Acta Mathematica et Informatica Universitatis Ostraviensis

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Acta Mathematica et Informatica Universitatis Ostraviensis, Vol. 1 (1993), No. 1, 59--66

Persistent URL: http://dml.cz/dmlcz/120474

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Maximal Reflectivity Preserving Subextensions

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Abstract. Let $\mathcal{K}$ be a category with a full reflective subcategory $\mathcal{R}$ and let $\mathcal{K}$ be embedded into a category $\mathcal{L}$ with the same objects. For some often used concrete categories $\mathcal{K}$'s it is proved that each subextension $\mathcal{M}/\mathcal{K}$ ($\mathcal{M} \neq \mathcal{K}$) of the extension $\mathcal{L}/\mathcal{K}$ breaks the reflectivity of $\mathcal{R}$.

Keywords: category, subcategory

1991 Mathematics Subject Classification: 18A40

0. Introduction

This paper deals with the following question. There is given a category $\mathcal{K}$ and its full reflective subcategory $\mathcal{R}$. The category $\mathcal{K}$ is embedded into a category $\mathcal{L}$ with the same objects. The main aim of this paper is to show for two very often used concrete categories $\mathcal{K}$'s that each subextension $\mathcal{M}/\mathcal{K}$ ($\mathcal{M} \neq \mathcal{K}$) of the extension $\mathcal{L}/\mathcal{K}$ breaks the reflectivity of $\mathcal{R}$ (Theorems 2.1 and 3.5).

In the first case the category $\mathcal{K}$ is the category of all completely regular spaces with their continuous mappings, in which the full subcategory $\mathcal{R}$ of all compact spaces is reflective. The reflection for topological space X is the embedding β_X of X into the *Čech-Stone compactification* βX. The category $\mathcal{K}$ is embedded into the category $\mathcal{L}$ of all completely regular spaces but with the *(set) mappings.*

In the other case the category $\mathcal{K}$ of all (partially) ordered sets is considered, where the morphisms are special isotonic mappings - the "c- mappings" $\left(f(M^{*+}) \subset f(M)^{*+}\right)$. The full subcategory $\mathcal{R}$ of complete lattices is reflective and the embedding γ_X of an ordered set X into the *Mac Neile completion* $\gamma(X)$ is a reflection for X. The morphisms of the category $\mathcal{L}$ are the *(set) mappings* as well.

It follows from these results that *the class of the morphisms generally used in the category of topological (completely regular) spaces - the class of the continuous mappings - cannot be extended by further (set) mappings without breaking the Čech-Stone compactification reflection.*

In a similar way, *we cannot add (in case of the ordered sets) to the class of the c mappings further (set) mappings if we want to preserve the Mac Neile completion reflection.*

The "absolute" view on this question is also mentioned (1.5) but it seems to be, from the point of view of this direction, less interesting.

In this paper basic knowledge of the category theory is used ([5], [3]) and we denote by $O(\mathcal{C})$ and $M(\mathcal{C})$ the class of objects of a category $\mathcal{C}$ and the class of morphisms of $\mathcal{C}$, respectively.

Further, we recall a notion of the reflective subcategory:

Let $\mathcal{R}$ be a full subcategory of a category $\mathcal{L}$. (In this paper only full reflective subcategories are considered.) An *$\mathcal{R}$-reflection* for a $\mathcal{K}$-object X is a morphism $\rho_X \in \mathrm{Hom}_{\mathcal{K}}(X, R(X))$ $(R(X) \in \mathrm{O}(R))$ such that for each $\mathcal{R}$ object Y and each morphism $f \in \mathrm{Hom}_{\mathcal{K}}(X, Y)$ there exists a unique morphism $f \in \mathrm{Hom}_{\mathcal{K}}(R(X), Y)$ with the property $\overline{f} \circ \rho_X = f$. The category $\mathcal{R}$ is called a *reflective subcategory of* $\mathcal{K}$ if there exists an $\mathcal{R}$ reflection for each $\mathcal{K}$-object.

