, \ldots, C_4) + 10y''(t, C_1, \ldots, C_4) + 9y(t, C_1, \ldots, C_4) = 0];$$

$$\Big\{[y(t, C_1, \dots, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \Big[t_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 3 \Big] \wedge$$
$$\wedge [t_k, t_{k+1} \in \langle t_k, t_{k+1} \rangle \to (t_k < t_{k+1})] \wedge [\varrho\langle t_k, t_{k+1} \rangle = \pi, N(t_k, t_{k+1}) = 6] \Big\} \Rightarrow$$
$$\Rightarrow \tilde{\exists} \bar{\eta}_1(t_k),$$

b) $\forall[t \in (-\infty, +\infty)] \wedge \forall \Big\{ C_i \in \boldsymbol{R} - \{0\}, i = 1, \dots, 4 \wedge \{[3C_1C_3 - C_2^2 = 0 \wedge$
$\wedge 2C_2^3 + 9C_1(3C_1C_4 - C_2C_3) \neq 0] \vee (C_2^2 - 3C_1C_3 < 0) \vee [C_2^2 - 3C_1C_3 > 0 \wedge$
$\wedge C_1 > 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) < 0] \vee$
$\vee [C_2^2 - 3C_1C_3 > 0 \wedge C_1 > 0 \wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] +$
$+ 9C_1(3C_1C_4 - C_2C_3) > 0] \vee [C_2^2 - 3C_1C_3 > 0 \quad C_1 < 0 \wedge$
$\wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) > 0] \vee$
$\vee [C_2^2 - 3C_1C_3 > 0 \wedge C_1 < 0 \wedge 2[C_2^3 - [C_2^2 - 3C_1C_3)^{3/2}] +$
$+ 9C_1(3C_1C_4 - C_2C_3) < 0] \Big\} \wedge \Big(\dfrac{C_2}{C_1} = p + q \wedge \dfrac{C_3}{C_1} = pq + r \wedge$
$\wedge \dfrac{C_4}{C_1} = pr \wedge \operatorname{sgn} \dfrac{C_4}{C_1} = \operatorname{sgn} p, p \neq 0, p + q \neq 0, pq + r \neq 0, r > 0 \wedge$
$\wedge q^2 - 4r < 0 \Big) \wedge \Big[\dfrac{1}{\sqrt{1 + p^2}} = \sin\alpha \wedge \dfrac{p}{\sqrt{1 + p^2}} = \cos\alpha,$
i.e. $\alpha = \operatorname{arctg} \dfrac{1}{p} \in \Big(-\dfrac{\pi}{2}, 0 \Big) \cup \Big(0, \dfrac{\pi}{2} \Big) \Big] \wedge \Big[\dfrac{2}{\sqrt{4 + q^2}} = \sin\beta \wedge \dfrac{q}{\sqrt{4 + q^2}} = \cos\beta,$
i.e. $\beta = \operatorname{arctg} \dfrac{2}{q} \in \Big(-\dfrac{\pi}{2}, 0 \Big) \cup \Big(0, \dfrac{\pi}{2} \Big) \Big] \Big\} \exists \Big[y(t, C_1, \dots, C_4) =$
$$= \sum_{i=1}^{4} C_i \sin^{4-i} t \cos^{i-1} t = C_1(\sin t + p\cos t)(\sin^2 t + q \sin t \cos t + r\cos^2 t) =$$
$$= \frac{C_1}{4} \sqrt{1 + p^2} \cos(t - \alpha) [(4 + q^2) \cos^2(t - \beta) + (4r - q^2) \cos^2 t] \in \mathbf{Y}_{1234}^{IV} \to$$
$$\to y^{(IV)}(t, C_1, \dots, C_4) + 10y''(t, C_1, \dots, C_4) + 9y(t, C_1, \dots, C_4) = 0 \Big];$$
$$\Big\{[y(t, C_1, \dots, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \Big[t_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 1 \Big] \wedge$$
$$\wedge [t_k, t_{k+1}, t_{k+2}, t_{k+3} \in \langle t_k, t_{k+3} \rangle \to (t_k < t_{k+1} < t_{k+2} < t_{k+3})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+3} \rangle = 3\pi, N(t_k, t_{k+1}, t_{k+2}, t_{k+3}) = 4] \Big\} \Rightarrow$$
$$\Rightarrow \exists \Big[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k + 3)\pi \Big]$$

