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THE SELECTION OF INPUT AND OUTPUT SCHEMES FOR A SYSTEM AND THE MODEL PROJECTION PROBLEMS

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A number of important control theory problems are involved in the selection of input-output schemes of a given system and one family of such problems is referred to as Model Projection Problems (MPP); these problems deal with the selection of effective sets of inputs, outputs out of larger potential sets of inputs, outputs respectively. The aim of this paper is to classify the different types of MPPs and discuss their relevance in the context of integrated system design. The dominant idea running through the present treatment of MPPs is that the suggested solutions aim at producing final models with inherently "good" control structure characteristics. Central to the present approach are problems of transformation of structural invariants. The overall objective of this paper is to demonstrate the importance of Control Theory tools in Early Process Design stages, which are not traditionally associated with control problems.

1. INTRODUCTION: PROBLEM MOTIVATION

The instrumentation of a process, that is the selection of measurement variables (outputs) and actuation variables (inputs) has a "micro" (local), as well as a "macro" (global) aspect. The "micro" role of instrumentation has been well developed and deals with the problem of measuring physical variables, and the implementation of action upon given physical variables; instrumentation theory and practice deals almost exclusively with the latter problems. The "macro" aspects of instrumentation stem from that designing an instrumentation scheme for a given process (classification of internal variables and selection of inputs and outputs) expresses the attempt of the "observer" (designer) to build bridges with the "internal mechanism" of the process, in order to observe it and/or act upon it. What is considered as the final system, on which control system design is to be performed, is the object obtained by the interaction of the "internal mechanism" and the specified overall instrumentation scheme. The internal mechanism, or model is associated with a given internal system interconnection and depends on the interconnection graph (topology) and the models for the subsystems; according to the accuracy and complexity of the subsystem models, we derive "progenitor models" of varying complexity and accuracy. The selection of systems of inputs and outputs is the final design decision that shapes the structural characteristics of the system model used for control design.

The selection of systems of inputs and outputs, as well as the shaping of overall system by appropriate interconnection of subsystems have been important issues in different areas of engineering design such as Electrical, Chemical, Aerospace Engineering etc. The system synthesis has been addressed as a topic in Network theory [25] and within the area of Process Synthesis of Chemical Engineering [2], [21], but it has not been addressed in generic terms, above the specific application areas; work in the area of large scale systems [24], [20] has been focused on analysis of composite system properties rather than synthesis methodologies. The problem of selection of systems of inputs, outputs has been considered within the area of Chemical Process Control [17], [3], [19], but it is still dominated by application area dependent heuristics and lacks any general theory; special issues in this area have been considered in [23], [17], but there has been no attempt to consider the overall problem of system model shaping as the result of input, output variable selection. This paper aims at providing a classification of the various problems arising in process synthesis and selection of input-output schemes and suggest a control theory based framework for their study.

The dominant idea running through this paper is that both process synthesis and selection of input, output schemes have a significant effect on the formation of the internal structural characteristics. Difficulties in control of the final system may be expressed in terms of certain structural characteristics of the final system model such as right half plane zeros, delays, high order infinite zeros, uncontrollability, unobservability etc. [22], [4], [5]. These structural characteristics are formed in an evolutionary manner as we go through the various stages of design of a process, which lead first to a progenitor model (result of process synthesis) and after selection of input, output schemes to a final system model [8], [6]. The aim of this paper is to examine a number of problems associated with the effect of selection of input, output schemes of a process on the resulting system model and demonstrate the significance of control theory based concepts, tools and criteria for directing the system structure evolutionary formation, along paths with inherently "good" control characteristics. This work is part of the overall effort to see instrumentation in a wider, global sense, with a significant role in the design of processes.