1. Reflectivity Preserving Subextensions

1.1. Definition. Let $\mathcal{R}$ be a reflective subcategory of a category $\mathcal{K}$ with $\rho_X \in \mathrm{Hom}_{\mathcal{K}}(X, R(X))$ an $\mathcal{R}$-reflection for a $\mathcal{K}$-object X, and let $\mathcal{K}$ be a subcategory of a category $\mathcal{L}$ with $\mathrm{O}(\mathcal{K}) = \mathrm{O}(\mathcal{L})$.

Let $\mathcal{M}$ be a subcategory of $\mathcal{L}$ and let $\mathcal{K}$ be a subcategory of $\mathcal{M}$. The category $\mathcal{M}$ is said to be a *reflectivity (of $\mathcal{R}$) preserving subextension of the extension $\mathcal{L}/\mathcal{K}$* if for each $X \in \mathrm{O}(\mathcal{K}) = \mathrm{O}(\mathcal{M})$, $Y \in \mathrm{O}(\mathcal{R})$, $f \in \mathrm{Hom}_{\mathcal{M}}(X, Y)$ there exists a unique morphism $\overline{f} \in \mathrm{Hom}\mathcal{M}(R(X), Y)$ such that $\overline{f} \circ \rho_X = f$. Briefly we can say that $\mathcal{M}$ is an *RP-subextension (of $\mathcal{L}/\mathcal{K}$)*.

If $\overline{\mathcal{R}}$ is a full subcategory of $\mathcal{M}$ with $\mathrm{O}(\overline{\mathcal{R}}) = \mathrm{O}(\mathcal{R})$, then $\mathcal{M}$ *is an RP-subextension if and only if $\overline{\mathcal{R}}$ is a reflective subcategory of $\mathcal{M}$ with $\overline{R}$-reflection $\rho_X\, X \to R(X)$ for each $X \in O(\overline{\mathcal{R}})$.*

Clearly, $\mathcal{K}$ *is also an RP-subextension of the extension $\mathcal{L}/\mathcal{K}$.*

If $\mathcal{M}$ is an RP-subextension of $\mathcal{L}/\mathcal{K}$ an $\mathcal{N}$ si not an RP-subextension for each subcategory $\mathcal{N}$ of $\mathcal{L}$, $\mathcal{M}$ is a subcategory of $\mathcal{N}$, $\mathcal{M} \neq \mathcal{N}$, then $\mathcal{M}$ is called a *maximal reflectivity (of $\mathcal{R}$) preserving subextension of the extension $\mathcal{L}/\mathcal{K}$* or briefly a *maximal RP-subextension (of $\mathcal{L}/\mathcal{K}$).*

The following two examples show variety of the number of maximal RP-subextensions.

1.2. Example. Let $\mathrm{O}(\mathcal{K}) = \{X, Y, Z\}$, $\mathrm{O}(\mathcal{R}) = \{Y, Z\}$,
$\mathrm{M}(\mathcal{K}) = \{i_X, i_Y, i_Z, \rho\}$, $\mathrm{M}(\mathcal{R}) = \{i_Y, i_Z\}$, where i_X, i_Y, i_Z are identities on X, Y, Z and ρ is a morphism from X to Y. In this way the categories $\mathcal{K}$ and $\mathcal{R}$ are defined, $\mathcal{R}$ is a reflective subcategory of $\mathcal{K}$, $\rho : X \to Y$ is an $\mathcal{R}$-reflection for X.

