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INTEGRAL EQUIVALENCE BETWEEN A NONLINEAR SYSTEM AND ITS NONLINEAR PERTURBATION

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In the present paper we consider differential systems of the forms

$$x' = A(t, x) + B(t, x) \tag{a}$$

$$y' = A(t, y), \tag{b}$$

where x, y, A, B, are real-valued n-vectors, and the functions $A(t, x)$, $B(t, x)$ are defined and continuous on $I \times R^n$, $I = \langle t_0, \infty)$, $t_0 \geqq 0$, R^n is the space of all real n-vectors. The paper [3] deals with the property of L^2-boundedness for solutions of ordinary differential equations. More specifically: there are determined some conditions under which all solutions of a perturbed linear differential equation belong to $L^2(0, +\infty)$ assuming the fact that all solutions of the unperturbed equation posses the same property. Our objective here is to give a more general concept. The problem we deal with in this paper is the integral equivalence of two systems (a) and (b). It is easy to see that if the two systems (a) and (b) are $(1, p)$-integrally equivalent (see Definition 1.) and some solution $y(t)$ of (b) is L^p-bounded, then the corresponding solution $x(t)$ of (a) is also L^p-bounded, and conversely. On the other hand, two systems (a) and (b) may be $(1, p)$-integrally equivalent although no solution of either of them is L^p-bounded.

Definition 1. *Let $\psi(t)$ be a positive continuous function on the interval $\langle t_0, +\infty)$ and let $p > 0$. We say that the systems* (a) *and* (b) *are (ψ, p)-integrally equivalent iff to each solution $x(t)$ of* (a) *there exists a solution $y(t)$ of* (b) *such that*

$$\psi^{-1}(t)|x(t) - y(t)| \in L^p(t_0, +\infty), \tag{c}$$

and conversely, to each solution $y(t)$ of (b) *there exists a solution $x(t)$ of* (a) *such that* (c) *holds.*

In [2] this problem is considered for special systems

$$A(t, x) = A(t)x.$$

This problem is solved here for general nonlinear systems (a) and (b), although the case $A(t, x)=Ax$ where A is a constant $n\times n$ matrix is not contained in our results, due to the fact that our hypotheses always require some kind of smallness of the vector $A(t, x)$ or $A(t, y+v)-A(t, y)$.

All functions considered throughout the paper will be continuous on their domains and the function $A(t, x)$ smooth enough to guarantee the existence of a solution $y(t)$, $t\in I_y$ of (b) which, unless otherwise stated, will be fixed. Let $B(a)$ be the Banach space of all bounded and continuous R^n-valued functions on $\langle a, +\infty)$ with the norm $|f|_B=\sup\limits_{t\geq a}|f(t)|$ where, for $x\in R^n$, $|x|=\sup|x_i|$.

Definition 2. *The sequence $f_n\in B(a)$ q-converges to $f\in B(a)$ if $\lim\limits_{n\to\infty}|f_n(t)-f(t)|=0$ for every $t\in\langle a, +\infty)$. This will be denoted by $f_n\xrightarrow{q}f$.*

Definition 3. *A set $M\subset B(a)$ is said to be uniformly bounded if $|f|_B\leq K$ for every $f\in M$, where K is some positive constant.*

Definition 4. *A set $M\subset B(a)$ is said to be equicontinuous if for every $\varepsilon>0$ there exists $\delta(\varepsilon)>0$ such that $f\in M$, t', $t''\geq a$* and *$|t'-t''|<\delta(\varepsilon)$ imply $|f(t')-f(t'')|<\varepsilon$.*

We shall need the following results in our considerations:

Lemma 1. *Let $g(t)\geq 0$ be a continuous function on $0\leq t<+\infty$ and such that*

$$\int_0^\infty s^{\frac{1}{p}}g(s)\,ds<+\infty,\quad p\geq 1.$$

Then

$$\int_t^\infty g(s)\,ds\in L^{p'}(0, +\infty),\quad p'\geq p.$$

Proof. Let $g_1(t)>0$ be a continuous function on $\langle 0, +\infty)$ such that

$$g(t)\leq g_1(t)\quad\text{and}\quad\int_0^\infty s^{\frac{1}{p}}g_1(s)\,ds<+\infty,\quad p\geq 1.$$

