Vladimír Vlček

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ON SPLITTING SOLUTIONS OF A CERTAIN N-th ORDER DIFFERENTIAL EQUATION

VLADIMÍR VLČEK

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Let us have a 2-*nd* order linear homogeneous differential equation

$$y''(t) + q(t)\,y(t) = 0 \tag{2}$$

with $q(t) \in \mathbf{C}^{(n-2)}(-\infty, +\infty)$, $n \in \mathbf{N}$, $n > 1$, $q(t) > 0$ on $\mathbf{I} = (-\infty, +\infty)$, whose basis $[u(t), v(t)]$ of the space of all solutions

$$y(t) = C_1 u(t) + C_2 v(t),$$

$C_i \in \mathbf{R}$ $(i = 1, 2)$, forms an ordered pair of functions $u(t)$, $v(t)$ oscillatory on the interval $\mathbf{I}$ in the sense of [2].

Consider a linear homogeneous differential equation of the n-*th* order

$$y^{(n)}(t) + \sum_{i=1}^{n} a_i(t)\,y^{(i-1)}(t) = 0, \tag{n}$$

where $a_i = a_i[q(t), q'(t), \ldots, q^{(n-2)}(t)]$, whose basis of the space of all solutions

$$y(t) = \sum_{i=1}^{n} C_i u^{n-i}(t)\,v^{i-1}(t), \tag{Y}$$

with $C_i \in \mathbf{R}$ $(i = 1, \ldots, n)$ forms an ordered n-tuple of functions

$$[u^{n-1}(t), u^{n-2}(t)\,v(t), \ldots, u(t)\,v^{n-2}(t), v^{n-1}(t)]. \tag{B}$$

For the fact that (n) is of the n-*th* order, any arbitrary zero of its nontrivial oscillatory solution $y(t)$ is of multiplicity $\nu \in \{1, \ldots, n-1\}$, i.e. $\nu = n - 1$ at most. Throughout this paper we shall assume all solutions both of (2) and of (n) to be nontrivial, only.

The bundles of oscillatory solutions of the differential equation (n)

A sufficient condition for the oscillatority of the solution $y(t)$ of (n) of an arbitrary order $n \in \mathbf{N}$, $n > 1$ with a basis (B), among whose zeros $t^* \in \mathbf{I}$ are such that

$$y(t^*) = u(t^*) \qquad \text{or} \qquad y(t^*) = v(t^*),$$

is that this solution $y(t)$ be in the form

$$y(t) = \sum_{i=1}^{n-1} C_i u^{n-i}(t)\, v^{i-1}(t), \qquad \text{where} \qquad \sum_{i=1}^{n-1} C_i^2 > 0, \tag{S*}$$

or

$$y(t) = \sum_{i=2}^{n} C_i u^{n-i}(t)\, v^{i-1}(t), \qquad \text{where} \qquad \sum_{i=2}^{n} C_i^2 > 0. \tag{S**}$$

With respect to the symmetrical distribution of both functions $u(t)$, $v(t)$ and their powers occuring in the general solution (Y) of (n) it suffices to study the zeros of oscillatory solutions of (n) from the bundle (S*), i.e. entirely such solutions among whose zeros always belong all zeros of the function $u(t)$.

Thereby: if the solution $y(t)$ of (n) is expressible in the form

$$y(t) = u^k(t)\, Y^*(t),$$

where $1 \leqq k \leqq n - 1$ ($k, n \in \mathbf{N}$, $n > 1$), where $u(t)$ is an oscillatory solution of (2) having all zeros simple and if at the same time

$$u(t_0) = 0, \qquad Y^*(t_0) \neq 0$$

holds at any point $t_0 \in \mathbf{I}$, then the point t_0 is a k-fold zero of the oscillatory solution $y(t)$ of (n).