Let $\mathcal{M}$ be a non-empty set, $M \cap \mathrm{M}(\mathcal{K}) = \emptyset$. We define the category $\mathcal{L}_{\mathcal{M}}$ as follows: $\mathrm{O}(\mathcal{L}_{\mathcal{M}}) = \mathrm{O}(\mathcal{K}) = \{X, Y, Z\}$, $\mathrm{M}(\mathcal{L}_{\mathcal{M}}) = \mathrm{M}(\mathcal{K}) \cup M \cup \{f\}$, where f is a symbol not belonging to the set $\mathrm{M}(\mathcal{K}) \cup M$. The elements from M are morphisms from Y to Z and f is a morphism from X to Z. For each $m \in M$ we have $m \circ \rho = f$.

We define a subcategory $\mathcal{L}_{\mathcal{M}}(m)$ of $\mathcal{L}_{\mathcal{M}}$ for each $m \in M$ as follows:

$$\mathrm{O}(\mathcal{L}_{\mathcal{M}}(m)) = \mathrm{O}(\mathcal{L}_{\mathcal{M}}) = \{X, Y, Z\},$$
$$\mathrm{M}(\mathcal{L}_{\mathcal{M}}(m)) = \mathrm{M}(\mathcal{K}) \cup \{m, f\}.$$

Then $\{\mathcal{L}_{\mathcal{M}}(m) : m \in M\}$ *is the set of all maximal reflectivity of* $\mathcal{R}$ *preserving subextensions of the extension* $\mathcal{L}_{\mathcal{M}}/\mathcal{K}$.

1.3. Example. Let I be a non-empty class and J a subclass of I. We define the following categories $\mathcal{K}$, $\mathcal{R}$, $\mathcal{L}$, $\mathcal{L}(\mathcal{J})$ as follows:

$$\mathrm{O}(\mathcal{K}) = \mathrm{O}(\mathcal{L}) = \mathrm{O}(\mathcal{L}(\mathcal{J})) = \{X, Y\} \cup \{Z_\iota \colon \iota \in I\},$$
$$\mathrm{O}(\mathcal{R}) = \{Y\} \cup \{Z_\iota \colon \iota \in I\}, \mathrm{M}(\mathcal{K}) = \{\rho\} \cup \{i_U \colon U \in \mathrm{O}(\mathcal{K})\},$$
$$\mathrm{M}(\mathcal{R}) = \{i_U \colon U \in \mathrm{O}(\mathcal{R})\}, \mathrm{M}(\mathcal{L}) = \mathrm{M}(\mathcal{K}) \cup \bigcup \{f_\iota, g_\iota, h_\iota\} (\iota \in I) \text{ and}$$
$$\mathrm{M}(\mathcal{L}(\mathcal{J})) = \mathrm{M}(\mathcal{K}) \cup \bigcup \{f_\iota, g_\iota\} (\iota \in I) \cup \bigcup \{f_\iota, h_\iota\} (\iota \in I - J).$$

The mentioned symbols are mutually different. The symbols i_ι are identities on U's, $\rho \in \mathrm{Hom}_{\mathcal{K}}(X, Y)$, $f_\iota \in \mathrm{Hom}_{\mathcal{L}}(X, Z_\iota)$, g_ι, $h_\iota \in \mathrm{Hom}_{\mathcal{L}}(Y, Z_\iota)$ and $g_\iota \circ \rho = h_\iota \circ \rho = f_\iota$ $(\iota \in I)$. The categories $\mathcal{R}$, $\mathcal{K}$, $\mathcal{L}(\mathcal{J})$ are subcategories of $\mathcal{L}$.

Then $\mathcal{R}$ is a reflective subcategory of $\mathcal{K}$, $\rho : X \to Y$ is an $\mathcal{R}$- reflection for X. *For each subclass* $J \subseteq I$ *the category* $\mathcal{L}(\mathcal{J})$ *is a maximal RP-subextension of* $\mathcal{L}/\mathcal{K}$ *and for each maximal RP-subextension* $\mathcal{M}$ *of* $\mathcal{L}/\mathcal{K}$ *there exists a subclass* $J \subset I$ *such that* $\mathcal{M} = \mathcal{L}(\mathcal{J})$.