c) $\forall[t\in(-\infty, +\infty)] \wedge \forall\{C_i \in \boldsymbol{R} - \{0\}, i = 1, \ldots, 4 \wedge (C_2^2 - 3C_1C_3 > 0) \wedge$
$\wedge \{[C_1 > 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) > 0 \wedge$
$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) = 0 \wedge (p < q)] \vee$
$\vee [C_1 > 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) = 0 \wedge$
$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) < 0 \wedge (p > q)] \vee$
$\vee [C_1 < 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) < 0 \wedge$
$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) = 0 \wedge (p < q)] \vee$
$\vee [C_1 < 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) = 0 \wedge$
$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) > 0 \wedge (p > q)]\} \wedge$

$$\wedge\left[\frac{C_2}{C_1} = p + 2q \wedge \frac{C_3}{C_1} = q(2p + q) \wedge \frac{C_4}{C_1} = pq^2 \wedge \operatorname{sgn}\frac{C_4}{C_1} = \operatorname{sgn} p, p \neq 0,\right.$$

$$\left. q \neq 0, p \neq q, p + 2q \neq 0, q + 2p \neq 0\right] \wedge \left[\frac{1}{\sqrt{1 + p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1 + p^2}} =\right.$$

$$\left. = \cos\alpha, \text{ i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right] \wedge \left[\frac{1}{\sqrt{1 + q^2}} = \sin\beta \wedge\right.$$

$$\left.\left. \wedge \frac{q}{\sqrt{1 + q^2}} = \cos\beta, \text{ i.e. } \beta = \operatorname{arctg}\frac{1}{q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)\right]\right\}$$

$$\exists[y(t, C_1, \ldots, C_4) = \sum_{i=1}^{4} C_i \sin^{4-i} t \cos^{i-1} t = C_1(\sin t + p\cos t)\times$$
$$\times(\sin t + q\cos t)^2 = C_1\sqrt{1 + p^2}(1 + q^2)\cos(t - \alpha)\cos^2(t - \beta) \in \mathbf{Y}^{IV}_{1234} \to$$
$$\to y^{(IV)}(t, C_1, \ldots, C_4) + 10y''(t, C_1, \ldots, C_4) + 9y(t, C_1, \ldots, C_4) = 0];$$

$$\left\{[y(t, C_1, \ldots, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \left\{\left[\left(t_k = \alpha + \frac{\pi}{2} + k\pi, \nu(t_k) = 1\right) \wedge\right.\right.\right.$$
$$\left.\wedge\left[t'_k = \beta + \frac{\pi}{2} + k\pi, \nu(t'_k) = 2\right)\right] \wedge$$

$$(1)\ \left(\frac{1}{p} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < \beta < \frac{\pi}{2}\right) \Rightarrow [t_k, t_{k+1}, t'_k \in \langle t_k, t_{k+1}\rangle \to$$
$$\to (t_k < t'_k < t_{k+1}) \wedge \varrho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_k) = 4] \wedge$$
$$\wedge [t_{k+1}, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t'_{k+1}) \wedge \rho\langle t'_k, t'_{k+1}\rangle = \pi,$$
$$N(t_{k+1}, t'_k, t'_{k+1}) = 5],$$

$$(2)\ \left(\frac{1}{q} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < \frac{\pi}{2}\right) \Rightarrow [t_k, t_{k+1}, t'_{k+1} \in \langle t_k, t_{k+1}\rangle \to$$
$$\to (t_k < t'_{k+1} < t_{k+1}) \wedge \rho\langle t_k, t_{k+1}\rangle = \pi, N(t_k, t_{k+1}, t'_{k+1}) = 4] \wedge$$
$$\wedge [t_k, t'_k, t'_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t_{k+1}) \wedge \varrho\langle t'_k, t'_{k+1}\rangle = \pi,$$
$$\left.\left.N(t_k, t'_k, t'_{k+1}) = 5]\right\}\right\} \Rightarrow \left\{\exists\left[\overline{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k + 1)\pi\right] \wedge \bar{\exists}\overline{\eta}_1(t'_k)\right\}$$

$$\text{d) } \forall[t\in(-\infty,+\infty)] \wedge \forall \Big\{ C_i \in \boldsymbol{R} - \{0\}, i = 1, \dots, 4 \wedge (C_2^2 - 3C_1C_3 > 0) \wedge$$