Lemma: Let $t_0 \in (-\infty, +\infty)$ be an arbitrary firmly chosen point whereat the function $u(t)$ from the basis $[u(t), v(t)]$ of the oscillatory differential equation (2) vanishes. Then every solution $y(t)$ of the differential equation (n) with the basis (B) vanishing together with the solution $u(t)$ of the differential equation (2) at the point t_0 is of the form

$$y(t) = \sum_{i=1}^{n-k} C_i u^{n-i}(t)\, v^{i-1}(t), \tag{S}$$

where $C_i \in \mathbf{R}$ ($i = 1, \ldots, n - k$) and where $C_{n-k} \neq 0$, $1 \leqq k \leqq n - 1$, $n > 1$, $n \in \mathbf{N}$, exactly if the point t_0 is a k-fold zero of the solution $y(t)$ of the differential equation (n).

An immediate consequence of the above Lemma is: if the point t_0 is a zero of the solution $y(t)$ of (n) with multiplicity $\nu = k$, $k \in \{1, \ldots, n - 1\}$, then all solutions $y(t)$ of (n) from (S) k-fold vanishing at t_0 together with the function $u(t)$ may be written as

$$y(t) = u^k(t)\, Y^*_{n-k}(t),$$

where

$$Y^*_{n-k}(t) = \sum_{i=1}^{n-k} C_i u^{n-k-i}(t)\, v^{i-1}(t), \qquad C_{n-k} \neq 0.$$

Corollaries of the Lemma

Every solution $y(t)$ of the differential equation (n) from an $(n-k)$-parametric bundle (S) always vanishes at all zeros of the function $u^k(t)$ all without exception being of the same multiplicity $\nu = k$, $k \in \{1, \ldots, n-1\}$ (the so-called strongly conjugate points from the bundle (S) of the solutions $y(t)$ of (n) – see Definition 1.3 in [1]).

Writting T_1 for the first (neighbouring) zero of the function $u(t)$ lying to the right of the point t_0, then every solution $y(t)$ of (n) from the bundle (S) has at least both points t_0, T_1 on the interval $\langle t_0, T_1 \rangle$, which are its (k-fold) zeros for all $k \in \in \{1, \ldots, n-1\}$.

Whether the solution $y(t)$ of (n) from the bundle (S) has besides both end points t_0, T_1 any more zeros (the so-called weakly conjugate points from the bundle (S) of the solutions $y(t)$ of (n), see Definition 1.3 in [1]) on the interval $\langle t_0, T_1 \rangle$, decides but the existence of the zeros of the $(n-k)$-parametric system of functions $Y^*_{n-k}(t)$ occurring in the bundle (S). This "moving" nature of the above zeros on the open interval (t_0, T_1), their occurrence, number and multiplicities are given by the arbitrariness of the $n-k$ real parameters $C_1, \ldots, C_{n-k}$ in the system of functions $Y^*_{n-k}(t)$ on one hand and by the varied possibilities of their interrelations by which the number, multiplicities and positions of those zeros between the points t_0 and T_1 are determined, on the other hand.

It may be generally stated here that the higher is the multiplicity k of the point t_0, the less – as regards the number $(n-k-1)$ – remains for the number of zeros of the functional system $Y^*_{n-k}(t)$ and for their possible multiplicities (whose maximum may be $n-k-1$).

The minimal number of zeros on $\langle t_0, T_1 \rangle$ may thus be expected when the value of $k \in \{1, \ldots, n-1\}$ is maximal, i.e. $k = n-1$. Then the only $(n-1)$-fold zeros of the solution $y(t)$ of (n) from the bundle (S) on the interval $\langle t_0, T_1 \rangle$ are but both end points t_0, T_1, i.e. the zeros of the function $u^{n-1}(t)$. This, however, is not the only possible case, when no zeros of the solution $y(t)$ of (n) from the bundle (S) exist on the open interval (t_0, T_1) as will be shown below.

If all solutions from the bundle (S) are vanishing together with the function $u(t)$ at the ν-fold point t_0, $\nu \in \{1, \ldots, n-1\}$, then the solution $y(t)$ of (n) from the bundle (S) may have at most an $\mu = (n-\nu-1)$-fold zero or $s \in \{1, \ldots, n-\nu-1\}$ zeros with the sum of their multiplicities $N \in \{1, \ldots, n-\nu-1\}$ on the open interval (t_0, T_1).