1.4. Note. We investigate this question about adding of further morphisms preserving reflectivity in a "relative" form. A little contribution to the "absolute" view of this problem is given in the following assertion.

1.5. Proposition. *Let* $\mathcal{R}$ *be a reflective subcategory of a category* $\mathcal{K}$. *Then there exists a category* $\mathcal{L}$ *such that* $\mathcal{K}$ *is a subcategory of* $\mathcal{L}$, $O(\mathcal{K}) = O(\mathcal{L})$, $\mathcal{K} \neq \mathcal{L}$, *and* $\mathcal{L}$ *is an RP-subextension of* $\mathcal{L}/\mathcal{K}$.

PROOF: Let $\Omega = \{\omega_{XY} : X, Y \in \mathrm{O}(\mathcal{K})\}$ be a class mutually different symbols not belonging to $\mathrm{M}(\mathcal{K})$. Define the category $\mathcal{L}$ in the following way:

$$\mathrm{O}(\mathcal{L}) = \mathrm{O}(\mathcal{K}), \mathrm{M}(\mathcal{L}) = \Omega \cup \mathrm{M}(\mathcal{K}),$$
$$\mathrm{Hom}_{\mathcal{L}}(X, Y) = \mathrm{Hom}_{\mathcal{K}}(X, Y) \cup \{\omega_{XY}\} \text{ for each } X, Y \in \mathrm{O}(\mathcal{K}),$$

for $X, Y, Z \in \mathrm{O}(\mathcal{L})$, $g \in \mathrm{Hom}_{\mathcal{L}}(X, Y)$, $f \in \mathrm{Hom}_{\mathcal{L}}(Y, Z)$ let the following qualities be satisfied:

$$\omega_{YZ} \circ g = \omega_{XZ} = f \circ \omega_{XY}$$

and let $\mathcal{K}$ be a subcategory of $\mathcal{L}$.

Then the category $\mathcal{L}$ possesses the required properites. □

2. The Čech-Stone Compactification

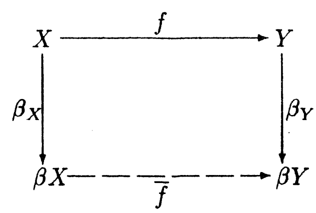
In this section we shall denote by

$\mathcal{K}$ the category of all completely regular spaces with the continuous mappings,
$\mathcal{R}$ the full subcategory of $\mathcal{K}$ of all compact spaces,
$\mathcal{L}$ the category of all completely regular spaces with the mappings.

It is a well-known result of *Čech* ([1]) and *Stone* ([6]) that $\mathcal{R}$ is a reflective subcategory of $\mathcal{K}$ and for a completely regular space X the embedding β_X from X into the *Čech-Stone compactification* βX is an $\mathcal{R}$-reflection for X (s. also Herrlich [2]).

2.1. Theorem. *The category $\mathcal{K}$ is a maximal reflectivity of $\mathcal{R}$ preserving subextension of the extension $\mathcal{L}/\mathcal{K}$.*

PROOF: I. Let $\mathcal{M}$ be a reflectivity of $\mathcal{R}$ preserving subextension of $\mathcal{L}/\mathcal{K}$ different from $\mathcal{K}$. Then there exist $X, Y \in \mathrm{O}(\mathcal{K})$ and $f \in \mathrm{Hom}_{\mathcal{M}}(X, Y) - \mathrm{Hom}_{\mathcal{K}}(X, Y)$. X, Y are completely regular spaces and f is a mapping from X into Y which is not continuous. Then there exists a mapping $\overline{f}$ from βX into βY such that the diagram

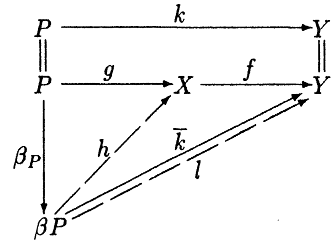


is commutative.