$$\wedge \{[C_1 > 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2} + 9C_1(3C_1C_4 - C_2C_3) > 0 \wedge$$

$$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) < 0] \vee$$

$$\vee [C_1 < 0 \wedge 2[C_2^3 + (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) < 0 \wedge$$

$$\wedge 2[C_2^3 - (C_2^2 - 3C_1C_3)^{3/2}] + 9C_1(3C_1C_4 - C_2C_3) > 0]\} \wedge$$

$$\wedge \Big(\frac{C_2}{C_1} = p + q + r \wedge \frac{C_3}{C_1} = pq + pr + qr \wedge \frac{C_4}{C_1} = pqr, p \neq 0,$$

$$q \neq 0, r \neq 0, p \neq q \neq r \neq p, p + q + r \neq 0, pq + pr + qr \neq 0 \Big) \wedge$$

$$\wedge \left[\frac{1}{\sqrt{1+p^2}} = \sin\alpha \wedge \frac{p}{\sqrt{1+p^2}} = \cos\alpha, \right.$$

$$\left. \text{i.e. } \alpha = \operatorname{arctg}\frac{1}{p} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right) \right] \wedge$$

$$\wedge \left[\frac{1}{\sqrt{1+q^2}} = \sin\beta \wedge \frac{q}{\sqrt{1+q^2}} = \cos\beta, \right.$$

$$\left. \text{i.e. } \beta = \operatorname{arctg}\frac{1}{q} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right) \right] \wedge$$

$$\wedge \left[\frac{1}{\sqrt{1+r^2}} = \sin\gamma \wedge \frac{r}{\sqrt{1+r^2}} = \cos\gamma, \right.$$

$$\left. \text{i.e. } \gamma = \operatorname{arctg}\frac{1}{r} \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right) \right] \Big\}$$

$$\exists [y(t, C_1, \dots, C_4) = \sum_{i=1}^{4} C_i \sin^{4-i} t \cos^{i-1} t =$$

$$= C_1 (\sin t + p\cos t)(\sin t + q\cos t)(\sin t + r\cos t) =$$

$$= C_1 \sqrt{(1+p^2)(1+q^2)(1+r^2)} \cos(t-\alpha)\cos(t-\beta)\cos(t-\gamma) \in \mathbf{Y}^{IV}_{1234} \to$$

$$\to y^{(IV)}(t, C_1, \dots, C_4) + 10y''(t, C_1, \dots, C_4) + 9y(t, C_1, \dots, C_4) = 0];$$

$$\Big\{ [y(t, C_1, \dots, C_4) = 0] \Leftrightarrow \forall(k \in \boldsymbol{C}) : \left[\left(t_k = \alpha + \frac{\pi}{2} + k\pi, v(t_k) = 1 \right) \wedge \right.$$

$$\wedge \left(t'_k = \beta + \frac{\pi}{2} + k\pi, v(t'_k) = 1 \right) \wedge$$

$$\left. \wedge \left(t''_k = \gamma + \frac{\pi}{2} + k\pi, v(t''_k) = 1 \right) \right] \wedge$$

$$(1) \left[\frac{1}{p} < \frac{1}{q} < \frac{1}{r}, \text{i.e. } -\frac{\pi}{2} < \alpha < \beta < \gamma < \frac{\pi}{2} \right] \Rightarrow$$

$$[t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1} \rangle \to (t_k < t'_k < t''_k < t_{k+1})] \wedge$$

$$\wedge [\rho\langle t_k, t_{k+1} \rangle = \pi, N(t_k, t_{k+1}, t'_k, t''_k) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_{k+1} < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(2) $$\left[\frac{1}{p} < \frac{1}{r} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \alpha < \gamma < \beta < \frac{\pi}{2}\right] \Rightarrow$$
$$\Rightarrow [t_k, t_{k+1}, t'_k, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_k < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_k) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_{k+1} < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_{k+1}, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_k < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t''_k, t''_{k+1}) = 4],$$

