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Applications of Mathematics, Vol. 45 (2000), No. 3, 161–176

Persistent URL: http://dml.cz/dmlcz/134434

DESTABILIZATION FOR QUASIVARIATIONAL INEQUALITIES OF REACTION-DIFFUSION TYPE*

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(Received May 12, 1998)

Abstract. We consider a reaction-diffusion system of the activator-inhibitor type with unilateral boundary conditions leading to a quasivariational inequality. We show that there exists a positive eigenvalue of the problem and we obtain an instability of the trivial solution also in some area of parameters where the trivial solution of the same system with Dirichlet and Neumann boundary conditions is stable. Theorems are proved using the method of a jump in the Leray-Schauder degree.

Keywords: reaction-diffusion system, unilateral conditions, quasivariational inequality, Leray-Schauder degree, eigenvalue, stability

MSC 2000: 35B35, 35K57, 35J85

1. Introduction

Many interesting natural phenomena can be described mathematically by the system of reaction-diffusion equations

$$\text{(RD)} \qquad \begin{aligned} u_t &= d_1\Delta u + f(u,v), \\ v_t &= d_2\Delta v + g(u,v) \quad \text{on} \quad [0,\infty)\times\Omega \end{aligned}$$

where f and g are some suitable functions such that $f(\bar{u},\bar{v}) = g(\bar{u},\bar{v}) = 0$. In chemical or biological models u and v usually describe some positive concentrations (see for example [1]), $[\bar{u},\bar{v}]$ is a trivial steady state. Without loss of generality we will suppose $\bar{u} = \bar{v} = 0$ because of the shift of coordinates.

* This research was supported by the grant No. 201/95/0630 of the Grant Agency of the Czech Republic.

We shall investigate the linearized reaction-diffusion system

$$\text{(RD}_\text{L}\text{)} \qquad \begin{aligned} u_t &= d_1\Delta u + b_{11}u + b_{12}v, \\ v_t &= d_2\Delta v + b_{21}u + b_{22}v \quad \text{on} \quad [0,\infty)\times\Omega \end{aligned}$$

where Ω is a bounded domain in $\mathbb{R}^m$ with a Lipschitzian boundary Γ. Furthermore, $d = [d_1, d_2] \in \mathbb{R}^2_+$ is a couple of positive numbers which will play the role of a parameter. Let us suppose that constants b_{11}, b_{12}, b_{21} and b_{22} satisfy the conditions

$$\text{(SIGN)} \qquad \begin{gathered} b_{11} > 0, \quad b_{12} < 0, \quad b_{21} > 0, \quad b_{22} < 0, \\ b_{11} + b_{22} < 0, \quad b_{11}b_{22} - b_{12}b_{21} > 0. \end{gathered}$$

These signs mean that we have the system of prey-predator (or activator-inhibitor) type.

Let Γ_D, Γ_N and Γ_U be disjoint open subsets in Γ such that

$$\operatorname{meas}(\Gamma\setminus(\Gamma_D\cup\Gamma_N\cup\Gamma_U)) = 0$$

and let Φ be a nonnegative function on the set $\Gamma\times\Gamma$ which will be specified later. We will prove that the trivial solution for the system (RD$_\text{L}$) with the unilateral boundary conditions

$$\text{(UC)} \qquad \begin{aligned} &\text{i)} \quad u = 0, \quad v = 0 \quad \text{on} \quad (0,\infty)\times\Gamma_D, \\ &\text{ii)} \quad \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \quad \text{on} \quad (0,\infty)\times\Gamma_N, \\ &\text{iii)} \quad \frac{\partial u}{\partial n} = 0, \quad \frac{\partial v}{\partial n} \geqslant 0 \quad \text{on} \quad (0,\infty)\times\Gamma_U, \\ &\text{iv)} \quad v(x) \geqslant -\int_\Gamma \Phi(x,y)\frac{\partial v(y)}{\partial n}\,\mathrm{d}\Gamma(y) \quad \text{and} \\ &\text{v)} \quad \left(v(x) + \int_\Gamma \Phi(x,y)\frac{\partial v(y)}{\partial n}\,\mathrm{d}\Gamma(y)\right)\frac{\partial v(x)}{\partial n} = 0 \quad \text{for} \quad x\in\Gamma_U \text{ and } t > 0 \end{aligned}$$