Specially: There may exist such a solution $y(t)$ of (n) having exactly $n - \nu - 1$ zeros with multiplicity $\mu = 1$ on the interval (t_0, T_1), which represents the maximal possible number of zeros of the solution $y(t)$ of (n) from the bundle (S) for every $k \in \{1, ..., n - 1\}$ on the interval $\langle t_0, T_1 \rangle$.

Thereby may however also exist such a solution $y(t)$ of (n) from the bundle (S) having no zero on the open interval (t_0, T_1), namely, if $n - k - 1$ is an even number.

It evidently holds: if the numbers $n - 1$ and k, $k \in \{1, ..., n - 1\}$, are both of the same parity (i.e. either both even, or both odd), when their difference is an even number, then the functional system $Y^*_{n-k}(t)$ – and so also the bundle (S) of the solutions $y(t)$ of (n) – need not have any zero on the open interval (t_0, T_1) at all.

However, if the numbers $n - 1$ and k are of different parity, when their difference is an odd number, then the functional system $Y^*_{n-k}(t)$ and together with it also the bundle (S) of the solutions $y(t)$ of (n) always have at least one zero on the open interval (t_0, T_1).

Potentially maximal both for the occurrence of zeros of the solutions $y(t)$ of (n) and for the number of zeros and the degree of their multiplicities evidently is the bundle

$$Y_{n-1}(t) = u(t) \sum_{i=1}^{n-1} C_i u^{n-i-1}(t) v^{i-1}(t), \qquad (S_1)$$

where $k = 1$, i.e. the bundle with the lowest possible multiplicity of both end points t_0, T_1 on the interval $\langle t_0, T_1 \rangle$ with the degree of the functional polynomial $Y^*_{n-1}(t)$ being maximal: $n - 2$. Exactly this bundle may contain the solution $y(t)$ of (n) with the maximal possible number of zeros (and consequently the densest distribution of zeros) on the interval $\langle t_0, T_1 \rangle$.

Factorization of the bundle (S) of the solutions $y(t)$ of (n)

From what has been said so far we can see that just the analysis of the $(n - k)$-parametric functional system $Y^*_{n-k}(t)$ will be of decisive importance for our further considerations. Here the point will be to find a mean enabling to penetrate into the structure of an arbitrary solution $y(t)$ of (n) from whatever type of the bundle (S) of these solutions for $k \in \{1, ..., n - 1\}$ giving at the same time a survey of the existence, the number and multiplicities of its zeros. It is the factorization of the functional system $Y^*_{n-k}(t)$ and thus also the bundle (S) or every solution $y(t)$ of (n) from this bundle.

Let us remark that the functional polynomial $Y^*_{n-k}(t)$ of the $(n - k - 1)st$ degree represents in fact an $(n - k)$-parametric space of all solutions of the differential equation of the $(n - k)th$ order, namely, of the same type as is the differential equation (n), i.e. the equation wherein the basis of the space of all their solutions constitutes the ordered $(n - k)$-tuple of functions

$$[u^{n-k-1}(t), u^{n-k-2}(t)\, v(t), \ldots, u(t)\, v^{n-k-2}(t), v^{n-k-1}(t)].$$

Hereby with the increasing k at the function $u^k(t)$ in the form of the bundle (S), the degree of the polynomial $Y^*_{n-k}(t)$ – and consequently also the order of the respective differential equation – is decreasing, until $k = n - 1$ when they vanish at all. That is where the superiority of the differential equation (n) is lying in convenient for interpreting its solutions in forms stepwise reduced up to the individual solutions of the differential equation (2).

The fact that $Y^*_{n-k}(t)$ is a homogeneous functional polynomial of the $(n - k - 1)st$ degree in the functions $u(t)$, $v(t)$ namely enables its interpreting (in agreement with the fundamental theorem of algebra) as a factorization into a product of the $n - k - 1$ functional binomials $c_{j1}u(t) + c_{j2}v(t)$ naturally with the complex coefficients c_{j1}, c_{j2} $(j = 1, 2, \ldots, n - k - 1)$ so that

$$\tilde{Y}^*_{n-k}(t) = \prod_{j=1}^{n-k-1} [c_{j1}u(t) + c_{j2}v(t)],$$

where $c_{j2} \neq 0$ for all $j = 1, 2, \ldots, n - k - 1$.

