Milan Sekanina

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GRAPHS AND BETWEENESS*

MILAN SEKANINA

1. In this paper, a graph (G, ϱ) is always connected, undirected, without loops and multiple edges, $G \neq \emptyset$. Thus, if a, b are vertices of (G, ϱ), $\{a, b\} \in \varrho$ exactly when a, b are connected by an edge. We often write G instead of (G, ϱ). If (G, ϱ) is a graph and $M \subset G$, then $(M, \varrho/M)$ means a full subgraph of (G, ϱ), i.e. $a, b \in M$, $\{a, b\} \in \varrho/M$ just if $\{a, b\} \in \varrho$. Often ϱ is used instead of ϱ/M. Let (M, ϱ) be a full subgraph of (G, ϱ). Then $\mathscr{K}(M)$ means the decomposition of (M, ϱ) into connected components. μ means the usual metric in (G, ϱ), i.e. $\mu(a, b)$ is the number of edges in a shortest path connecting the vertices a and b. $\mathscr{E}(G, \varrho)$ is the system of all 2-components of the graph (G, ϱ). Here a 2-component is a maximal full subgraph of (G, ϱ) containing for any two distinct vertices a, b belonging to it at least one circle in which a and b are lying. By [6], § 15 one easily sees that the following assertion is true.

Proposition. *If* $X, Y \in \mathscr{E}(G, \varrho)$, $X \neq Y$, *then* $\operatorname{card}(X \cap Y) \leqq 1$. *If* $X \in \mathscr{E}(G, \varrho)$ *and* $Y \in \mathscr{K}(G - X)$, *there is exactly one* $y \in X$ *such that* $\mu(y, Y) = 1$. *We shall call* y *the projection of* Y *in* X.

We shall say that a vertex b of (G, ϱ) lies between vertices a and c when b belongs to any path connecting a with c. We write $[a, b, c]$ in this case.

1.1. a) For $x, y \in G$ we have $[x, x, x]$, $[x, x, y]$, $[x, y, y]$.

b) For $x, y, z \in G$ we have

$$[x, z, y] \Rightarrow [y, z, x].$$

1.2. Let (M, ϱ_1) be a connected subgraph of a graph (G, ϱ), $a, b, c \in M$, $[a, b, c]$ in (G, ϱ). Then $[a, b, c]$ in (M, ϱ_1).

1.3. Let (G, ϱ), (G_1, ϱ_1) be two graphs, $f: G \to G_1$ a map such that

$$[x, z, y] \text{ in } (G, \varrho) \Rightarrow [f(x), f(z), f(y)] \text{ in } (G_1, \varrho_1).$$

Then f is called a b-mapping.

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1.4. Let $f : (G, \varrho) \to (G_1, \varrho_1)$, $g : (G_1, \varrho_1) \to (G_2, \varrho_2)$ be b-mappings. Then gf is a b-mapping, too.

1.5. Graphs (G, ϱ) together with the class of all b-mappings form a category. This category will be denoted by $\mathscr{G}$.

1.5 is evident by 1.4 and the fact that identity mappings are b-mappings.

1.6. Notions of the category theory will be used in the sense of reference [3]. Especially, for a category $\mathscr{C}$, $[a, b]_{\mathscr{C}}$ is the set of all morphisms from a to b.

The following assertion is clear.

1.7. In a 2-connected graph (G, ϱ) $[a, b, c]$ holds only in cases described by 1.1.a. Therefore $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} = G^G$ (the set of all mappings of G in G).

1.8. Let $X \in \mathscr{E}(G, \varrho)$, $Y \in \mathscr{K}(G - X)$, y the projection of Y in X. Let $f : G \to G$ be defined as follows:

$$f(x) = y \quad \text{for} \quad x \in Y,$$

$$f(x) = x \quad \text{otherwise.}$$

Then f is a b-mapping.

Proof. Let $[a, b, c]$ in (G, ϱ).