The general types of problems related to the selection of input, output schemes for a process may be classified as [8]: (i) Model Orientation Problems (MOP), (ii) Model Expansion Problems (MEP) and (iii) Model Projection Problems (MPP). The first deals with the classification of internal variables into inputs and outputs [26] and it is a problem where issues from implicit systems theory [16] are of relevance. The second family of problems deals with the selection of additional measurements for reconstruction of unmeasurable internal variables [18], as well as selection of additional inputs, outputs which enhance the existing model. The problems considered here are those referred to as Model Projection type and they are defined on systems, where Model Orientation issues have been previously resolved; it is assumed that for the given process there is a large number of potential inputs and a large dimension input-output model. What it is referred as a Model Projection Problem, is the selection of a subset of process inputs and outputs, which will result in a smaller dimension model which will then be used for Control System Design. This problem frequently arises in the design of large processes and it is due to the cost of the large number of measurements and actuation variables, as well as the fact that it may be difficult to use for further analysis such a large dimension input output model. Defining a smaller set of effective inputs, outputs which is "adequate" for the further control design needs, is an important issue which is addressed here. We shall distinguish the family of MPPs into the classes of external and internal types of problems and for each family we shall discuss some representative problems and issues. In particular, it is shown that the nature of MPPs is closely related to problems of transformation of structural invariants and in the case of constant MPPs these structure transformation issues are formulated in a pencil setting. Of course any MPP defined on a given process is governed by practical rules and heuristics associated with the nature of the given problem; however, here we shall examine the theoretical issues and thus the selection of input, output subsets will be assumed to be unconstrained. The aim here is to define issues and problems; a survey of relevant results and a more detailed treatment may be found in [7] and the relevant references.

The paper is structured as follows: In Section 2, the different types of the Process Progenitor Models are considered and the Model Projection Problems are introduced. The basic properties of Internal Progenitor Models, as far as structural characteristics, are considered in Section 3, where the implications of the "completeness" assumption in the formation of interconnections are also discussed. Section 4 deals with the Model Projection Problems on External Progenitor Models, where it is shown that such problems are equivalent to Kronecker structure transformation Problems on matrix pencils. The paper provides a framework for the discussion of many new problems arising in structuring composite systems and introduces some important new concepts and results.

2. PROCESS PROGENITOR MODELS AND THE FAMILY OF THE MODEL PROJECTION PROBLEMS

Most of the time, a process is synthesised by connecting subprocesses (subsystems) and the two fundamental ingredients of the composite system model are:

(i) The topology (graph) of system interconnections $\mathcal{F}$.

(ii) The family $\mathcal{T}$ of subsystem models.

It is assumed that each subsystem Σ_k is represented by a proper rational transfer function matrix $G_k(s) \in R^{q_k \times p_k}(s)$, that is the subsystems S_k are regular state space and are both controllable and observable or more generally stabilisable and detectable [27], [15]. Furthermore, in forming composite structures we assume that there are no loading effects, that is each subsystem transfer function remains unchanged after the connections. In forming the composite system structure we make the following assumptions [1]:

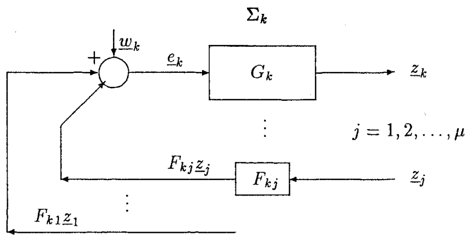

Fig. 1. Interconnection structure for subsystem Σ_k.

The completeness Interconnection Assumptions. For each subsystem Σ_k ($G_k \in R_{pr}^{q_k \times p_k}(s)$), $k = 1, \ldots, \mu$ we have the interconnection structure shown in Figure 1: to *each subsystem* Σ_k with *input* $\underline{e}_k$ and *output* $\underline{z}_k$ we associate a *summing node* with the following characteristics:

(i) its output is the subsystem input $\underline{e}_k$

(ii) its inputs are:

- (a) an *exogenous input* $\underline{w}_k$ (always assignable, or disturbance signal)
- (b) Other inputs, which are feedbacks of the form $F_{kj}\underline{z}_j, j = 1, 2, \ldots, \mu$ to the k-th summing node, where F_{kj} is assumed to be a real matrix (some of them may be zero)
- (c) $\underline{w}_k$ has as many independent coordinates as those needed to define a basis for $col.sp\{[F_{k1}; \cdots ; F_{k\mu}]\}$ and has subsystem outputs $\underline{y}_k = \underline{z}_k$, where $\underline{z}_k$ contains all subsystems Σ_k variables which feed to the other subsystems.