The above assumption follows from the essential requirement $C_{n-k} \neq 0$ laid upon the last from the $n - k$ parameters $C_1, \ldots, C_{n-k} \in \mathbf{R}$ occuring in a "summation" form of the functional system $Y^*_{n-k}(t)$ (cf. Lemma), where, on mutual comparing both forms $Y^*_{n-k}(t)$ and $\tilde{Y}^*_{n-k}(t)$, we obtain

$$C_{n-k} = \prod_{j=1}^{n-k-1} c_{j2}.$$

Since all coefficients $C_1, \ldots, C_{n-k}$ occuring in the $(n - k)th$ parametric functional system $Y^*_{n-k}(t)$ are real, it holds for the coefficients c_{j1}, c_{j2} being in the "product" form $\tilde{Y}_{n-k}(t)$: to any pair of complex (imaginary) coefficients c_{j1}, c_{j2}, $j \in \{1, 2, \ldots, n - k - 1\}$ involved in a functional twoparametric binomial $c_{j1}u(t) + c_{j2}v(t)$ there necessarily exists a binomial among the remaining functional binomials, with such a pair of imaginary coefficients c_{m1}, c_{m2}, $m \in \{1, 2, \ldots, n - k - 1\}$ that both ordered pairs (c_{j1}, c_{j2}) and (c_{m1}, c_{m2}) are complex conjugate. We constitute that both ordered pairs (c_{11}, c_{12}), (c_{21}, c_{22}) of imaginary coefficients occuring successively in the twoparametric functional binomials $c_{11}u(t) + c_{12}v(t)$, $c_{21}u(t) + c_{22}v(t)$ are complex conjugate exactly if c_{11} is complex conjugate to c_{21} and c_{12} is complex conjugate to c_{22}.

Besides, there is required a linear independence of the alone coefficients in the complex pairs, so that

$$\operatorname{Re} c_{j1} \operatorname{Im} c_{j2} - \operatorname{Im} c_{j1} \operatorname{Re} c_{j2} \neq 0; \qquad j = 1, 2.$$

In consequence of the above property of coefficients the product of such two functional binomials with complex conjugate coefficients is a real function having no zero on the interval $\langle t_0, T_1 \rangle$ and even on the whole interval $\mathbf{I} = (-\infty, +\infty)$. It is here either always positive or always negative (the latter alternative follows from the fact that a negative number may eventually be pointed out from the complex conjugate pair of coefficients).

It holds: writing

$$c_{11} = a_1 + a_2\mathrm{i}, \qquad c_{12} = b_1 + b_2\mathrm{i}, \qquad c_{21} = a_1 - a_2\mathrm{i}, \qquad c_{22} = b_1 - b_2\mathrm{i},$$

where $a_j, b_j \in \mathbf{R}$ $(j = 1, 2)$, so that

$$|c_{11}| = |c_{21}| = \sqrt{a_1^2 + a_2^2}, \qquad |c_{12}| = |c_{22}| = \sqrt{b_1^2 + b_2^2},$$

whereby $a_1b_2 - a_2b_1 \neq 0$, then we get for the product of two complex functional binomials

$$\begin{aligned}
&[c_{11}u(t) + c_{12}v(t)]\,[c_{21}u(t) + c_{22}v(t)] = \\
&= (a_1^2 + a_2^2)\,u^2(t) + 2(a_1b_1 + a_2b_2)\,u(t)\,v(t) + (b_1^2 + b_2^2)\,v^2(t) = \\
&= |c_{j1}|^2u^2(t) + 2\,|c_{j1}|\,|c_{m2}|\,u(t)\,v(t)\cos\omega + |c_{m2}|^2v^2(t)\cos^2\omega + \\
&\quad + |c_{m2}|^2v^2(t) - |c_{m2}|^2v^2(t)\cos^2\omega = \\
&= [|c_{j1}|\,u(t) + |c_{m2}|\,v(t)\cos\omega]^2 + [|c_{m2}|\,v(t)\sin\omega]^2 > 0,
\end{aligned}$$

where $\omega = |\arg c_{j1} - \arg c_{m2}| \neq k\pi$; $j, m = 1, 2$; $k = 0, \pm 1, \pm 2, \ldots$; the inequality obtained holds for all $t \in \mathbf{I} = (-\infty, +\infty)$.