is unstable (the definition will be specified later) even for some set of parameters $d = [d_1, d_2]$ where the trivial solution for the problem (RD$_\text{L}$) with the classical boundary conditions

$$\text{(CC)} \qquad \begin{gathered} u = 0, \quad v = 0 \quad \text{on } (0,\infty)\times\Gamma_D, \\ \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \quad \text{on } (0,\infty)\times(\Gamma_N\cup\Gamma_U) \end{gathered}$$

is stable. In fact, we will find a positive eigenvalue of the stationary eigenvalue problem

$$
\text{(EP}_\text{C}) \qquad \begin{aligned} d_1\Delta u + b_{11}u + b_{12}v &= \lambda u,\\ d_2\Delta v + b_{21}u + b_{22}v &= \lambda v \end{aligned}
$$

with boundary conditions (UC) also in some area of parameters d where all eigenvalues of the problem ((EP$_\text{C}$), (CC)) have negative real parts.

In our interpretation, boundary conditions (UC) simulate some kind of regulation by a semipermeable membrane where the flow-rate through the membrane depends on the pumping from the reservoir surrounding the domain Ω.

Unfortunately, the results presented here apply to the "linear" system (RD$_\text{L}$) only. The approach to (RD) remains open because stability of the trivial solution for the problem ((RD), (UC)) can occur even in the case of instability for its "linearization" ((RD$_\text{L}$), (UC)) (see [2]).

The destabilizing effect of unilateral conditions given by inequalities for systems of reaction-diffusion type was first proved in papers of P. Drábek and M. Kučera (see [3]). They considered a more special type of boundary conditions than we do—the ones which we would obtain if we took $\Phi \equiv 0$ in our considerations. This kind of problem leads to variational inequalities and the method of the proof in [3] uses a penalty technique, the main result is also the existence of a positive eigenvalue.

Related results for the inequalities were also proved—the existence of a bifurcation point on any curve going from the domain of instability to the domain of stability of the classical problem even in this area of stability. The destabilizing effect was shown also on simple examples in [4].

The method of the Leray-Schauder degree used here was first introduced to the area of eigenvalues and bifurcation problems for variational inequalities in paper [5] by P. Quittner with the same results. This method was also used in [6] for the destabilization for quasivariational inequalities of the same type as here but in the sense of bifurcation. The present paper combines results and considerations of [5] and [6].

On the other hand, in the case of unilateral conditions given for the activator (component u in our system) instead of for the inhibitor v, some kind of the stabilizing effect in the sense of bifurcations was proved. Most recently it can be seen in [7] for boundary conditions leading to a system of differential inclusions but the proof uses a completely different method. The destabilization for inclusions has been also already proved (see [8], [9]).

You can recently find also more details about the progress on the field in [10].

Let us assume

$$(\Gamma) \qquad \operatorname{meas}(\Gamma_D) > 0, \qquad \operatorname{meas}(\Gamma_U) > 0.$$

Let $\mathbb{V} = \{\varphi \in H^1(\Omega);\ \varphi = 0 \text{ on } \Gamma_D \text{ in the sense of traces}\}$ be the Hilbert space with the scalar product $\langle \varphi, \psi \rangle = \int\limits_\Omega \nabla\varphi\nabla\psi \,\mathrm{d}x$ and its corresponding norm $\|\,.\,\|$. Condition (Γ) ensures that $\|\,.\,\|$ is a norm equivalent to the usual one on the space $\mathbb{V}$.

Let $H^1_L(\Omega) = \big\{\varphi \in H^1(\Omega);\ \Delta\varphi \in L^2(\Omega)\big\}$ and $\mathbb{H} = H^1_L(\Omega) \cap \mathbb{V}$ be the Hilbert spaces with the scalar product $(\varphi, \psi) = \int\limits_\Omega (\nabla\varphi\nabla\psi + \Delta\varphi\Delta\psi)\,\mathrm{d}x$ (see [11]) and with its corresponding norm $|\,.\,|$ or the corresponding norm on $\mathbb{H}\times\mathbb{H}$.