An interconnected system satisfying the above assumptions will be called a *complete* composite system and shall be denoted by $\Sigma_c(\mathcal{T}, \mathcal{F})$. In the present study we restrict ourselves to the case where all subsystems are represented by proper transfer functions; however the above definition of completeness is also valid in the case where we have non-proper transfer functions, or singular subsystems. The implications of the above assumptions are that

$$\hat{\underline{e}}_k = \hat{\underline{w}}_k + \sum_{k=1}^{\mu} F_{kj}\hat{\underline{z}}_j, \qquad \hat{\underline{z}}_k = G_k(s)\hat{\underline{e}}_k, \tag{1}$$

where $\hat{\underline{e}}_k, \hat{\underline{w}}_k, \hat{\underline{z}}_k$ denote the Laplace transforms of the corresponding vector signals. If n_i denotes the McMillan degree of $G_i(s)$, then by aggregation we may define the global quantities

$$q = \sum_{k=1}^{\mu} q_k, \; p = \sum_{k=1}^{\mu} p_k, \; n = \sum_{k=1}^{\mu} n_k$$

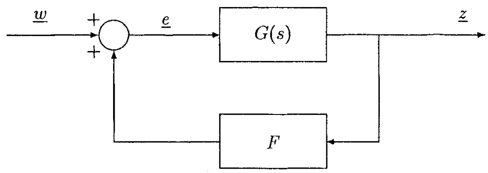

Fig. 2. Equivalent feedback configuration.

$$\begin{aligned}
&\underline{e} = [\underline{e}_1^t, \ldots, \underline{e}_\mu^t]^t, \ \underline{z} = [\underline{z}_1^t, \ldots, \underline{z}_\mu^t]^t, \\
&\underline{w} = [\underline{w}_1^t, \ldots, \underline{w}_\mu^t]^t, \ \underline{x} = [\underline{x}_1^t, \ldots, \underline{x}_\mu^t]^t, \\
&F \equiv [F_{kj}]_{k,j\in\tilde{\mu}} \in R^{p\times q}, \ G(s) \equiv \text{ bl.diag}\{G_k(s), k \in \tilde{\mu}\} \in R_{\text{pr}}^{q\times p}(s)
\end{aligned} \tag{2}$$

where $\underline{x}_k$ denotes the state vector of the $S_k(A_k, B_k, C_k, D_k)$ minimal realisation of $G_k(s)$. Using the aggregate expressions we may express (1).as

$$\hat{\underline{e}} = \hat{\underline{w}} + F\hat{\underline{z}}, \ \hat{\underline{z}} = G(s)\hat{\underline{e}} \tag{3}$$

which describes the composite system as a feedback system as shown in Figure 2.

The matrix F is a representation of the topology $\mathcal{F}$ of the interconnections and will be called the *interconnection matrix* of Σ_c; the aggregate system is denoted by Σ_c and it is represented by the aggregate transfer function $G(s)$. With the composite system Σ_c we define the following two transfer functions

$$H_{ew}(s) : \underline{w} \to \underline{e}, \ H_{ew}(s) = (I - FG(s))^{-1} \in R(s)^{p\times p} \tag{4}$$

$$H_{zw}(s) : \underline{w} \to \underline{z}, \ H_{zw}(s) = G(s)(I - FG(s))^{-1} \in R(s)^{q\times p} \tag{5}$$