Remark: However, if we denote the real coefficients

$$A = a_1^2 + a_2^2, \qquad B = 2(a_1b_1 + a_2b_2), \qquad C = b_1^2 + b_2^2$$

in a quadratic functional trinomial

$$(a_1^2 + a_2^2)\,u^2(t) + 2(a_1b_1 + a_2b_2)\,u(t)\,v(t) + (b_1^2 + b_2^2)\,v^2(t),$$

where $a_1b_2 - a_2b_1 \neq 0$, then its nonfactorability into a product of two real functional binomials follows from the fact that

$$B^2 - 4AC = -4(a_1b_2 - a_2b_1)^2 < 0.$$

Two functional binomials with complex conjugate pairs of coefficients (c_{j1}, c_{j2}), (c_{m1}, c_{m2}), $j \neq m$; $j, m \in \{1, 2, \ldots, \mathrm{n} - \mathrm{k} - 1\}$ will be termed the mutual associated binomials.

The number of always two mutually associated functional binomials in the functional system $\tilde{Y}^*_{n-k}(t)$ is always even.

The presence of any pair of mutually associated binomials in the functional system $\tilde{Y}^*_{n-k}(t)$ decreases always by *2* the maximal possible number of the $n - k - 1$ zeros of this system on the open interval (t_0, T_1). If v is the multiplicity of such a pair, then its presence in the functional system $\tilde{Y}^*_{n-k}(t)$ decreases by $2v$ the total highest possible number of zeros on the interval (t_0, T_1). In particular, if $n - k - 1$ is an even number, it may happen that with $2v = n - k - 1$ (no matter whether $2v$ is a multiplicity of one pair or a sum of multiplicities of more distinct pairs of mutually associated functional binomials), the functional system $\tilde{Y}^*_{n-k}(t)$ has no zero on the interval (t_0, T_1). For the occurrence (and a possibly multiplicity) of zeros of the functional system $\tilde{Y}^*_{n-k}(t)$ – and thus also of the bundle (S) of the solutions $y(t)$ relative to (n) – will be only such a functional binomial

$$y_j(t) = c_{j1}u(t) + c_{j2}v(t),$$

$j \in \{1, 2, \dots, n - k - 1\}$, of interest, wherein both coefficients c_{j1}, c_{j2} are real (with $c_{j2} \neq 0$).

However in this case $y_j(t)$ always means a solution of the differential equation (2) linearly independent with the solution $u(t)$ of the same differential equation on the interval $\mathbf{I} = (-\infty, +\infty)$. An immediate consequence of this is the fact that the zeros of the any function $y_j(t)$ mutually separate with the zeros of the function $u(t)$ on $\mathbf{I}$.

Specially: between two points t_0 and T_1 in $\langle t_0, T_1 \rangle$ there lies exactly by one zero of each function $y_j(t)$, $j \in \{1, 2, \dots, n - k - 1\}$. Then it is true that any simple zero of $y_j(t) = c_{j1}u(t) + c_{j2}v(t)$, $c_{j2} \neq 0$, on an open interval (t_0, T_1) differs from the (also simple) zero of $y_m(t) = c_{m1}u(t) + c_{m2}v(t)$, $c_{m2} \neq 0$, on the same interval (t_0, T_1) if

$$\begin{vmatrix} c_{j1} & c_{j2} \\ c_{m1} & c_{m2} \end{vmatrix} \neq 0; \qquad j, m \in \{1, 2, \dots, n - k - 1\}, j \neq m$$

and vice versa: if both functions $y_j(t)$, $y_m(t)$ on the interval $\mathbf{I} = (-\infty, +\infty)$ are linearly independent, then their (simple) zeros are different from each other on $\mathbf{I}$ and especially on (t_0, T_1).

To the linear dependence of two functions $y_j(t)$, $y_m(t)$ corresponds the double-multiplicity of their common zero on (t_0, T_1).