Let $H^{\frac12}(\Gamma)$ be the space of traces of all functions from the Sobolev space $H^1(\Omega)$ and let $H^{-\frac12}(\Gamma)$ be its dual (see for example [11]). For any fixed $v \in \mathbb{H}$ we also denote

$$K_v = \left\{\varphi \in \mathbb{V};\ \varphi(x) \geqslant -\int\limits_\Gamma \Phi(x,y)\frac{\partial v(y)}{\partial n}\,\mathrm{d}\Gamma(y) \text{ on } \Gamma_U \text{ in the sense of traces}\right\}$$

where $\frac{\partial v}{\partial n} \in H^{-\frac12}(\Gamma)$ (see [11]) and $\int\limits_\Gamma \varphi(z)\psi(z)\,\mathrm{d}\Gamma(z)$ is understood for $\varphi \in H^{\frac12}(\Gamma)$ and $\psi \in H^{-\frac12}(\Gamma)$ in the sense of duality between $H^{\frac12}(\Gamma)$ and $H^{-\frac12}(\Gamma)$.

Under this notation, by the solution of the problem ((RD$_\mathrm{L}$), (UC)) we can understand a pair $[u,v]$ such that for all $T>0$ we have

$$[u,v] \in L^2((0,T),\mathbb{H}) \times L^2\big((0,T),\mathbb{H}\big),\ \ [u_t,v_t] \in L^2((0,T),\mathbb{V}^\star) \times L^2((0,T),\mathbb{V}^\star)$$

(see [12]) and for almost all $t>0$

$$\begin{aligned}
&\text{i)} && v(t,.\,) \in K_{v(t,.)},\\
&\text{ii)} && \int\limits_\Omega [u_t(t,x)\varphi(x) + d_1\nabla_x u(t,x)\nabla\varphi(x)\\
& && \qquad - \big(b_{11}u(t,x) + b_{12}v(t,x)\big)\varphi(x)]\,\mathrm{d}x = 0 \quad \forall\varphi\in\mathbb{V},\\
&\text{iii)} && \int\limits_\Omega \big[v_t(t,x)\big(\psi(x)-v(t,x)\big) + d_2\nabla_x v(t,x)\nabla_x\big(\psi(x)-v(t,x)\big)\\
& && \qquad - \big(b_{21}u(t,x) + b_{22}v(t,x)\big)\big(\psi(x)-v(t,x)\big)\big]\,\mathrm{d}x \geqslant 0 \quad \forall\psi\in K_v
\end{aligned}$$

hold.

By stability of a stationary solution $\overline{U}$ of the problem ((RD$_\mathrm{L}$), (UC)) with initial conditions with respect to the norm $\|.\|$ we mean the following property: For any $\varepsilon > 0$ there is $\delta > 0$ such that any solution $U = [u, v]$ (solution in the sense of the previous definition) satisfying $\|U(t_0) - \overline{U}\| < \delta$ for some t_0 is defined on the interval $[t_0, \infty)$ and $\|U(t) - \overline{U}\| < \varepsilon$ holds for all $t \in [t_0, \infty)$.

$\overline{U}$ is unstable if it is not stable. At this point we have to say that only instability is investigated in the article. In fact, the solution of the evolution problem will appear only once (see Corollary 1) and it will be very smooth and unstable with respect to any reasonable norm. The generality of the definition of a weak solution of the evolution problem is not a crucial point in the paper.

Moreover, a weak solution of the problem ((EP$_\mathrm{C}$), (UC)) is a pair $[u, v] \in \mathbb{H} \times \mathbb{H}$ satisfying the quasivariational inequality (see [6])

(I$_\lambda$)

$$\text{i)} \quad v \in K_v,$$

$$\text{ii)} \quad \int_\Omega [d_1 \nabla u \nabla \varphi - (b_{11} u + b_{12} v - \lambda u)\varphi] \,\mathrm{d}x = 0 \quad \forall \varphi \in \mathbb{V},$$

$$\text{iii)} \quad \int_\Omega [d_2 \nabla v \nabla(\psi - v) - (b_{21} u + b_{22} v - \lambda v)(\psi - v)] \,\mathrm{d}x \geqslant 0 \quad \forall \psi \in K_v.$$