It is sufficient to prove that

$$\int_t^\infty g_1(s)\,ds\subset L^{p'}(0, +\infty),\quad p'\geq p.$$

For $p\geq 1$ we have

$$\int_0^\infty\int_t^\infty s^{\frac{1}{p}-1}g_1(s)\,ds\,dt=\int_0^\infty\int_0^s s^{\frac{1}{p}-1}g_1(s)\,dt\,ds=\int_0^\infty s^{\frac{1}{p}}g_1(s)\,ds<+\infty.\tag{1}$$

We have to prove that

$$\int_0^\infty \left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^p \mathrm{d}t < +\infty.$$

In view of (1) it suffices to show that

(2)

$$\lim_{t\to\infty} \frac{\left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^p}{\int_t^\infty s^{\frac{1}{p}-1} g_1(s)\,\mathrm{d}s} \text{ is finite.}$$

The function

$$\int_t^\infty s^{\frac{1}{p}-1} g_1(s)\,\mathrm{d}s$$

is non-negative non-increasing and by (1)

$$\lim_{t\to\infty} \int_t^\infty s^{\frac{1}{p}-1} g_1(s)\,\mathrm{d}s = 0.$$

We can use l'Hospital's rule

$$0 \leqq \lim_{t\to\infty} \frac{\left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^p}{\int_t^\infty s^{\frac{1}{p}-1} g_1(s)\,\mathrm{d}s} =$$

$$= \lim_{t\to\infty} \frac{-pg_1(t)\left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^{p-1}}{-t^{\frac{1}{p}-1} g_1(t)} = \lim_{t\to\infty} p\left(t^{\frac{1}{p}} \int_t^\infty g_1(s)\,\mathrm{d}s\right)^{p-1}$$

$$\leqq \lim_{t\to\infty} p\left(\int_t^\infty s^{\frac{1}{p}} g_1(s)\,\mathrm{d}s\right)^{p-1} = 0,$$

thus

$$\int_t^\infty g_1(s)\,\mathrm{d}s \in L^p(0, +\infty).$$

Since

$$\int_t^\infty g_1(s)\,\mathrm{d}s \to 0 \quad \text{for} \quad t\to\infty,$$

there exists $T > 0$ such that

$$\int_t^\infty g_1(s)\,\mathrm{d}s < 1 \quad \text{for} \quad t > T.$$

Then for $p'>p$ we have

$$\left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^{p'}<\left(\int_t^\infty g_1(s)\,\mathrm{d}s\right)^{p} \quad \text{for} \quad t>T,$$

which completes the proof.
If $p=1$, the present Lemma 1 reduces to Lemma 2 in [2].

Theorem 1. (M. Švec [1]) *Suppose that $N\subset B(a)$ is a non-void convex, q-closed set and T: $N\to B(a)$ a q-continuous operator ($f_m\xrightarrow{q}f$, f_m, $f\in N$ imply that $\lim_{m\to\infty}|Tf_m-Tf|_B=0$) such that TN is a uniformly bounded and equicontinuous set with $TN\subset N$. Then T has at least on fixed point in N.*

For $r>0$ and $a\in I_y$ we give two conditions:

Condition *$C_1(l, r, a)$. The vector $D(t, u)$ satisfies Condition $C_1(l, r, a)$ if there exists a non-negative function $l(t)$ such that $t\in\langle a, +\infty)$ and $|u|\leq r$ imply*

$$|D(t, u+y)|\leq l(t), \quad y\in R^n$$

and

$$\int_a^\infty t^{\frac{1}{p}}\, l(t)\,\mathrm{d}t<+\infty.$$

Condition *$C_2(l, g, r, a)$. The vector $D(t, u)$ satisfies condition $C_2(l, g, r, a)$ if there exist non-negative functions l, g such that*

$$t\geq a, \quad |u|\leq r, \quad |v|\leq r$$

imply

$$|D(t, u+y)-D(t, v+y)|\leq l(t)\,g(|u-v|), \quad y\in R^n$$

and

$$\int_a^\infty t^{\frac{1}{p}} l(t)\,\mathrm{d}t<+\infty.$$

It is evident that *Condition C_1* implies *Condition C_2*. The converse is true if

$$\int_a^\infty t^{\frac{1}{p}}|D(t, y)|\,\mathrm{d}t<+\infty \quad \text{for some} \quad y\in R^n.$$