1. If $\{a, b, c\} \cap Y = \emptyset$, then clearly $[f(a), f(b), f(c)]$.

2. Let card $\{a, b, c\} \cap Y \geqq 1$. If $\{a, c\} \subset Y$, then by connectivity of Y we have $b \in Y$ and hence $[f(a), f(b), f(c)]$ for card $\{a, b, c\} \cap Y \geqq 2$ in general, therefore $[f(a), f(b), f(c)]$. For the connectivity of X we cannot have $a, c \in X$, $b \in Y$. Hence let $a \in Y$, $b, c \in X$ (the case $a, b \in X$, $c \in Y$ is dual). Then $f(a) = y$, $f(b) = b$, $f(c) = c$. Let $y = c_1, c_2, \ldots, c$, $y = d_1, d_2, \ldots d_s, a$ be some paths in (G, ϱ). Then $a, d_s, \ldots, d_2, y, c_2, \ldots, c$ is a path from a to c. As $d_s, \ldots, d_2 \in Y$, b must be an element of $\{y, c_2, \ldots, c\}$. Therefore $[f(a), f(b), f(c)]$.

1.9. Let $X \in \mathscr{E}(G, \varrho)$. Let $\mathscr{Y}$ be a system of some $Y_i \in \mathscr{K}(G - X)$. Let y_i be always the projection of Y_i in X. Define $f : G \to G$ so that

$$f(x) = y_i \quad \text{for} \quad x \in Y_i \quad \text{(for all } i\text{)},$$

$$f(x) = x \quad \text{otherwise.}$$

Then f is a b-mapping.

The proof follows from 1.8. by composing suitable b-mappings.

1.10. Let $a, b \in (G, \varrho)$. Then there exists such a b-mapping $f : G \to G$ for which $f(G) = \{a, b\}$, $f(a) = a$, $f(b) = b$.

Proof. If there exists a 2-component of (G, ϱ) containing both a and b, we get 1.10 immediately by 1.2, 1.7, 1.8.

Suppose $a \in X \in \mathscr{E}(G, \varrho)$, $b \notin X$, $b \in Y \in \mathscr{K}(G - X)$. For $x \in Y$ define $f(x) = b$; $f(x) = a$ otherwise. Let $[x, y, z]$ in (G, ϱ). Everything will be proved if

$f(x) = f(z) \Rightarrow f(x) = f(y)$. Suppose we have $f(x) = f(z) \neq f(y)$. We have to distinguish two cases.

1) $f(x) = f(z) = a$, $f(y) = b$. Then $x, z \in G - Y$, $y \in Y$, a contradiction with the connectivity of $G - Y$.

The second case

2) $f(x) = f(z) = b$, $f(y) = a$ is similar.

1.11. (Corollary). Let T be a two-vertex tree with vertices a_1, b_1. Let $a, b \in \in (G, \varrho)$, $a \neq b$. There exists such a b-mapping $f: G \to T$ for which

$$f(a) = a_1, \quad f(b) = b_1.$$

1.12. Let $a, b, a_1, b_1 \in (G, \varrho)$, $a \neq b$. Then there exists such a b-mapping $f: G \to G$ such that $f(G) = \{a_1, b_1\}$, $f(a) = a_1$, $f(b) = b_1$.

2. A tree is a graph (G, ϱ) without circles (i.e. (G, ϱ) has only one-element 2-components). Thus, one-vertex trees are considered, too.

2.1. Let (G, ϱ) be a tree, $a \neq b \neq c \neq a$ are vertices of G, $[a, b, c]$ in (G, ϱ). Then there exists a b-mapping $f: G \to G$ such that $f(G) = \{a, b, c\}$, $f(a) = a$, $f(b) = b$, $f(c) = c$.

Proof. Let $a = a_0, a_1, \ldots, a_i = b, \ldots, a_s = c$ be the path connecting a with c. If $x \in X \in \mathscr{K}(G - \{a_0, \ldots, a_s\})$ and $\mu(X, a_i) = 1$, we put $f(x) = a_i$. Let $f(a_i) = a_i$ for $i = 0, 1, \ldots, s$. Similarly as in 1.8. one sees that f is a b-mapping. Let $g: \{a_0, \ldots, a_s\} \to \{a_0, a_i, a_s\}$ be defined by $g(a_0) = g(a_1) = \ldots$ $g(a_{i-1}) = a_0$, $g(a_i) = a_i$, $g(a_{i+1}) = \ldots = g(a_s) = a_s$. g is a b-mapping of $(\{a_0, \ldots, a_s\}, \varrho)$ into itself and gf is a demanded mapping.

