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INTERPOLATIVE FUZZY ALGORITHMS FOR INTELLIGENT CONTROLLERS AND DECISION MAKING

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The interpolation principle for fuzzy algorithms is introduced. The fuzzy interpolative algorithm creation procedures for different rules of inference are suggested. The best results are achieved for the rule of inference according to a degree of matching fuzzy input and a rule.

The application of fuzzy sets theory methods in control is based on the implementation of linguistic rules. An algorithm describing systems behaviour consists of a set of such rules; each of them is a verbal description of a control strategy. Control rules most frequently used are usually in the form of fuzzy conditional rules:

$$R: \text{ IF } A, \quad \text{THEN } B = A \Rightarrow B, \tag{1}$$

where $A \Rightarrow B$ is an implication on the Cartesian product of two universes $U_A \times U_B$. The fuzzy algorithm is:

$$\begin{aligned} R^N: \ & \text{IF } A_1, \text{ THEN } B_1 \text{ ELSE} \\ & \text{IF } A_2, \text{ THEN } B_2 \text{ ELSE} \\ & \cdots\cdots\cdots\cdots\cdots\cdots \\ & \text{IF } A_N, \text{ THEN } B_N. \end{aligned} \tag{2}$$

The most widespread interpretation of (1) in fuzzy control is $R = A \times B$, where $\times$ is the Cartesian product of the two fuzzy sets A and B. The connective ELSE in the fuzzy algorithm (2) is treated as OR operation, so $R^N = \bigcup_{i=1}^N R_i = \bigcup_{i=1}^N (A_i \times B_i)$.

It is reasonable to demand a fuzzy algorithm designed to be satisfied with the following claim (interpolation principle): each fuzzy conditional statement describes any aspect of control, so what is true for one rule must be true for the whole algorithm if it includes the rule.

As a result of analysis of different interpretations of fuzzy implications in decision making for fuzzy systems control it can be shown that traditional approaches sometimes do not satisfy interpolation principle which for an algorithm $A = \{Y_i \Rightarrow Y_i\}$, $i = 1, \ldots, N$ and a rule of inference f is:

$$f(X_k, \{X_i \Rightarrow Y_i\}) = Y_k, \quad X_k \in \{X_i\}, \quad Y_k \in \{Y_i\}, \qquad i = 1, \ldots, N.$$

The reason for interpolation principle nonsatisfaction in traditional approaches is in an interdependence of fuzzy algorithm rules.

Note that the concept of fuzzy interpolation was introduced in [2], but in that form it is rather a fuzzy approximation.

The fuzzy interpolative algorithm generation procedures are based on the following theorem.

Theorem. Let there be a fuzzy rule $X_1 \Rightarrow Y_1$ and an antecedent X_2. A fuzzy algorithm $A = \{X_k \Rightarrow Y_k\}$, $k = 1,2$ is an interpolative one iff the fuzzy conclusion satisfies the following condition:

$$\check{Y}_2 \subseteq Y_2 \subseteq \hat{Y}_2,$$

where fuzzy sets $\check{Y}_2$ and $\hat{Y}_2$ are lower and upper bounds of fuzzy conclusion respectively.

Here for fuzzy sets $X_1 = \|{}^1x_n\|$; $X_2 = \|{}^2x_n\|$; $Y_1 = \|{}^1y_j\|$; $Y_2 = \|{}^2y_j\|$; $n = 1,\ldots,n_m$; $j = 1,\ldots,j_m$ in the case of

– max-min composition

$$\begin{aligned}\check{Y}_2 &= Y_1 \cap (X_1 \circ X_2);\\ {}^2\hat{y}_j &= \begin{cases} 1, & \bigvee_i ({}^1x_i \wedge {}^2x_i) \le {}^1y_j; \\ {}^1y_j, & \bigvee_i ({}^1x_i \wedge {}^2x_i) > {}^1y_j. \end{cases}\end{aligned}$$