If the mapping $\overline{f}$ is continuous, then the mapping f is also continuous. Hence $\overline{f}$ is not continuous.

II. Therefore we can assume that X, Y are compact spaces. There exist $x \in X$ and a neighbourhood V of $f(x)$ in Y such that $U = f^{-1}(V)$ is not a neighbourhood of x in the space X, from which it follows $x \in \mathrm{cl}_X(X - U)$. Put $P = X - U$ with descrete topology and let g be the identity embedding from P into X. Since g is a continuous mapping from the space P into the space X, there exists a continuous mapping h from βP into X with the property $h \circ \beta_P = g$.

III. The set $h(\beta P)$ is closed in X (because it is compact) and it contains the set $P = X - U$. Since $x \in \mathrm{cl}_X(X - U)$, it holds $x \in h(\beta P)$, therefore there exists $z \in \beta P$ such that $h(z) = x$. Put $\overline{k} = f \circ h$ and $k = f \circ g$. Then $k \in \mathrm{Hom}_{\mathcal{M}}(P, Y)$, $\overline{k} \in \mathrm{Hom}_{\mathcal{M}}(\beta P, Y)$ and $\overline{k} \circ \beta_P = k$. As k is continuous, there exists a continuous mapping l from βP into Y such that $l \circ \beta_P = k$ (s. diagram).



The category $\mathcal{M}$ is an RP-subextension of $\mathcal{L}/\mathcal{K}$, hence $\overline{k} = l$ so that $\overline{k}$ is continuous. But $\overline{k}(z) = f(x)$ and $\overline{k}^{-1}(V) = h^{-1}(U) \subset \beta P - P$, hence $\overline{k}^{-1}(V)$ is not a neighbourhood of z in βP, which is a contradiction. □

3. The Mac Neille Completion

Let $P = (P, \leq)$ be an ordered set. For $M \subseteq P$ put

$$M^* = \{p \in P : p \geq m \quad \text{for each } m \in M\},$$
$$M^+ = \{p \in P : p \leq m \quad \text{for each } m \in M\},$$
$$\mathrm{c}M = M^{*+}.$$

Then c is a closure operator on P. We denote by $\nu(P)$ the set of all c-closed sets of P ordered by inclusion $\subseteq$ ($\nu(P) = (\nu(P), \subseteq)$). For $p \in P$ put

$$\nu_P(p) = c\{p\} = \{p\}^{*+} = \{x \in P : x \leq p\} == (p] \in \nu(P).$$

ν_P defines a mapping from P into $\nu(P)$. The ordered set $\nu(P)$ is a complete lattice and ν_P is an embedding of $(P, \leq)$ into $(\nu(P), \subseteq)$. The lattice $\nu(P)$ is called the *Mac Neille completion* of the ordered set P (Mac Neille [4]).

We call a mapping f from an ordered set X into an ordered set Y a c-mapping if for each $M \subseteq X$ we have

$$f(\mathrm{c}M) \subseteq \mathrm{c}f(M),$$

which is equivalent to the property that *for each* c*-closed subset* N *of* Y *the set* $f^{-1}(N)$ *is also* c*-closed.* Clearly, there holds:

3.1. Proposition. *(a) Every* c*-mapping is isotonic.*

(b) Let L_1, L_2 *be complete lattices. Than a mapping* f *from* L_1 *into* L_2 *is a* c*-mapping if and only if for each* $M \subseteq L_1$ *the equality*

$$f(\sup{}_{L_1} M) = \sup{}_{L_2} f(M)$$

is valid.