(3) $$\left[\frac{1}{q} < \frac{1}{p} < \frac{1}{r}, \text{ i.e. } -\frac{\pi}{2} < \beta < \alpha < \gamma < \frac{\pi}{2}\right] \Rightarrow$$
$$\Rightarrow [t_k, t_{k+1}, t'_{k+1}, t''_k \in \langle t_k, t_{k+1}\rangle \to (t_k < t''_k < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_k) = 4],$$

$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t_k < t''_k < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_{k+1}, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t'_{k+1} < t_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(4) $$\left[\frac{1}{q} < \frac{1}{r} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \beta < \gamma < \alpha < \frac{\pi}{2}\right] \Rightarrow$$
$$\Rightarrow [t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_{k+1} < t''_{k+1} < t_{k+1})] \wedge$$
$$[\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$

$$[t_k, t'_k, t'_{k+1}, t''_k \in \langle t'_k, t'_{k+1}\rangle \to (t'_k < t''_k < t_k < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_k) = 4],$$

$$[t_k, t'_{k+1}, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \to (t''_k < t_k < t'_{k+1} < t''_{k+1})] \wedge$$
$$\wedge [\varrho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_{k+1}, t''_k, t''_{k+1}) = 4],$$

(5) $$\left[\frac{1}{r} < \frac{1}{p} < \frac{1}{q}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \alpha < \beta < \frac{\pi}{2}\right] \Rightarrow$$
$$[t_k, t_{k+1}, t'_k, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \to (t_k < t'_k < t''_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_k, t''_{k+1}) = 4],$$

$$[t_{k+1}, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t''_{k+1} < t_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\varrho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_{k+1}, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow [t''_k < t_k < t'_k < t''_{k+1})] \wedge$$
$$\wedge [\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t''_k, t''_{k+1}) = 4],$$

$$(6)\ \left[\frac{1}{r} < \frac{1}{q} < \frac{1}{p}, \text{ i.e. } -\frac{\pi}{2} < \gamma < \beta < \alpha < \frac{\pi}{2}\right] \Rightarrow$$
$$[t_k, t_{k+1}, t'_{k+1}, t''_{k+1} \in \langle t_k, t_{k+1}\rangle \rightarrow (t_k < t''_{k+1} < t'_{k+1} < t_{k+1})] \wedge$$
$$\wedge [\rho\langle t_k, t_{k+1}\rangle = \pi, \mathrm{N}(t_k, t_{k+1}, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t'_{k+1}, t''_{k+1} \in \langle t'_k, t'_{k+1}\rangle \rightarrow (t'_k < t_k < t''_{k+1} < t'_{k+1})] \wedge$$
$$\wedge [\rho\langle t'_k, t'_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t'_{k+1}, t''_{k+1}) = 4],$$
$$[t_k, t'_k, t''_k, t''_{k+1} \in \langle t''_k, t''_{k+1}\rangle \rightarrow (t''_k < t'_k < t_k < t''_{k+1})] \wedge$$
$$\left.[\rho\langle t''_k, t''_{k+1}\rangle = \pi, \mathrm{N}(t_k, t'_k, t''_k, t''_{k+1}) = 4]\right\} \Rightarrow$$

$$\left\{\exists\left[\bar{\eta}_1(t_k) = \alpha + \frac{\pi}{2} + (k+1)\pi\right] \wedge \exists\left[\bar{\eta}_1(t'_k) = \beta + \frac{\pi}{2} + (k+1)\pi\right] \wedge\right.$$
$$\left.\wedge \exists\left[\bar{\eta}_1(t''_k) = \gamma + \frac{\pi}{2} + (k+1)\pi\right]\right\}.$$

Shrnutí

KONJUGOVANÉ BODY ŘEŠENÍ ITEROVANÉ LINEÁRNÍ DIFERENCIÁLNÍ ROVNICE 4. ŘÁDU

Vladimír Vlček

V článku je vyšetřována existence a hledán tvar konjugovaných bodů příslušejících k nulovým bodům každého (netriviálního) oscilatorického řešení rovnice (1) ze všech patnácti podprostorů prostoru jejích řešení.

Резюме

СОПРЯЖЕННЫЕ ТОЧКИ РЕШЕНИЙ ИТЕРИРОВАННОГО ЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ 4-го ПОРЯДКА

Владимир Влчек

В работе ращается вопрос о существовании (и форме) сопряжённых точек принадлежащих нулевым точкам каждого (нетривиального) колеблющегося решения уравнения (1) из всех пятнадцати подпространств пространства его решений.