– Δ-composition [1]

$$\begin{aligned}\check{Y}_2 &= X_2 \Delta (X_1 \times Y_1);\\ {}^2\hat{y}_j &= \begin{cases} 1, & \bigvee_{\substack{i:\,{}^1x_i \ne 1\\ {}^2x_i = 1}} {}^1x_i \le {}^1y_j; \quad \bigvee_{i:\,{}^1x_i = 1} {}^2x_i \le {}^1y_j; \\ 1^*, & \bigvee_{\substack{i:\,{}^1x_i \ne 1\\ {}^2x_i = 1}} {}^1x_i > {}^1y_j; \quad \bigvee_{i:\,{}^1x_i = 1} {}^2x_i \le {}^1y_j; \\ {}^1y_j, & \bigvee_{\substack{i:\,{}^1x_i \ne 1\\ {}^2x_i = 1}} {}^1x_i \le {}^1y_j; \quad \bigvee_{i:\,{}^1x_i = 1} {}^2x_i > {}^1y_j; \\ {}^1y_j, & \bigvee_{\substack{i:\,{}^1x_i \ne 1\\ {}^2x_i = 1}} {}^1x_i > {}^1y_j; \quad \bigvee_{i:\,{}^1x_i = 1} {}^2x_i > {}^1y_j, \end{cases}\end{aligned}$$

where 1^* is the nearest to 1 real number, $1^* < 1$.

– rule of inference according to a degree of matching fuzzy input and a rule (here a response Y for input X is determined like $Y = \bigcup_{i=1}^{N} [(X \cap X_i)\, \Delta\, (X_i \times Y_i)]$):

$$\begin{aligned}\check{Y}_2 &= (X_1 \Delta X_2)\, \Delta\, (X_1 \times Y_1);\\ {}^2\hat{y}_j &= \begin{cases} 1, & \bigvee_n ({}^1x_n \wedge {}^2x_n \wedge {}^2x_n) \le {}^1y_j \\ 1^*, & \bigvee_n ({}^1x_n \wedge {}^2x_n \wedge {}^2x_n) > {}^1y_j. \end{cases}\end{aligned}$$

If $\check{Y}^{\circ}$, $\check{Y}^{\wedge}$, $\check{Y}^{\vee}$, $\hat{Y}^{\circ}$, $\hat{Y}^{\wedge}$ and $\hat{Y}^{\vee}$ are lower and upper bounds determined for max-min and Δ-compositions and for the rule of inference according to a degree of matching fuzzy input and a rule, respectively, then

$$\check{Y}^{\vee} \subseteq \check{Y}^{\wedge} \subseteq \check{Y}^{\circ}$$
$$\hat{Y}^{\vee} \subseteq \hat{Y}^{\wedge} \subseteq \hat{Y}^{\circ}.$$

Therefore the most severe restrictions are imposed in the case of max-min composition, and the most loyal ones are in the case of the rule of inference according to a degree of matching fuzzy input and a rule. So the last rule has the best abilities to express decision-maker's opinion.

A procedure for creating an interpolative algorithm which does not depend upon implemented rule of inference is the following.

1. Let there be a fuzzy interpolative algorithm $A_{n-1} = \{X_i \Rightarrow Y_i\}$, $i = 1, \dots, n-1$ and an antecedent of a new rule X_n.

2. A lower bound $\check{Y}_n$ of a conclusion Y_n of a new rule $X_n \Rightarrow Y_n$ for all rules of the algorithm is determined as the union of all lower bounds $\check{Y}_{ni}$, $i = 1, \dots, n-1$, achieved for each rule separately:

$$\check{Y}_n = \bigcup_{i=1}^{n-1} \check{Y}_{ni}.$$

3. An upper bound $\hat{Y}_n$ of a conclusion Y_n of a new rule $X_n \Rightarrow Y_n$ for all rules of the algorithm is determined as the intersection of all upper bounds $\hat{Y}_{ni}$, $i = 1, \dots, n-1$, achieved for each rule separately:

$$\hat{Y}_n = \bigcup_{i=1}^{n-1} \hat{Y}_{ni}.$$

4. A conclusion satisfying the restriction $\check{Y}_n \subseteq Y_n \subseteq \hat{Y}_n$ and relevant to a decision-maker's domain is suggested.

The restrictions imposed by this procedure are not very severe and they can be considered as a guide to achieve the desired result rather than something limiting decision-maker's initiative.

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