2.2. (Corollary). Let T' be a three-vertex tree with the vertices a_1, b_1, c_1 for which $[a_1, b_1, c_1]$ in T'. Let (G, ϱ) be a tree, $a, b, c \in G$, $a \neq b \neq c \neq a$ and $[a, b, c]$ in (G, ϱ). There exists a b-mapping f from (G, ϱ) to T' such that $f(a) = a_1$, $f(b) = b_1$, $f(c) = c_1$.

2.3. (Corollary). Let $[a, b, c]$, $[a', b', c']$ in a tree (G, ϱ), $a \neq b \neq c \neq a$, $a' \neq b' \neq c' \neq a'$. There exists such a b-mapping $f: G \to G$ for which $f(G) = \{a', b', c'\}$ and $f(a) = a'$, $f(b) = b'$, $f(c) = c'$.

In [4] the following two propositions have been proved.

2.4. Let (G, ϱ) be a tree, $a, b, c \in G$. Then there exists exactly one $d \in G$ such that $[a, d, b]$, $[b, d, c]$, $[a, d, c]$. We write $\omega(a, b, c) = d$ and the ternary operation ω is called the intersection operation.

2.5. Let (G, ϱ) be a tree. Then

1) $[a, b, c] \Leftrightarrow \omega(a, b, c) = b$.

2) $\omega(a, a, b) = \omega(a, b, a) = \omega(b, a, a) = a$.

3) ω is a symmetrical operation.

Similarly as in 2.1. the following can be proved.

2.6. Let (G, ϱ) be a tree. Let $a, b, c \in G$ and $\omega(a, b, c) \notin \{a, b, c\}$. There exists such a b-mapping $f: G \to G$ for which $f(G) = \{a, b, c, \omega(a, b, c)\}$, $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(\omega(a, b, c)) = \omega(a, b, c)$.

2.7. (Corollary). Let (G, ϱ) be a tree, $a, b, c, a_1, b_1, c_1 \in G$, $\omega(a, b, c) \notin \{a, b, c\}$, $\omega(a_1, b_1, c_1) \notin \{a_1, b_1, c_1\}$. There exists a b-mapping $f: G \to G$ such that $f(G) = \{a_1, b_1, c_1, \omega(a_1, b_1, c_1)\}$, $f(a) = a_1$, $f(b) = b_1$, $f(c) = c_1$, $f(\omega(a, b, c)) = \omega(a_1, b_1, c_1)$.

3. The notions from the theory of universal algebras are taken from [2]. The category of all algebras with one fundamental operation of a fixed arity (generally denoted by α) will be denoted by $\mathscr{A}_\alpha$. In considerations on morphisms of $\mathscr{A}_\alpha$ (and of $\mathscr{G}$, as well) we identify these mappings with the carrying mappings of the supports of relevant structures.

3.1. Let (G, ϱ), (G, ϱ_1) be trees, ω denote (for both of them) the intersection operation. Then

$$[(G, \omega), (G_1, \omega)]_{\mathscr{A}_\alpha} = [(G, \varrho), (G_1, \varrho_1)]_{\mathscr{G}}.$$

Proof. Let $f \in [(G, \omega), (G_1, \omega)]_{\mathscr{A}_\alpha}$ and $[a, b, c]$ in (G, ϱ). $\omega(a, b, c) = b$ therefore $f(b) = \omega(f(a), f(b), f(c))$, and so $[f(a), f(b), f(c)]$ in (G_1, ϱ_1).