Now we are able to prove

Theorem 2. *Suppose that the vectors $A(t, u)$, $B(t, u)$ satisfy Condition $C_2(l_1, g, r, t_y)$, or $C_1(l_2, r, t_y)$, respectively. Then for each solution $y(t)$ of the system* (b) *there exists a solution $x(t)$ of* (a) *such that*

$$|x(t)-y(t)|\in L^p(t_y, +\infty).$$

Proof. Let $t_1 \geqslant t_y$ be such that

$$\int_{t_1}^{\infty} (Kl_1(t) + l_2(t))\, \mathrm{d}t \leqslant r, \quad K = \sup_{|u| \leqslant 2r} g(|u|).$$

Then the operator

$$T:\ S_r \to B(t_1)\ (S_r = \{f \in B(t_1):\ |f|_B \leqslant r)$$

defined by

$$(Tf)(t) = -\int_t^{\infty} (A(s, f(s) + y(s)) - A(s, y(s)))\, \mathrm{d}s -$$

$$-\int_t^{\infty} B(s, f(s) + y(s))\, \mathrm{d}s$$

is well defined because of

$$\int_t^{\infty} |A(s, f(s) + y(s)) - A(s, y(s))|\, \mathrm{d}s + \int_t^{\infty} |B(s, f(s) + y(s)|\, \mathrm{d}s$$

$$\leqslant \int_{t_1}^{\infty} (Kl_1(t) + l_2(t))\, \mathrm{d}t \leqslant r < +\infty.$$

By the standard method it is easy to show that all hypotheses of Theorem 1 are satisfied. Thus T has at least one fixed point in S_r. This fixed point $v(t)$ has the property that the function

$$x(t) = v(t) + y(t)$$

satisfies (a). Therefore we have to prove that

$$|v(t)| \in L^p(t_1, +\infty). \tag{3}$$

Using Minkowski's inequality we obtain

$$\int_{t_1}^{\infty} |v(t)|^p\, \mathrm{d}t\Big)^{\frac{1}{p}} \leqslant$$

$$\leqslant \Big(\int_{t_1}^{\infty} \Big(\int_t^{\infty} |A(s, v(s) + y(s) - A(s, y(s))|\, \mathrm{d}s\Big)^p \mathrm{d}t\Big)^{\frac{1}{p}}$$

$$+ \Big(\int_{t_1}^{\infty} \Big(\int_t^{\infty} |B(s, v(s) + y(s))|\, \mathrm{d}s\Big)^p \mathrm{d}t\Big)^{\frac{1}{p}} \leqslant$$

$$\leqslant \Big(\int_{t_1}^{\infty} \Big(\int_t^{\infty} l_1(s) g(|v(s)|)\, \mathrm{d}s\Big)^p \mathrm{d}t\Big)^{\frac{1}{p}} +$$

$$+ \Big(\int_{t_1}^{\infty} \Big(\int_t^{\infty} l_2(s)\, \mathrm{d}s\Big)^p \mathrm{d}t\Big)^{\frac{1}{p}} \leqslant$$

$$\leq K\left(\int_{t_1}^{\infty}\left(\int_t^{\infty} l_1(s)\,\mathrm{d}s\right)^p \mathrm{d}t\right)^{\frac{1}{p}}+$$

$$+\left(\int_{t_1}^{\infty}\left(\int_t^{\infty} l_2(s)\,\mathrm{d}s\right)^p \mathrm{d}t\right)^{\frac{1}{p}}.$$

By the assumptions of the theorem and Lemma 1 we get (3). The proof of the theorem is complete.

Theorem 3. *Suppose the functions $A(t,u)$, $B(t,u)$ satisfy Condition $C_2(l_1, i, r, t_y)$, $C_2(l_2, i, r, t_y)$, where i is the identity function on $R^+ = \langle 0, +\infty)$, respectively. Moreover, assume that*

$$\int_{t_y}^{\infty} |B(t, y(t)|\,\mathrm{d}t < +\infty.$$

Then the solution whose existence is ensured by Theorem 2 is unique.

Proof. It is easy to verify that the operator T in this case is a strict contraction mapping.