The composite system will be called *well formed*, if all transfer functions are well defined and will be called *well- posed*, if all closed-loop transfer functions are well defined and proper, [15], [1] etc. The complete, well posed composite system $\Sigma_c(G(s); F)$ which is described as in Figure (2.2), is a mixed internal- external composite representation. If all potential inputs and outputs have been used at the subsystem level, then the feedback configuration $\Sigma_c(G(s); F)$ of Figure (2.2) will be called an *internal progenitor model* (PM) of the composite system. The transfer functions $H_{uw}(s)$, or $H_{yw}(s)$ associated with such a system and where F is not explicitly stated (as in the factorisations (4), (5)) will be called an *external progenitor model* (EPM) of the composite system. Internal, or external progenitor models represent the degree of modelling (assumed complexity and accuracy) of the composite system as a function of the modelling activity at the subsystem level. In an ideal design, unconstrained by resources and effort all possible inputs and outputs should be used; economic and technical reasons, however, force us frequently to select a subset of the potential inputs, outputs as effective, operational inputs, outputs. Developing criteria and techniques for selection of an effective input, output scheme as projections of the potential input, output vectors of IPM, or EPM is what we define *Model Projection Problems* of their ideal, unconstrained forms are formally defined below.

Definition 2.1. Let $\Sigma_c(G(s);F)$, be an IPM, where $G(s), F$ are defined as in (2), (3), $\underline{z} \in R^q, \underline{w}, \underline{e} \in R^p$ and let $H(s)$ be the $\underline{w} \to \underline{z}$ transfer function, or EPM. We may define:

(i) An *External Model Projection Problem* (EMPP) is equivalent to selecting the sensor, actuator maps $K(s) \in R^{m\times q}(s)$, $L(s) \in R^{p\times l}(s)$ $(m \le q,\, l \le p)$ such that the *External Effective Model* (EEM), with $\underline{y} \in R^m, \underline{u} \in R^l$ effective input, outputs respectively, is described by $\hat{\underline{y}} = K(s)\,\hat{\underline{z}}$, $\hat{\underline{e}} = L(s)\hat{\underline{u}}$, $\hat{\underline{z}} = H(s)\,\hat{\underline{e}}$ or

$$\hat{\underline{y}} = \Theta(s)\,\hat{\underline{u}}, \quad \Theta(s) = K(s)\,H(s)\,L(s) \tag{6}$$

has a transfer function $\Theta(s)$ with desirable properties.

(ii) An *Internal Model Projection Problem* (IMPP) is defined as an EMPP with the additional conditions that the effective inputs, outputs are defined as aggregates of the effective inputs, outputs at the subsystem level i.e. $\hat{\underline{y}}_i = K_i(s)\,\hat{\underline{z}}_i$, $\hat{\underline{e}}_i = L_i(s)\,\hat{\underline{u}}_i$, $\underline{y}_i \in R^{m_i}, \underline{u}_i \in R^{l_i}$ where $K_i(s) \in R^{m_i\times q_i}(s)$, $L_i(s) \in R^{p_i\times l_i}(s)$ and thus

$$\hat{\underline{y}} = \Theta(s)\,\hat{\underline{u}}, \qquad \Theta(s) = \text{bl.diag}\{K_i(s)\}\,H(s)\,\text{bl.diag}\{L_i(s)\} \tag{7}$$

In practice, the matrices $K(s)$, $L(s)$ are not completely free, their structure is constrained and their dynamics express those of the actuators, sensors used. In this paper we shall assume both $K(s)$, $L(s)$ to be constant and unconstrained (apart from the constrained imposed by IMPP); these problems are referred to as *Constant*-EMPP, (CEMPP), -IMPP (CIMPP). Note that MPPs are generalised and constrained Model Matching Problems, since a pair $(K(s),\, L(s))$ is sought, of constrained structure and dynamics, to produce a desirable transfer function $\Theta(s)$. The solvability of such problems is closely related to what is the desirable $\Theta(s)$, and determining the range of properties of $\Theta(s)$ achieved under various $(K(s),\, L(s))$ pairs.