Let us introduce an operator $A\colon \mathbb{V} \to \mathbb{V}$ by $\langle Au, \varphi\rangle = \int_\Omega u\varphi \,\mathrm{d}x$ $\forall u, \ \varphi \in \mathbb{V}$, the projection $P_v\colon \mathbb{V} \to K_v$ on the convex closed set K_v defined for any fixed $v \in \mathbb{H}$ by

$$\forall z \in \mathbb{V} \quad P_v z \in K_v, \quad \|P_v z - z\| = \min_{y \in K_v} \|y - z\|$$

(see [13] for more details) and an operator $T\colon \mathbb{R}^2_+ \times \mathbb{R} \times \mathbb{H} \times \mathbb{H}^2 \to \mathbb{H} \times \mathbb{H}$ defined for all $d = [d_1, d_2] \in \mathbb{R}^2_+$, $\lambda \in \mathbb{R}$, $f \in \mathbb{H}$ and $U = [u, v] \in \mathbb{H} \times \mathbb{H}$ by

$$T(d, \lambda, f, U) = \begin{bmatrix} u - d_1^{-1}(b_{11} Au + b_{12} Av - \lambda Au - f) \\ v - P_v\big(d_2^{-1}(b_{21} Au + b_{22} Av - \lambda Av)\big) \end{bmatrix}.$$

Then the inequality (I$_\lambda$) can be rewritten (see also [6]) as

(EP$_\mathrm{W}$)

$$\begin{aligned} d_1 u - b_{11} Au - b_{12} Av + \lambda Au &= 0, \\ d_2 v - P_{d_2 v}\big(b_{21} Au + b_{22} Av - \lambda Av\big) &= 0 \end{aligned}$$

or simply

$$T(d, \lambda, 0, U) = 0.$$

Simultaneously, the weak formulation for the stationary version of the problem ((RD$_\mathrm{L}$), (CC)) is

$$\text{(CP}_\mathrm{W}\text{)}\qquad \begin{aligned} d_1u - b_{11}Au - b_{12}Av &= 0,\\ d_2v - b_{21}Au - b_{22}Av &= 0. \end{aligned}$$

Moreover, we denote by $E_{I_d}(\lambda) = \{U = [u,v] \in \mathbb{H} \times \mathbb{H};\ T(d,\lambda,0,U) = 0\}$ the set of all eigenvectors of the problem (EP$_\mathrm{W}$). Let σ_{I_d} be the set of all $\lambda \in \mathbb{R}$ such that there is a nontrivial solution of (EP$_\mathrm{W}$), i.e. $E_{I_d}(\lambda) \neq \{[0,0]\}$. It means that σ_{I_d} is the set of all real eigenvalues of the problem (EP$_\mathrm{W}$). The set of all solutions of (CP$_\mathrm{W}$) belonging to $\mathbb{H} \times \mathbb{H}$ will be denoted by E_d where $d = [d_1, d_2]$. Finally, the set of all eigenvalues of the problem ((EP$_\mathrm{C}$), (CC)) will be denoted by σ_d. Let us note that the eigenvalues of this problem are the same as for its weak formulation because the corresponding eigenvectors are sufficiently smooth (see [3]).

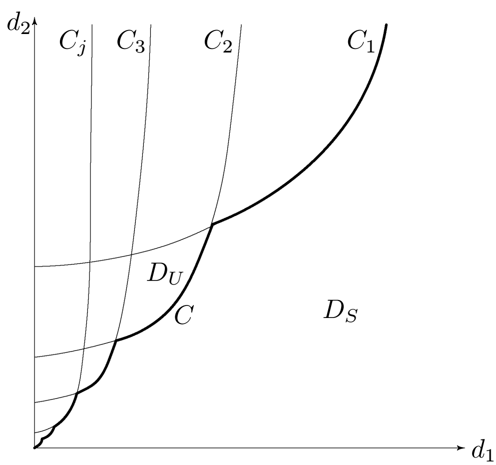

Figure 1. The domain of stability D_S, the domain of instability D_U and the envelope of hyperbolas C for the system (CP$_\mathrm{W}$).