Theorem 4. *Suppose that $x(t)$, $t \in I_x = \langle t_x, +\infty)$ is a fixed solution of* (a) *and that $A(t,u)$ satisfies Condition $C_2(l_1, g, r, t_x)$ with $y(t)$ replaced by $x(t)$ throughout. Moreover, assume that*

$$\int_{t_x}^{\infty} t^{\frac{1}{p}} |B(t, x(t))|\,\mathrm{d}t < +\infty.$$

Then there exists a solution $y(t)$ of (b) *such that*

$$|x(t) - y(t)| \in L^p(t_x, +\infty).$$

Proof. We consider now the operator $T\colon S_r \to B(t_1)$ such that

$$(Tf)(t) = -\int_t^{\infty} (A(s, f(s) + x(s)) - A(s, x(s)))\,\mathrm{d}s$$

$$-\int_t^{\infty} B(s, x(s)\,\mathrm{d}s$$

and the proof follows as in Theorem 2.

Previous Theorems imply the following theorem and corollary:

Theorem 5. *Assume that*

$$\left.\begin{array}{l}|A(t,u) - A(t,v)| \leq l_1(t)|u-v| \\ |B(t,u) - B(t,v)| \leq l_2(t)|u-v|\end{array}\right. \quad |u|, |v| < +\infty \text{ and } t \in \langle t_0, +\infty)$$

where

$$\int_{t_0}^{\infty} t^{\frac{1}{p}} l_1(t)\,\mathrm{d}t < +\infty, \quad \int_{t_0}^{\infty} t^{\frac{1}{p}} l_2(t)\,\mathrm{d}t < +\infty,$$

$$\int_{t_0}^{\infty} t^{\frac{1}{p}} |B(t, 0)|\,\mathrm{d}t < +\infty.$$

Then between the set of bounded solutions of the system (a) *and that of* (b) *there is a* $(1, p)$*-integral equivalence.*

Corollary 1. *Consider the system*

$$x' = B(t, x) + Q(t), \tag{a$_2$}$$

where $Q(t)$ *is a continuous vector-valued function defined on* I *and* $B(t, u)$ *satisfies Condition* $C_1(l_1, r, t_0)$ *for every function*

$$y(t) = d + \int_{t_0}^{t} Q(s)\,\mathrm{d}s,$$

where d *is an* n*-vector. Then for every* n*-vector* d *there exists a solution* $x(t)$ *of* (a_2) *such that*

$$\left| x(t) - \left(d + \int_{t_0}^{t} Q(s)\,\mathrm{d}s \right) \right| \in L^p(t_0, +\infty).$$

Consider now the following n-th order equations

$$x^{(n)} = A(t, x, x', \ldots, x^{(n-1)}) + B(t, x, x', \ldots, x^{(n-1)}) \tag{a$_3$}$$

$$y^{(n)} = A(t, y, y', \ldots, y^{(n-1)}) \tag{b$_3$}$$

where A, B are real-valued functions defined and continuous on $I \times R^n$, $I = \langle t_0, +\infty)$, $t_0 \geq 0$.

Theorem 6. *Suppose that the function* $A(t, u)$ *satisfies the Lipschitz-like condition*

$$|A(t, x_1, x_2, \ldots, x_n) - A(t, y_1, y_2, \ldots, y_n)| \leq \sum_{i=1}^{n} \lambda_i(t)\, g_i(|x - y|)$$

for any $x, y \in R^n$ *with* $|x - y| \leq r$ *(*r *is some positive constant), where*

$$\int_{t_0}^{\infty} t^{n-1+\frac{1}{p}} \lambda_i(t)\,\mathrm{d}t < +\infty \quad \text{for} \quad i = 1, 2, \ldots, n.$$

Assume also that for any solution $y(t)$ of (b₃)*, any vector $v \in R^n$ with $|v| \leq r$ and any $t \in I_y$ we have*

$$|B(t, y+v_1, y'+v_2, \dots, y^{(n-1)}+v_n)| \leq L(t),$$

$$\int_{t_y}^{\infty} t^{n-1+\frac{1}{p}} L(t)\, dt < +\infty.$$

Then for every solution $y(t)$ of (b_3) *there is a solution $x(t)$ of* (a_3) *with the property*

$$|x^{(i)}(t) - y^{(i)}(t)| \in L^{p'}(t_0, +\infty), \quad i=0, 1, \dots, n-1;$$

$$p' \geq p.$$

Proof. Let

$$v_k = x^{(k-1)}(t) - y^{(k-1)}(t) \quad \text{for} \quad k=1, \dots, n$$

and

$$B_0(t, v(t)) =$$
$$= A(t, y+v_1(t), \dots, y^{(n-1)}+v_n(t)) - A(t, y, \dots, y^{(n-1)}) +$$
$$+ B(t, y+v_1(t), \dots, y^{(n-1)}+v_n(t)).$$