Generally: the function $y_j^\mu(t)$, $\mu \in \{1, 2, \dots, n - k - 1\}$, has an μ-fold zero on (t_0, T_1).

Thus the maximal number of zeros of the solution $y(t)$ from the bundle (S) relative to the differential equation (n) on the open interval (t_0, T_1) may be $n - k - 1$, whereby all zeros are simple altogether. This situation is relevant to the factorization of the functional system $\tilde{Y}^*_{n-k}(t)$ into $n - k - 1$ functional binomials with all real coefficients, always in twos (in pairs) linearly independent on $\mathbf{I}$.

For every $k \in \{1, 2, \dots, n - 1\}$, $n \in \mathbf{N}$, $n > 1$, exactly such a solution is called a totally split up solution $y(t)$ of the differential equation (n) from the bundle (S) having the form

$$y(t) = u^k(t) \prod_{j=1}^{n-k-1} [c_{j1}u(t) + c_{j2}v(t)],$$

where $c_{j1}, c_{j2} \in \mathbf{R}$, $c_{j2} \neq 0$, for all $j \in \{1, 2, \dots, n - k - 1\}$ with

$$\begin{vmatrix} c_{j1} & c_{j2} \\ c_{m1} & c_{m2} \end{vmatrix} \neq 0$$

for $j, m \in \{1, 2, \dots, n - k - 1\}$, $j \neq m$.

Such a solution $y(t)$ has always for all $k \in \{1, 2, \dots, n - 1\}$ the maximal possible number of zeros being simple and separating themselves on the interval $\langle t_0, T_1 \rangle$. Hereat the sum of multiplicities of all these simple zeros on the open interval (t_0, T_1) is exactly $\mu = n - k - 1$.

If we denote $y_0(t) = u(t)$, $y_j(t) = c_{j1}u(t) + c_{j2}v(t)$, $c_{j2} \neq 0$, $j \in \{1, 2, \dots, n-k-1\}$, then the totally split up solution $y(t)$ of the differential equation (n) from the bundle (S) may be more briefly written as

$$y(t) = y_0^k(t) \prod_{j=1}^{n-k-1} y_j(t).$$

Any simple zero of this solution on the open interval (t_0, T_1) belongs by one always to only one from the functions $y_j(t)$. Then every from these functions, obtained by a certain choice of the corresponding constants $c_{j1}, c_{j2} \in \mathbf{R}$ $(j = 1, 2, \dots, n-k-1)$ in a twoparametric system of functions

$$c_{j1}u(t) + c_{j2}v(t), \qquad c_{j2} \neq 0,$$

always represents any solution of the differential equation (2). All thus obtained solutions of (2) are always among themselves in pairs linearly independent, whereby each of them is linearly independent with the solution $y_0(t) = u(t)$ of this equation on $\mathbf{I} = (-\infty, +\infty)$.

Specially: It holds for $k = 1$ that a totally split up solution

$$y(t) = y_0(t) \prod_{j=1}^{n-1} y_j(t)$$

has the maximal number of simple zeros (namely n) of all solutions $y(t)$ of (n) on the interval $\langle t_0, T_1 \rangle$ at all. It represents the case of the densest possible distribution of zeros that the solution $y(t)$ of (n) on $\langle t_0, T_1 \rangle$ may reach.

All of this leads us to believe that we can use various types of solutions $y(t)$ of the differential equation (n) from the bundle (S) for all possible $k \in \{1, 2, \dots, n-k-1\}$ in solving k-point $(1 \leqq k \leqq n)$ boundary value problems for this differential equation, wherein before all the totally split up solutions are applied in achieving the maximal number of zeros on the given interval.

An example of the factorized solutions

Below we show an example concerning a bundle of solutions of a seventh order differential equation with all type forms for factoring the solutions inclusive their totally split up solutions for every $k \in \{1, \dots, 6\}$ – expressed by means of always two and two linearly independent solutions of differential equation (2).