For the system (CP$_\mathrm{W}$) under the assumption (SIGN) the following result is known:

Theorem 0 (see [3], for the one-dimensional case and the Neumann boundary conditions it was done earlier in [14], [15]). *Let $\{e_j\}_{j=1,\dots}$, be an orthonormal system of eigenvectors of the problem $-\Delta u = \lambda u$ with the boundary conditions* (CC) *in* $\mathbb{V}$ *corresponding to the eigenvalues κ_j. Let C_j, $j = 1, 2, \dots$, be the hyperbolas*

$$d_2 = -\frac{b_{12}b_{21}/\kappa_j}{b_{11} - \kappa_j d_1} + \frac{b_{22}}{\kappa_j}$$

and C their envelope (see Fig. 1). Then $E_d \neq \{[0,0]\}$ if and only if there is some j such that $d \in C_j$. Let k be the multiplicity of the eigenvalue κ_j. Furthermore, if $d \in C_j \cap C$, $d \notin C_{j-1}$, $d \notin C_{j+k}$ then $E_d = \mathcal{L}in\{[e_i, \alpha_j(d)e_i],\ i = j, \ldots, j+k-1\}$ where $\alpha_j(d) \neq 0$ is a continuous function on $\mathbb{R}^2_+$. If $d \in C \cap C_j \cap C_{j+k}$, $d \notin C_{j-1}$ and m is the multiplicity of the eigenvalue κ_{j+k} then $E_d = \mathcal{L}in\{[e_i, \alpha_j(d)e_i],\ i = j, \ldots, j+k-1,\ [e_i, \alpha_{j+k}(d)e_i],\ i = j+k, \ldots, j+k+m-1\}$. Moreover, if d is to the right (or to the left) from the hyperbola C_j then the eigenvalue of the problem ((EP$_\mathrm{C}$), (CC)) *corresponding to the eigenvector of the type $[e_j, \alpha_j(d)e_j]$ is negative (or positive, respectively). If $d \in D_S = \{d \in \mathbb{R}^2_+;\ d$ is to the right from $C\}$ (domain of stability) then all eigenvalues $\lambda \in \sigma_d$ have negative real parts. If $d \in D_U = \{d \in \mathbb{R}^2_+;\ d$ is to the left from $C\}$ (domain of instability) then there is at least one positive eigenvalue $\lambda \in \sigma_d$. The problem* ((EP$_\mathrm{C}$), (CC)) *has also some complex eigenvalues but their real parts are always negative for all $d \in \mathbb{R}^2_+$.*

For technical reasons, let us assume that $\Phi \in \mathcal{C}^2(\overline{\Omega} \times \overline{\Omega})$ is a given function with the following property:

$$(\Phi) \qquad \begin{aligned} &\text{For every } x \in \Gamma_U\colon\ \Phi(x,y) = 0\ \forall y \in \Gamma_D \text{ and } \Phi(x,y) \geqslant 0\ \forall y \in \Gamma_U,\\ &\text{for every } x \in \Gamma_D\colon\ \Phi(x,y) = 0\ \forall y \in \Gamma. \end{aligned}$$

Furthermore, for any d^0 in $\mathbb{R}^2_+$ we formulate the following essential assumption:

$$(\mathrm{v}) \qquad \exists [u_0, v_0] \in E_{d^0} \text{ and } \exists \varepsilon > 0 \text{ such that } v_0 \geqslant \varepsilon \text{ on } \Gamma_U.$$

Theorem 1. *Let $d^0 \in C$ and let conditions* (Γ), (Φ), (SIGN), (v) *hold. Then there is a neighbourhood $\mathcal{W}(d^0)$ of d^0 such that for all $d \in \mathcal{W}(d^0) \cap D_S$ there is a positive eigenvalue $\lambda \in \sigma_{I_d}$.*