Let $f \in [(G, \varrho), (G_1, \varrho_1)]_{\mathscr{G}}$ and $\omega(a, b, c) = d$ in (G, ω). We have $[a, d, b]$, $[b, d, c]$, $[a, d, c]$, hence $[f(a), f(d), f(b)]$, $[f(b), f(d), f(c)]$, $[f(a), f(d), f(c)]$ and therefore $\omega(f(a), f(b), f(c)) = f(d)$.

3.2. Let $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$. Then α is an idempotent operation.

This follows from the fact that $[(G, \varrho), (G, \varrho)]_{\mathscr{G}}$ contains all constant mappings.

3.3. Let $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$, where α is a binary operation. Then α is a projection.

Proof. Let $\{a, b\} \subset G$, $a \neq b$. By 1.10. $\{a, b\}$ is a subalgebra in (G, α). Let, e.g. $\alpha(a, b) = a$. Let $a_1, b_1 \in G$. By 1.12 $f(a) = a_1, f(b) = b_1$ for certain $f \in [(G, \varrho), (G, \varrho)]_{\mathscr{G}}$. Therefore $\alpha(a_1, b_1) = a_1$ and so α is a projection.

3.4. Let (G, ϱ) be a tree, α essentially ternary on G, and $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$. If card $G = 2$, then (G, α) is a Post algebra, if card $G > 2$, we have $\alpha = \omega$. In both cases,

$$[(G, \varrho), (G, \varrho)]_{\mathscr{G}} = [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}.$$

Proof. If card $G = 2$, $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} = G^G$ and this implies by [2], § 6 that (G, α) is a Post algebra. Let card $G > 2$, $a, b, c \in G$, $a \neq b \neq c \neq a$ $[a, b, c]$ in (G, ϱ). By 2.1. $\{a, b, c\}$ is a subalgebra in (G, α). Suppose $\alpha(a, b, c) = a$. In the sequel, we use 1.12 and 2.3 without any further reference. E.g., $\alpha(a, b, c) = a$ by 1.12 implies $\alpha(a, b, b) = a$, $\alpha(a, a, c) = a$. As $\alpha(x, y, y)$, $\alpha(x, x, y)$ are projections, we have $\alpha(x, y, y) = x$, $\alpha(x, x, y) = x$.

By 2.3. we have $\alpha(c\ b, a) = c$.

1. Suppose $\alpha(b, a, c) = a$, whence $\alpha(x, y, x) = y$.

1.1. If $\alpha(a, c, b) = a$, then $\alpha(x, y, x) = x$, a contradiction.

1.2. If $\alpha(a, c, b) = b$, then $\alpha(x, y, y) = y$, a contradiction.

1.3. If $\alpha(a, c, b) = c$, then $\alpha(x, y, y) = y$ again.

2. Suppose $\alpha(b, a, c) = c$. Then $\alpha(x, x, y) = y$, a contradiction. Therefore $\alpha(b, a, c) = b$, hence $\alpha(b, c, a) = b$ and $\alpha(x, y, x) = x$, therefore $\alpha(c, a, b)$ can be c or b, but $\alpha(x, y, y) = x$ implies $\alpha(c, a, b) = c$. Therefore, if $x_1, y_1, z_1 \in \{a, b, c\}$, we have $\alpha(x_1, y_1, z_1) = x_1$. By 2.3 this is simultaneously true for all triples a', b', c' with $[a', b', c']$.

Let now $a', b', c' \in G$, $\omega(a', b', c') \notin \{a', b', c'\}$. By 2.6 $\{a', b', c', \omega(a', b', c')\}$ is a subalgebra. Suppose $\alpha(a', b', c') = b'$. Then (e.g., by 1.8) $\alpha(a', \omega(a', b', c'), c') = \omega(a', b', c')$, which is a contradiction, because $[a', \omega(a', b', c'), c']$. In the same way it turns out that $\alpha(a', b', c') = \omega(a', b', c')$, $\alpha(a', b', c') = c'$ are contradictory, too. Therefore $\alpha(a', b', c') = a'$, and so $\alpha(x, y, z) = x$, which is a contradiction to the supposition on α.