The bundle of all solutions $y(t)$ relative to the seventh order diff. equation (with a basis of the type considered) vanishing together with the function $u(t)$ at an arbitrary firmly chosen point $t_0 \in \mathbf{I} = (-\infty, +\infty)$ is of the form

$$y(t) = y_0^k(t)\prod_{j=1}^{6-k} y_j(t), \qquad k \in \{1, \ldots, 6\},$$

(where for $k = 6$ we pose $y_1(t) \equiv 1$, etc., at right).

1. For $k = 6$ is $y(t) = Cy_0^6(t)$ (*)
2. For $k = 5$ is $y(t) = Cy_0^5(t)\, y_1(t)$ (*)
3. For $k = 4$ is a) $y(t) = Cy_0^4(t)$
   b) $y(t) = Cy_0^4(t)\, y_1^2(t)$
   c) $y(t) = Cy_0^4(t)\, y_1(t)\, y_2(t)$ (*)
4. For $k = 3$ is a) $y(t) = Cy_0^3(t)\, y_1(t)$
   b) $y(t) = Cy_0^3(t)\, y_1^3(t)$
   c) $y(t) = Cy_0^3(t)\, y_1(t)\, y_2^2(t)$
   d) $y(t) = Cy_0^3(t)\, y_1(t)\, y_2(t)\, y_3(t)$ (*)
5. For $k = 2$ is a) $y(t) = Cy_0^2(t)$
   b) $y(t) = Cy_0^2(t)\, y_1^2(t)$
   c) $y(t) = Cy_0^2(t)\, y_1(t)\, y_2(t)$
   d) $y(t) = Cy_0^2(t)\, y_1^4(t)$
   e) $y(t) = Cy_0^2(t)\, y_1^2(t)\, y_2^2(t)$
   f) $y(t) = Cy_0^2(t)\, y_1(t)\, y_2(t)\, y_3^2(t)$
   g) $y(t) = Cy_0^2(t)\, y_1(t)\, y_2(t)\, y_3(t)\, y_4(t)$ (*)
6. For $k = 1$ is a) $y(t) = Cy_0(t)\, y_1(t)$
   b) $y(t) = Cy_0(t)\, y_1^3(t)$
   c) $y(t) = Cy_0(t)\, y_1(t)\, y_2^2(t)$
   d) $y(t) = Cy_0(t)\, y_1(t)\, y_2(t)\, y_3(t)$
   e) $y(t) = Cy_0(t)\, y_1^5(t)$
   f) $y(t) = Cy_0(t)\, y_1(t)\, y_2^4(t)$
   g) $y(t) = Cy_0(t)\, y_1(t)\, y_2(t)\, y_3^3(t)$
   h) $y(t) = Cy_0(t)\, y_1(t)\, y_2^2(t)\, y_3^2(t)$
   i) $y(t) = Cy_0(t)\, y_1(t)\, y_2(t)\, y_3(t)\, y_4^2(t)$
   j) $y(t) = Cy_0(t)\, y_1(t)\, y_2(t)\, y_3(t)\, y_4(t)\, y_5(t)$, (*)

where throughout $C \in \mathbf{R} - \{0\}$ means an arbitrary constant (parameter).

The solutions denoted by (*) are totally split up for the corresponding $k \in \in \{1, \ldots, 6\}$.

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Author's address:
RNDr. Vladimír Vlček, CSc.
přírodovědecká fakulta UP
Gottwaldova 15
Olomouc
771 46

O ROZLOŽITELNOSTI ŘEŠENÍ JISTÉ DIFERENCIÁLNÍ ROVNICE N-TÉHO ŘÁDU

Souhrn

Práce se zabývá rozkladem libovolného svazku oscilatorických řešení jisté diferenciální rovnice n-tého řádu na součin oscilatorických řešení diferenciálních rovnic nižších řádů (téhož typu) až do rovnice 2. řádu včetně.

О РАЗЛОЖИМОСТИ РЕШЕНИЙ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ N-ГО ПОРЯДКА ОПРЕДЕЛЕННОГО ТИПА

Резюме

В работе изучается разложение любого пучка колеблющихся решений дифференциального уравнения n-го порядка определенного типа в произведение колеблющихся решений дифференциальных уравнений низших порядков (того же самого типа) до уравнения 2-го порядка включительно.