3. INTERNAL PROGENITOR MODELS, THEIR PROPERTIES AND THE STRUCTURAL IMPPs

In this section we consider the properties of Internal Progenitor Models, in relation to those of the aggregate system and examine in particular the effect to total loss of subsystem inputs, or outputs on the resulting system properties. This study aims at qualifying the effect of deviating from the completeness assumption and examines this under structural changes represented by the loss of subsystem inputs and/or outputs. By assuming that $S_k(A_k, B_k, C_k, D_k)$ is the state space model of the k-th subsystem then we may define the aggregate system state equations by $S_a(\bar{A}, \bar{B}, \bar{C}, \bar{D}) : \dot{\underline{x}} = \bar{A}\underline{x} + \bar{B}\underline{e}$, $\underline{z} = \bar{C}\underline{x} + \bar{D}\underline{e}$ where, $\bar{A} \equiv \text{diag}\{A_i, i \in \tilde{\mu}\}$, $\bar{B} \equiv \text{bl.diag}\{B_i, i \in \tilde{\mu}\}$, $\bar{C} \equiv \text{bl.diag}\{C_i, i \in \tilde{\mu}\}$, $\bar{D} \equiv \text{bl.diag}\{D_i, i \in \tilde{\mu}\}$. Given that the interconnection equations are described by $\underline{e} = \underline{w} + F\underline{z}$ and assuming that the system is well posed, i.e. $|\Delta| \neq 0$, where $\Delta = (I - F\bar{D})$ then the composite system state equations are given by

$$S_c(A, B, C, D) : \dot{\underline{x}} = A\underline{x} + B\underline{w},\ \underline{y} = C\underline{x} + D\underline{w} \tag{8}$$

$$A = \bar{A} + \bar{B}\Delta F\bar{C},\ B = \bar{B}\Delta,\ C = (I + \bar{D}\Delta F)\bar{C},\ D = \bar{D}\Delta. \tag{9}$$

Given that the aggregate and the composite system are output feedback equivalent we have the following result.

Theorem 3.1. The aggregate system $S_a(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ and the complete composite system $S_c(A, B, C, D)$ have the following relationship between their structural characteristics:

(i) They have the same controllability (observability) properties, as these are defined by the set of controllability (observability) indices and input (output) decoupling zeros.

(ii) They have the same zero structure properties as these are defined by the finite zeros, orders of infinite zeros and output nulling column, row minimal indices.

Remark 3.1. Given that for the aggregate system the above types of Kronecker invariants are expressed as a union of the corresponding sets defined on the subsystems, the structural characteristics of the complete composite system are independent of the underlined interconnection graph, as this is expressed by the interconnection matrix F.

Thus the "completeness" of the interconnection scheme is essential in the transference of the subsystem structural characteristics to the composite system structure. In the following, it will be shown that breaking of the completeness assumption, by loss of inputs, outputs at the subsystem level, makes the nature of the underlined graph essential, as far as determining the resulting properties of the composite system.

Remark 3.2. Although the minimal indices and decoupling zeros of the aggregate and the complete composite system are the same, the more refined controllability, observability properties as those depending on the values of singular values of the corresponding controllability, observability Grammians are different. Furthermore, the nature of the interconnection matrix F is crucial, as far as determining the stability properties of the composite, complete system in terms of those of the aggregate system.

An interconnecting structure, represented by F, that results in a stable complete composite system will be referred to as *stabilising interconnection structure.* We may thus examine the deviations from completeness as loss of inputs, outputs. To examine the effect of loss of inputs, outputs on the complete composite system we have to introduce some tools from the matrix pencil characterisation of system properties [10], [11] and we shall restrict ourselves to the study of effects on the zero structure.