Similarly for $\alpha(a, b, c) = c$ and for

$$\alpha(a, b, c) = b, \quad \alpha(b, a, c) = a.$$

Suppose $\alpha(a, b, c) = b$, $\alpha(b, a, c) = c$. $\alpha(a, b, c) = b$ implies $\alpha(x, x, y) = x$, $\alpha(b, a, c) = c$ implies $\alpha(x, x, y) = y$, a contradiction.

Therefore we must have $\alpha(a, b, c) = \alpha(b, a, c) = b$. Then $\alpha(x, x, y) = \alpha(y, x, x) = \alpha(x, y, x) = x$ and this implies $\alpha(a, c, b) = b$. Thus for $x_1, y_1, z_1 \in \{a, b, c\}$ we have $\alpha(x_1, y_1, z_1) = \omega(x_1, y_1, z_1)$ and this holds for all triples a', b', c' with $[a', b', c']$ and by upper arguments for all triples in general.

3.5. If (G, ϱ) is not a tree, α a ternary operation on G and $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$, then α is a projection.

Proof. Let $X \in \mathscr{E}(G, \varrho)$, card $X \geqslant 3$. Let $f : G \to X$ from 1.6 for $\mathscr{Y} = \mathscr{K}(G - X)$. By 1.2. and 1.7. $[(X, \varrho), (X, \varrho)]_{\mathscr{G}} = X^X$. Therefore X is a subalgebra in (G, α) and α restricted to X (notation α/X) is a projection.

Let, e.g. α/X be the projection to the first coordinate, i.e. for $a, b, c \in X$ we have $\alpha(a, b, c) = a$. Therefore $\alpha(x, x, y) = \alpha(x, y, x) = \alpha(x, y, y) = x$ on X. But as card $X \geqq 3$ and $\alpha(x, x, y)$, $\alpha(x, y, x)$, $\alpha(x, y, y)$ are projections on the whole G by 3.3., the mentioned equalities hold on G. Suppose we have $a, b, c \in G$ so that $a \neq \alpha(a, b, c)$. Let f be a b-mapping from 1.10. for which $\alpha(a, b, c)$ stands for b.

It is $\alpha(a, b, c) = f(\alpha(a, b, c)) = \alpha(f(a), f(b), f(c)) = \alpha(a, f(b), f(c))$. But card $\{a, f(b), f(c)\} \leqslant 2$ and therefore $\alpha(a, f(b), f(c)) = a$, a contradiction.

3.6. Let (G, ϱ) be a graph and α be an essentially n-ary operation on G for some $n \geqq 3$ such that $\alpha(x_1, \ldots, x_n) = x_1$, whenever card $\{x_1, \ldots, x_n\} < n$. Then there cannot hold

$$[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}.$$

Proof. Suppose the upper inclusion to be true. As α is essentially n-ary, α is not a projection and there exist $a_1, \ldots, a_n \in G$ such that $\alpha(a_1, \ldots, a_n) \neq a_1$. Let f be a b-mapping from 1.10, where $\alpha(a_1, \ldots, a_n)$ stands for b. We have $\alpha(f(a_1), \ldots, f(a_n)) = f(\alpha(a_1, \ldots, a_n)) = \alpha(a_1, \ldots, a_n)$. But card $\{f(a_1), \ldots, f(a_n)\} \leqq 2$, i.e. $\alpha(f(a_1), \ldots, f(a_n)) = f(a_1) = a_1$, a contradiction.

By 3.1.—3.6, taking in account Lemma 20 from [5] we get the final result of this section.

3.7. Let $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} \subset [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$, card $G \geqq 3$, where α is an operation on G. Then

1) if (G, ϱ) is a tree and α is essentially n-ary for certain $n \geqq 2$, ω is an algebraic operation in the algebra (G, α) and $[(G, \varrho), (G, \varrho)]_{\mathscr{G}} = [(G, \alpha), (G, \alpha)]_{\mathscr{A}_\alpha}$.

2) if (G, ϱ) is not a tree, then α is a projection.