Corollary 1. *Let $d^0 \in C$ and let conditions* (Γ), (Φ), (SIGN), (v) *hold again. Then there is a neighbourhood $\mathcal{W}(d^0)$ of d^0 such that for all $d \in \mathcal{W}(d^0) \cap D_S$ the trivial solution of* ((RD$_\mathrm{L}$), (UC)) *is unstable. In fact, if $[u_0, v_0]$ is a nontrivial solution of the problem* (EP$_\mathrm{W}$) *for some $\lambda > 0$ then $U(t) = [e^{\lambda t}\tau u_0, e^{\lambda t}\tau v_0]$ is a solution of the evolution problem* ((RD$_\mathrm{L}$), (UC)) *for any $\tau > 0$.*

Corollary 2. *Let $d\colon (-\infty, \infty) \to \mathbb{R}^2_+$ be a continuous curve such that $d(0) \in C$, $E_{I_{d(s)}}(0) = \{[0,0]\}$ for any $s \in (0,1)$ and $E_{I_{d(1)}}(0) \neq \{[0,0]\}$. Let all conditions of Theorem 1 be fulfilled (set $d^0 = d(0)$). Then also for all $s \in (0,1)$ there is a positive eigenvalue $\lambda \in \sigma_{I_{d(s)}}$ and the trivial solution of the problem* ((RD$_\mathrm{L}$), (UC)) *for the parameter $d(s)$ is unstable.*

Remark. There is also a bifurcation point in D_S on such a curve even in the case of a nonlinear system ((RD), (UC)) under some additional conditions on the nonlinearity (see [6]).

3. Proof of the main result

Lemma 1. *The operator $A\colon \mathbb{H} \to \mathbb{H}$ and the operator from $\mathbb{R}_+ \times \mathbb{R} \times \mathbb{H} \times \mathbb{H}$ to $\mathbb{H}$ defined by $[d_2, \lambda, u, v] \mapsto P_v\bigl(d_2^{-1}(b_{21}Au + b_{22}Av - \lambda Av)\bigr)$ are completely continuous.*

P r o o f (similar to the proof of Lemma 2.1 in [6]). Let u, $v \in \mathbb{H}$, $d = [d_1, d_2] \in \mathbb{R}_+^2$, $w = P_v\bigl(d_2^{-1}(b_{21}Au + b_{22}Av - \lambda Av)\bigr)$. Obviously $w \in K_v$ and also

$$\bigl\langle w - d_2^{-1}(b_{21}Au + b_{22}Av - \lambda Av), \psi - w\bigr\rangle \geqslant 0 \quad \forall \psi \in K_v.$$

Choosing $\psi = w \pm \varphi$ for any $\varphi \in \mathcal{D}(\Omega)$ ($\psi \in K_v$ again) we obtain

$$\int_\Omega \bigl[\nabla w \nabla \varphi - d_2^{-1}(b_{21}u + b_{22}v - \lambda v)\varphi\bigr]\,\mathrm{d}x = 0 \quad \forall \varphi \in \mathcal{D}(\Omega).$$

Thus $\Delta w = -d_2^{-1}(b_{21}u + b_{22}v - \lambda v)$ in the sense of distributions and consequently $w \in \mathbb{H}$. Now let $u_n \rightharpoonup u$, $v_n \rightharpoonup v$ in $\mathbb{H}$, $d_n \to d_2 > 0$, $\lambda_n \to \lambda$,

$$w_n = P_{v_n}\bigl(d_n^{-1}(b_{21}Au_n + b_{22}Av_n - \lambda_n Av_n)\bigr),\ z_n = d_n^{-1}(b_{21}Au_n + b_{22}Av_n - \lambda_n Av_n)$$

and $z = d_2^{-1}(b_{21}Au + b_{22}Av - \lambda Av)$. Since A is completely continuous in $\mathbb{V}$ and since $\mathbb{H}$ is continuously embedded in $\mathbb{V}$ we obtain $z_n \to z$ in $\mathbb{V}$.