Definition 3.1. Let $S(A, B, C)$ be a linear system, $A \in R^{n\times n}$, $B \in R^{n\times l}$, $C \in R^{m\times n}$, $\text{rank}(B) = l$, $\text{rank}(C) = m$, $N \in R^{(n-l)\times n}$ be a basis matrix for $\mathcal{N}_l(B)$, (left annihilator of B) and $M \in R^{n\times(n-m)}$ be a basis matrix for $\mathcal{N}_r(C)$ (right annihilator of C). We may define the following restricted system pencils:

(i) The input-state restriction pencil $R(s) = sN - NA \in R^{(n-l)\times n}[s]$.
(ii) The state-output restriction pencil $T(s) = sM - AM \in R^{n\times(n-m)}[s]$.
(iii) The zero pencil $Z(s) = sNM - NAM \in R^{(n-l)\times(n-m)}[s]$.

Remark 3.3. [10], [11] The pencil $R(s)$, $T(s)$ completely characterise the controllability, observability feedback invariant properties whereas $Z(s)$ defines completely the zero structure of the system $S(A, B, C)$.

As far as the zero structure of the systems resulting by loss of inputs, outputs we have the following result that may be readily established.

Theorem 3.2. Let $\Sigma_c(S_i \in \tilde{\mu}; F)$, be any complete composite system, where all subsystems are strictly proper, i.e. $S_i = S_i(A_i, B_i, C_i)$ and let $R_i(s) = sN_i - N_iA_i$, $T_i(s) = sM_i - A_iM_i$, $Z_i = sN_iM_i - N_iA_iM_i$, $Q_i(s) = sI - A_i$, $i = 1, 2, \ldots, \mu$ be the pencils associated with the ith subsystem. Assuming that total loss of inputs, and/or outputs may take at the subsystem level, then for the resulting composite system $\tilde{S}(\tilde{A}, \tilde{B}, \tilde{C})$ the zero pencil $\tilde{Z}(s)$ may be expressed as

$$\tilde{Z}(s) = \text{bl.diag}\{X_1(s); \ldots; X_i(s); \ldots; X_\mu(s)\} \tag{10}$$

where each $X_i(s)$ block is associated with the ith subsystem and it is of the type:

(i) If all inputs and outputs of the ith subsystem are present, then $X_i(s) = Z_i(s)$.
(ii) If all inputs are lost at the ith subsystem, then $X_i(s) = T_i(s)$.
(iii) If all outputs are lost at the ith subsystem, then $X_i(s) = R_i(s)$.
(iv) If all inputs and all outputs are lost at the ith subsystem, then $X_i(s) = Q_i(s)$.

The cases considered here are extreme cases of the general IMPP. Graph analysis may provide some useful tools for approaching the general IMPP and in particular the study of deviations from completeness on the controllability, observability properties.

4. MODEL PROJECTION PROBLEMS ON EXTERNAL PROGENITOR MODELS AND THE MATRIX PENCIL TRANSFORMATION PROBLEM

In this section we consider problems of the model projection type, which are defined on External Progenitor Models represented by a proper transfer function matrix $H(s) \in R_{pr}^{q\times p}(s)$, or by a minimal state space realisation $S(A, B, C, D)$. It should be noted that the EPM assumption implies that we make no special assumption about an internal natural graph for the system. All problems considered here on EPMs may also be stated for IPMs with the only difference that in the latter case the projection matrices have to be block diagonal. The constant EMPP (C-EMPP), defined on the system Σ with $H(s)$, or $S(A, B, C, D)$ model may be stated as follows: Find $K \in R^{m\times q}$, $L \in R^{p\times l}$, $m \leq q$, $l \leq p$, $\text{rank}(K) = m$, $\text{rank}(L) = l$ such that

$$\Theta(s) = KH(s)L \in R^{m\times l}_{pr}(s), \tag{11}$$

where $\Theta(s)$ is some "desirable" model to be specified, or equivalently in state space terms

$$\tilde{S}_\Theta(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) : \tilde{A} = A,\ \tilde{B} = BL,\ \tilde{C} = KC,\ \tilde{D} = KDL \tag{12}$$

where $\tilde{S}_\Theta$ is a realisation of $\Theta(s)$, or a desirable model. The system Σ_Θ obtained from Σ under the (K, L) projecting pair will be referred to as an *input-output projected system* and the whole family of such systems that corresponds to all (K, L) possible pairs will be denoted by $\{\Sigma\}$. The EMPP will be called *full*, if $m < q$, $l < p$ and will be called *left-, right-partial* if $m = q$, $l = p$ respectively; if both $m = q$ and $l = p$, then it is called *trivial*. Some typical issues and problems within the overall framework of C-EMPP are:

Matrix Pencil Transformation Problem. (MPTP) Transform the C-EMPP to an equivalent problem of the matrix pencil setup.

Least Dimension Problems. (LDP) [7] Define the lowest bounds for the number of effective inputs and outputs, which are necessary for certain control property to hold true in the projected model.

Zero Assignment Problems. (ZAP) [9] Define the conditions on EPM such that the projected model has a given zero structure.

The first of the three problems is examined next, whereas the others are considered in the references. For the case of strictly proper systems, the full C-EMPP (and thus also the partial) may be studied as an equivalent matrix pencil theory problem. In fact, let us assume that $S(A, B, C)$ is the progenitor model, $\text{rank}(B) = p$, $\text{rank}(C) = q$ and let $(B^\dagger, N)$, $(C^\dagger, M)$ be pairs of left inverse, left annihilator for B, right inverse, right annihilator for C respectively ($B^\dagger B = I_p$, $NB = 0$, $CC^\dagger = I_q$, $CM = 0$). We first note:

Lemma 4.1. Let $K \in R^{m\times q}$, $L \in R^{p\times l}$, $\text{rank}(K) = m < q$, $\text{rank}(L) = l < p$ and let Q, R be such that

$$KR = K[K^\dagger, K^\perp] = [I_m, 0],\ R \in R^{q\times q},\ |R| \neq 0 \tag{13}$$

$$QL = \begin{bmatrix} L^\dagger \\ L^\perp \end{bmatrix} L = \begin{bmatrix} I_l \\ 0 \end{bmatrix},\ Q \in R^{p\times p},\ |Q| \neq 0. \tag{14}$$

For any $\tilde{C} = KC$, $\tilde{B} = BL$ pair, there exist matrices $\tilde{Q}$, $\tilde{R} \in R^{n\times n}$, $|\tilde{Q}| \neq 0$, $|\tilde{R}| \neq 0$, such that

$$\tilde{C}\tilde{R} = \tilde{C}[C^\dagger K^\dagger, C^\dagger K^\perp, M] = [I_m, 0_{q-m}, 0_{n-q}],\ R \in R^{q\times q},\ |R| \neq 0 \tag{15}$$

$$\tilde{Q}\tilde{B} = \begin{bmatrix} L^\dagger B^\dagger \\ L^\perp B^\dagger \\ N \end{bmatrix} \tilde{B} = \begin{bmatrix} I_l \\ 0_{p-l} \\ 0_{n-p} \end{bmatrix} \tag{16}$$

and for any pair $\tilde{C}, \tilde{B}$ defined as above, the $(\tilde{C}^\dagger, \tilde{M})$, $(\tilde{N}, \tilde{B}^\dagger)$ pairs are:

$$\tilde{B}^\dagger = L^\dagger B^\dagger,\ \tilde{C}^\dagger = C^\dagger K^\dagger,\ \tilde{M} = [M, C^\dagger K^\perp],\ \tilde{N} = \begin{bmatrix} N \\ L^\perp B^\dagger \end{bmatrix}. \tag{17}$$

Using the above we may describe the essential pencils of the input-output projected system $S_\theta(\tilde{A}, \tilde{B}, \tilde{C})$ as shown below:

Proposition 4.1. Let $S(A, B, C)$ be a progenitor model, (K, L), a projecting pair and let $\tilde{S}(\tilde{A}, \tilde{B}, \tilde{C})$ be the resulting input-output projecting system. For the $\tilde{S}(\tilde{A}, \tilde{B}, \tilde{C})$ the following properties hold true:

(i) The pencils $R(s) = sN - NA$, $\tilde{R}(s) = s\tilde{N} - \tilde{N}\tilde{A}$ of $S(A, B)$, $\tilde{S}(\tilde{A}, \tilde{B})$ are related as:

$$\tilde{R}(s) = \begin{bmatrix} R(s) \\ sL^\perp B^\dagger - L^\perp B^\dagger A \end{bmatrix}. \tag{18}$$

(ii) The pencils $T(s) = sM - AM$, $\tilde{T}(s) = s\tilde{M} - \tilde{A}\tilde{M}$ of $S(A, C)$, $\tilde{S}(\tilde{A}, \tilde{C})$ are related as:

$$\tilde{T}(s) = \left[T(s); sC^\dagger K^\perp - AC^\dagger K^\perp\right]. \tag{19}$$

(iii) The pencils $Z(s) = sNM - NAM$, $\tilde{Z}(s) = s\tilde{N}\tilde{M} - \tilde{N}\tilde{A}\tilde{M}$ of $S(A, B, C)$, $\tilde{S}(\tilde{A}, \tilde{B}, \tilde{C})$ are related as:

$$\tilde{Z}(s) = \begin{bmatrix} Z(s) & sNC^\dagger K^\perp - NAC^\dagger K^\perp \\ sL^\perp B^\dagger M - L^\perp B^\dagger AM & sL^\perp B^\dagger C^\dagger K^\perp \end{bmatrix}. \tag{20}$$

Given that $\tilde{R}(s)$, $\tilde{T}(s)$, $\tilde{Z}(s)$ define the controllability, observability, zero properties of $\tilde{S}(\tilde{A}, \tilde{B}, \tilde{C})$; Proposition (4.1) implies that that the C-EMPP is equivalent to augmentation of existing pencils and thus it is a problem of transformation of Kronecker invariants defined below:

Definition 4.1. Let $sF - G \in R^{m\times n}[s]$. Determining the Kronecker structure of the pencils

$$[sF - G;\ A(s)],\ \begin{bmatrix} sF - G \\ B(s) \end{bmatrix},\ \begin{bmatrix} sF - G & A(s) \\ B(s) & C(s) \end{bmatrix}, \tag{21}$$

where $A(s)$, $B(s)$ and $C(s)$ are given dimension but otherwise free pencils, as function of the Kronecker structure of $sF - G$, will be called a *Kronecker Structure Transformation Problem* (KSTP), by column, row augmentation. If the pencils $A(s)$, $B(s)$, $C(s)$ are not free, but come from certain families, then the corresponding KSTP are called *restricted-KSTP* (R-KSTP).

It should be noted that KSTP, also emerges in the study of generalised dynamic cover problems of the geometric theory [13], [14]; in fact C-EMPP is equivalent to a series of generalised dynamic cover problems, or equivalent KSTPs. The detailed study of KSTP is given in [14].

5. CONCLUSIONS

The paper has provided a formulation of a number of control problems which arise in the selection of effective sets of inputs and outputs out of potential sets of inputs and outputs. This general area of problems is part of the overall effort to integrate the Early Process Design Stages with the late, Control Design stage using tools from Control Theory and Design [17, 3, 19]. In particular, these problems refer to the general area of investigating issues related to the Global Instrumentation of a process. The common theme running through all of this work is that of transformation of invariants either of the state space framework or the transfer function. In fact different types of structural invariants characterise alternative system properties and thus the essence of the present approach is to view the selection of effective sets of inputs and outputs as a process characterising the evolution of system structure and properties, starting from some form of progenitor model and structure. The matrix pencil transformation problem is studied in detail in [14]. Although it has been assumed that model orientation issues [26] have been sorted out before we started the present types of investigation, this problem is by no means trivial and is currently under investigation. The present paper has provided a brief account of some of the issues and a more detailed account is given in [7].

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