Then it suffices to prove that the system

$$v_1 = -\int_t^{\infty} \frac{(t-s)^{n-1}}{(n-1)!} B_0(s, v(s))\, ds = B_1(t, v)$$

$$v_2 = -\int_t^{\infty} \frac{(t-s)^{n-2}}{(n-2)!} B_0(s, v(s))\, ds = B_2(t, v)$$

$$\vdots$$

$$v_n = -\int_t^{\infty} B_0(s, v(s))\, ds = B_n(t, v)$$

has a solution $v(t) \in L^{p'}(t_0, +\infty)$.

Theorems 2—4 can be extendedd to n-th order systems if all of the integral conditions considered are replaced by those having integrands multiplied by

$$t^{n-1+\frac{1}{p}}.$$

In this case one would have to replace $B(a)$ by Banach space $B_{n-1}(a)$ of all $(n-1)$-times continuously differentiable functions with bounded derivatives and with the norm

$$|f|_{B_{n-1}} = \max_{0 \leq i \leq n-1} \sup_{t \geq a} \{|f^{(i)}(t)|\}.$$

We shall give some results for $2n$d-order systems

$$(a_4) \qquad x'' = A(t)x + F(t, x)$$

$$(b_4) \qquad y'' = A(t)y$$

with $n \times n$ matrix $A(t)$ and the n-vector function $F(t, x)$.

Theorem 7. *Let $A(t)$ and $F(t, x)$ satisfy the conditions*

$$\int_{t_y}^{\infty} t^{\frac{1}{p}} |B(t)| \, dt < +\infty, \quad \int_{t_y}^{\infty} t^{1+\frac{1}{p}} |C(t)A(t)| \, dt < +\infty,$$

$$\int_{t_y}^{\infty} t^{1+\frac{1}{p}} \lambda(t) \, dt < +\infty,$$

where

$$B(t) = \int_t^{\infty} A(s) \, ds$$

$$(\textit{assumed to converge})$$

$$C(t) = \int_t^{\infty} B(s) \, ds$$

and

$$|F(t, v + y(s))| \leqslant \lambda(t) \quad \text{for any} \quad v \in R^n \quad \text{with} \quad |v| \leqslant r$$

and for fixed solution $y(t)$, $t \in I_y = \langle t_y, +\infty)$ of (b_4). Then there exists a solution $x(t)$ of the system (a_4) such that

$$|x(t) - y(t)| \in L^{p'}(t_0, +\infty), \quad p' \geqslant p.$$

Proof. If we put

$$v(t) = x(t) - y(t),$$

then we have

$$v''(t) = A(t)v(t) + F(t, v(t) + y(t)).$$

Thus, we only have to prove the existence of a solution

$$(4) \qquad v(t) \in L^{p'}(t_y, +\infty)$$

of this equation. This can be done by considering the operator

$$(Tf)(t) = -C(t)f(t) + 2\int_t^{\infty} B(s)f(s) \, ds$$

$$+ \int_t^{\infty} (s - t)C(s)A(s)f(s) \, ds$$

$$+ \int_t^{\infty} (s - t)(E - C(s))F(s, f(s) + y(s)) \, ds$$

defined on the ball of bounded functions in $B_1(a)$ with the norm $\leqslant r$ for $t \geqslant t_1$ (t_1 some suitable number, $t_1 \geqslant t_y$). It is easy to show that T has at least one fixed point $v(t)$ in

$$S_r = \{f \in B_1(t_1):\ |f|_{B_1} \leqslant r\}.$$

Now we have to show (4). Since

$$|C(t)| \leqslant \int_t^\infty |B(s)|\, ds \to 0 \quad \text{as} \quad t \to +\infty$$

there exists t_0 such that $|C(t)| \leqslant 1$ for $t \geqslant t_0$.
Then

$$\left(\int_{t_1}^\infty |v(t)|^p\, dt\right)^{\frac{1}{p}} \leqslant 3r\left(\int_{t_1}^\infty \left(\int_t^\infty |B(s)|\, ds\right)^p dt\right)^{\frac{1}{p}}$$
$$+ r\left(\int_{t_1}^\infty \left(\int_t^\infty s|C(s)A(s)|\, ds\right)^p dt\right)^{\frac{1}{p}}$$
$$+ 2\left(\int_{t_1}^\infty \left(\int_t^\infty s\lambda(s)\, ds\right)^p dt\right)^{\frac{1}{p}} < +\infty$$

according to Lemma 1. The proof of the theorem is completed.