(c) Let P *be an ordered set. Then we have for* $M \subseteq P$ *:*

$$\nu_P(\mathrm{c}M) = \mathrm{c}\nu_P(M) \cap \nu_P(P),$$

hence *the mapping* $\nu_P : P \to \nu(P)$ *is a* c*-mapping.*

Further we denote by

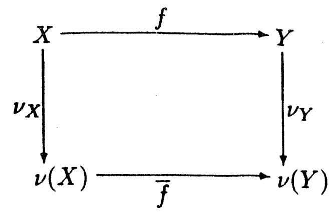
$\mathcal{K}$ the category of all ordered sets with the c-mappings,
$\mathcal{R}$ the full subcategory of $\mathcal{K}$ of all complete lattices,
$\mathcal{L}$ the category of all ordered sets with the mappings.

This is well known (s. Herrlich [2], 8.3 (10)):

3.2. Theorem. $\mathcal{R}$ *is a reflective subcategory of* $\mathcal{K}$ *and* $\nu_X : X \to \nu(X)$ *is an* $\mathcal{R}$*-reflection for each* $\mathcal{K}$*-object* X.

Further, we mention two Lemmas.

3.3. Lemma. *Let* X, Y *be ordered sets and* f *a mapping from* X *into* Y. *Let* $\overline{f}$ *be a mapping from* $\nu(X)$ *into* $\nu(Y)$ *such that the following diagram*



is commutative.

If $\overline{f}$ is a c*-mapping, then f is also a* c*-mapping.*

PROOF: Let $N \subseteq Y$ be c-closed and let $M = f^{-1}(N)$. According to 3.1(c) we have

$$\nu_Y(N) = \mathrm{c}\nu_Y(N) \cap \nu_Y(Y).$$

So that, since $\overline{f} \circ \nu_X = \nu_Y \circ f$,

$$\overline{f}^{-1}(\mathrm{c}\nu_Y(N)) \cap \nu_X(X) = \nu_X(M).$$

If $\overline{f}$ is a c-mapping, the set $\overline{f}^{-1}(\mathrm{c}\nu_Y(N))$ is a c-closed, hence

$$\mathrm{c}\nu_X(M) \cap \nu_X(X) = \nu_X(M),$$

and according to 3.1 (c) $\nu_X(M) = \nu_X(\mathrm{c}M)$. Thus $M = \mathrm{c}M$ and the result follows. □

3.4. Lemma. *Let P, Z be ordered sets, Z has a least element ω and let $\xi \in Z$, $a \in P$. Put*

$$f(p) = \begin{cases} \xi & \text{for } p \in P - (a] \\ \omega & \text{for } p \in (a]. \end{cases}$$

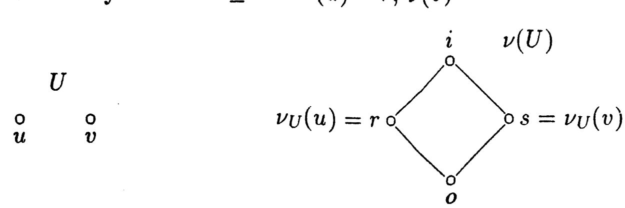
Then f is a c*-mapping from P into Z.*

PROOF: Each c-closed set N of Z contains the element ω. Therefore $f^{-1}(N) = (a]$ for $\xi \notin N$ and $f^{-1}(N) = P$ for $\xi \in N$ so that $f^{-1}(N)$ is c-closed. □

3.5. Theorem. *The $\mathcal{K}$ is a maximal reflectivity of R preserving subextension of the extension $\mathcal{L}/\mathcal{K}$.*

PROOF: I. Let $\mathcal{M}$ be an RP-subextension of $\mathcal{L}/\mathcal{K}$ different from $\mathcal{K}$. Sccording to 3.3 we can assume that there exist complete lattices X and Y and a mapping f from X to Y which is not a c-mapping. First, we show that f is isotonic.

Suppose a, $b \in X$, $a \leq b$ and $f(a)$ non $\leq f(b)$. Put $\alpha = f(a)$, $\beta = f(b)$, $\gamma = \sup_Y\{\alpha, \beta\}$, $\delta = \inf_Y\{\alpha, \beta\}$. Then $\alpha \neq \delta$. Let $U = \{u, v\}$ be a two-element antichain. Then $\nu(U) = \{o, r, s, i\}$, where $r = \{u\}$, $s = \{v\}$, $o = \emptyset$ and $i = U$, $\nu(U)$ is ordered by inclusion $\subseteq$ and $\nu(u) = r$, $\nu(v) = s$.