4. Let $\mathscr{T}$ be the full subcategory of $\mathscr{G}$ consisting of all trees. Let F be the embedding of $\mathscr{T}$ in $\mathscr{A}_\alpha$ where α is ternary given by $F[(G, \varrho)] = (G, \omega)$ and $F(f) = f$ for mappings. Let now (G, ϱ) be an arbitrary graph from $\mathscr{G}$. Let $F'(G, \varrho)$ be the algebra from $\mathscr{A}_\alpha$ generated by the set $\mathbf{G}$ which is equal to G if (G, ϱ) is a tree and to $G \times \{(G, \varrho)\}$ in other cases (we write $\mathbf{x}$ instead of x or $\langle x, (G, \varrho)\rangle$ in the sequel) with the following defining relations

$$\alpha(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathbf{v} \quad \text{iff} \quad x, y, z, v \in G \quad \text{and}$$
$$[x, v, y],\ [y, v, z],\ [x, v, z] \quad \text{in} \quad (G, \varrho) \qquad (*)$$

4.1. If (G, ϱ) is a tree, $F'(G, \varrho) = (G, \omega)$. Clear.

4.2. For $w \in F'(G, \varrho)$ we have $\alpha(w, w, w) = w$ iff $w \in \mathbf{G}$ (the equality is meant as an equality of elements in $F'(G, \varrho)$ not an identity of terms).

Proof. (*) and 1.1 imply $\alpha(\mathbf{x}, \mathbf{x}, \mathbf{x}) = \mathbf{x}$ for $x \in G$. Let w be a term from $F'(G, \varrho)$ having the minimal length among all terms which are equivalent to w. If this length is not 1, then terms equivalent to $\alpha(w, w, w)$ are of the form $\alpha(w_1, w_2, w_3)$, where w_1, w_2, w_3 are terms equivalent to w, therefore of a length greater or equal to the length of w and so the length of $\alpha(w_1, w_2, w_3)$ is greater than the length of w. Therefore $\alpha(w, w, w) \neq w$.

4.3. Let $f \in [F'(G, \varrho), F'(G_1, \varrho_1)]_{\mathscr{A}_\alpha}$. Then $f(\mathbf{G}) \subset \mathbf{G}_1$.

The proof follows immediately by 4.2.

4.4. Let $f \in [F'(G, \varrho), F'(G_1, \varrho_1)]_{\mathscr{A}_\alpha}$. Define $f' : G \to G_1$ by

$$f'(x) = y = f(\mathbf{x}) = \mathbf{y}. \quad \text{Then} \quad f' \in [(G, \varrho), (G, \varrho_1)]_{\mathscr{G}}.$$

Proof. Let $[x, y, z]$ in (G, ϱ). Then $\alpha(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathbf{y}$ and so $\alpha(f(\mathbf{x}), f(\mathbf{y}), f(\mathbf{z})) = f(\mathbf{y})$ in $F'(G_1, \varrho_1)$. Hence $[f'(x), f'(y), f'(z)]$ in (G_1, ϱ_1).

4.5. Let $f_1 \in [(G, \varrho), (G_1, \varrho_1)]_{\mathscr{G}}$. Then there exists exactly one $f \in [F'(G, \varrho), F'(G_1, \varrho_1)]_{\mathscr{A}_\alpha}$ such that $f' = f_1$ (notation as in 4.4).

Proof. Let $f_1^*(\mathbf{x}) = \mathbf{y}$ iff $f_1(x) = y$. By (*) and by the definition of the b-mapping f^* preserves all defining relations. Therefore there exists exactly one f with the demanded properties.

f from 4.5. will be denoted by $F'(f_1)$. An immediate consequence of 4.4 and 4.5 is

4.6. The functor $F' : \mathscr{G} \to \mathscr{A}_\alpha$ is a full embedding, which extends $F : \mathscr{T} \to \to \mathscr{A}_\alpha$.

4.7. It can be easily proved that F' is the left Kan extension of F (see [1], chapter X).

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Katedra algebry a geometrie
Přírodovědecké fakulty UJEP
Janáčkovo nám. 2a
662 95 Brno