Theorem 8. *Suppose that $y(t)$, $t \in I_y$ is a solution of the system*

$$y'' = Q(t).$$

Let $A(t)$ and $F(t, x)$ satisfy the conditions

$$\int_{t_y}^\infty t^{\frac{1}{p}} |y(t)B(t)|\, dt < +\infty, \quad \int_{t_y}^\infty t^{\frac{1}{p}} |y(t)C(t)A(t)|\, dt < +\infty$$

and

$$\int_{t_y}^\infty t^{1+\frac{1}{p}} \lambda(t)\, dt < +\infty.$$

Then there exists a solution of the system

$$x'' = A(t)x + F(t, x) + Q(t)$$

for which we have

$$|x(t) - y(t)| \in L^{p'}(\max(t_x, t_y), +\infty), \quad p' \geqslant p.$$

Proof. In this case it suffices to show that the integral equation

$$v(t) = -C(t)v(t) + 2\int_t^\infty B(s)v(s)\, ds$$

$$+\int_t^\infty (s-t)\,C(s)\,v(s)\,\mathrm{d}s$$

$$+\int_t^\infty (s-t)(E+C(s))(A(s)y(s)+F(s,\,v(s)+y(s))\,\mathrm{d}s$$

has a solution $v(t)$ which belongs to $L^{p'}(\max(t_x, t_y), +\infty)$, $p'\geq p$.

Theorems 7, 8 guarantee the existence of a unique solution $x(t)$ if $F(t, u)$ satisfies a Lipschitz condition of the form

$$|F(t,u)-F(t,v)|\leq\lambda_1(t)|u-v|,\quad u, v\in R^n$$

where

$$\int_{t_y}^\infty t^{1+\frac{1}{p}}\lambda_1(t)\,\mathrm{d}t<+\infty.$$

The following result deals with second order scalar equations of the forms

$$(\mathrm{a}_5)\qquad x''=a(t)x$$

and

$$(\mathrm{b}_5)\qquad y''=(a(t)+p(t))y.$$

Let

$$b(t)=\int_t^\infty a(s)\,\mathrm{d}s\quad\text{(assumed to converge)}$$

and

$$c(t)=\int_t^\infty b(s)\,\mathrm{d}s.$$

Theorem 9. *Let the functions a, b, c, p satisfy the conditions*

$$\int_{t_y}^\infty t^{\frac{1}{p}}|b(t)|\,\mathrm{d}t<+\infty,\quad \int_{t_y}^\infty t^{1+\frac{1}{p}}|a(t)c(t)|\,\mathrm{d}t<+\infty,$$

$$\int_{t_y}^\infty t^{1+\frac{1}{p}}|p(t)|\,\mathrm{d}t<+\infty,$$

where $I_y=\langle t_y, +\infty)$ is the domain of a fixed solution $y(t)$ of the equation (b_5) *such that*

$$y(t)\in L^p(t_y, +\infty).$$

Then there exists a unique solution $x(t)$ of (a_5) *such that*

$$x(t)\in L^p(t_x, +\infty).$$

Proof. Let

$$(5)\qquad u(t)=x(t)-y(t)$$

be the unique solution of the differential equation

$$u'' = a(t)\,u(t) - p(t)\,y(t)$$

such that

$$u(t) \in L^p(\max(t_x, t_y), +\infty).$$

Now the assertion of the theorem is obtained from (5) and the Minkowski Inequality.

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ИНТЕГРАЛЬНАЯ ЭКВИВАЛЕНТНОСТЬ НЕЛИНЕЙНОЙ СИСТЕМЫ И СИСТЕМЫ, ПОЛУЧЕННОЙ ВОЗМУЩЕНИЕМ ЕЕ

Alexander Haščák

Резюме

В статье даются достаточные условия для (ψ, p)-интегральной эквивалентности нелинейной системы дифференциальных уравнений и возмущенной нелинейной системы.