Define the following mappings :

$$g : U \to Y,\ g(u) = \alpha,\ g(v) = \beta,$$
$$\overline{g} : \nu(U) \to Y,\ \overline{g}(r) = \alpha,\ \overline{g}(s) = \beta,\ \overline{g}(i) = \gamma,\ \overline{g}(o) = \delta,$$
$$\phi : \nu(U) \to X,\ \phi(r) = \phi(o) = a,\ \phi(s) = \phi(i) = b,$$
$$h = f \circ \phi : \nu(U) \to Y.$$

Then g, $\overline{g}$, ϕ are c-mappings, hence g, $\overline{g}$, $\phi \in M(M)$ and then $h \in M(M)$.

Since $h(r) = \alpha$, $h(s) = \beta$ and $h(o) = \alpha \neq \delta = \overline{g}(o)$, we have $h \circ \nu_U = \overline{g} \circ \nu_U = g$, while $h \neq \overline{g}$.

The mapping f is isotonic. According to 3.1 (b) there exists $M \subseteq X$ such that $f(\sup_X M) \neq \sup_Y f(M)$. Since f is isotonic, we can suppose that M has the following property:

$$m \in M,\ x \in X,\ x \leq m \Rightarrow x \in M.$$

Put $\mu = \sup_X M$ and $\sigma = \sup_Y f(M)$. We denote the least element of Y by ω. Then $\omega \leq \sigma < f(\mu)$ and $\mu \notin M$.

II: Let $M = \emptyset$. Then μ is the least element of X. We denote the set of all negative integers ordered in a usual way by P. Then $\nu(P) = \nu_P(P) \cup \{\emptyset\}$ and $\emptyset$ is the least element of $\nu(P)$. Put

$$f_1(t) = \mu \text{ for } t \in \nu(P), \quad f_2(t) = \begin{cases} f(\mu) & \text{for } t \in \nu_P(P) \\ \omega & \text{for } t = \emptyset. \end{cases}$$

Clearly, $f_1 : \nu(P) \to X$ is a c-mapping and according to 3.4 $f_2 : \nu(P) \to Y$ as well. For $p \in P$ we get $f \circ f_1 \circ \nu_P(p) = f(\mu) = f_2 \circ \nu_P(p)$, but $f \circ f_1(\emptyset) = f(\mu) \neq \omega = f_2(\emptyset)$, which is a contradiction.

III. Let $M \neq \emptyset$. We denote the set $M \cup \{\mu\}$ by N. The sets M and N are ordered sets by order induced by that of X. There exists an isomorphism i from $\nu(M)$ on N such that $i\nu_M(m) = m$ for each $m \in M$.

Put

$$f_1(t) = \begin{cases} f(\mu) & \text{for } t \in Y - (\sigma] \\ \omega & \text{for } t \in (\sigma], \end{cases} \quad , \quad f_2(t) = \begin{cases} f(\mu) & \text{for } t \in Y - (f(\mu)] \\ \omega & \text{for } t \in (f(\mu)]. \end{cases}$$

According to 3.4 f_1, f_2 are c-mappings from Y into Y. Put $F = f_1 \circ f \circ j \circ i$, $G = f_2 \circ f \circ j \circ i$ where j is the identity embedding from N into X. Then, F, $G \in \mathrm{Hom}_M(\nu(M), Y)$, $F \circ \nu_M = G \circ \nu_M$, but $F(i^{-1}(\mu)) = f(\mu) \neq \omega = G(i^{-1}(\mu))$, which is a contradiction. The Theorem is proved. □

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(*Received* March 27, 1993)