Now let us define

$$f_n(x) = \int_\Omega \bigl(\Phi(x,y)\Delta v_n(y) + \nabla_y \Phi(x,y)\nabla v_n(y)\bigr)\,\mathrm{d}y,$$
$$f(x) = \int_\Omega \bigl(\Phi(x,y)\Delta v(y) + \nabla_y \Phi(x,y)\nabla v(y)\bigr)\,\mathrm{d}y.$$

According to the Green formula we obtain in the sense of traces also

$$f_n(x) = \int_\Gamma \Phi(x,y)\frac{\partial v_n(y)}{\partial n}\,\mathrm{d}\Gamma(y), \quad f(x) = \int_\Gamma \Phi(x,y)\frac{\partial v(y)}{\partial n}\,\mathrm{d}\Gamma(y).$$

Moreover, denote by $x = [x_1, x_2, \ldots, x_m]$ the coordinates in $\mathbb{R}^m$. Computing derivatives with respect to the parameter we obtain

$$\frac{\partial^2 f_n}{\partial x_i \partial x_j}(x) = \int_\Omega \biggl(\frac{\partial^2 \Phi(x,y)}{\partial x_i \partial x_j}\Delta v_n(y) + \nabla_y \frac{\partial^2 \Phi(x,y)}{\partial x_i \partial x_j}\nabla v_n(y)\biggr)\,\mathrm{d}y.$$

The function Φ and its partial derivatives are continuous on the compact $\overline{\Omega}$ so they are bounded. Since the sequences Δv_n and ∇v_n weakly converge in L^2 there are also constants k_0, k_1, k_2 such that $|\frac{\partial^2 f_n}{\partial x_i \partial x_j}(x)| \leqslant k_2$ independently of x and n for any $i, j = 1, 2, \ldots, m$ and similarly $|\frac{\partial f_n}{\partial x_i}(x)| \leqslant k_1$ and $|f_n(x)| \leqslant k_0$. So the sequence $\{f_n\}$ is bounded in $H^2(\Omega)$ and we can take a subsequence such that

$$f_n \to f \text{ in } H^1(\Omega). \tag{1}$$

Now let $\psi \in K_v$ be fixed. According to (1) there is a sequence $\psi_n \in K_{v_n}$ such that $\psi_n \to \psi$ in $\mathbb{V}$ (take $\psi_n = \psi - f_n + f$). Since P_{v_n} is a projection we obtain

$$\|z_n - w_n\| = \|z_n - P_{v_n} z_n\| \leqslant \|z_n - \psi_n\|$$

and consequently $w_n = (w_n - z_n) + z_n$ are bounded in $\mathbb{V}$. After a restriction to a subsequence we can assume $w_n \rightharpoonup y \in \mathbb{V}$ and analogously as above we obtain for any $\psi \in K_v$

$$\|z - y\| \leqslant \liminf_{n\to\infty} \|z_n - w_n\| \leqslant \limsup_{n\to\infty} \|z_n - w_n\| \leqslant \|z - \psi\|. \tag{2}$$

In addition, $w_n = P_{v_n} z_n \in K_{v_n}$ and using (1) we have $y \in K_v$. Now $y = P_v z = w$ and we put $\psi = y$ $(= w)$ into (2). So the terms are equal, $\lim\limits_{n\to\infty} \|z_n - w_n\| = \|z - w\|$ and we have proved $w_n \to w$ in $\mathbb{V}$. Analogously as above $w_n \in \mathbb{H}$, $\Delta w_n = -d_n^{-1}(b_{21} u_n + b_{22} v_n - \lambda_n v_n)$ and also $\Delta w_n \to \Delta w$ in $L^2(\Omega)$. Thus $w_n \to w$ in $\mathbb{H}$ and the complete continuity is proved. The proof of the property for the operator $A\colon \mathbb{H} \to \mathbb{H}$ proceeds in the same way (see also [6]). □

Remark. In fact, in the proof of Lemma 1 we have proved that for any completely continuous operator A:

$$\text{If} \quad v_n \rightharpoonup v, \ z_n \rightharpoonup z \text{ in } \mathbb{H}, \quad \text{then} \quad P_{v_n} A z_n \to P_v A z \text{ in } \mathbb{H}.$$

Lemma 2 (see [6], Lemma 2.2). *Let $d^0 \in C$ be such that the condition* (v) *is fulfilled. Then $E_{I_{d^0}}(0) \subset E_{d^0}$.*

Lemma 3. *Let $d = [d_1, d_2] \in \mathbb{R}^2_+$. Then there is $k \in \mathbb{R}$ such that*

$$\deg\bigl(T(d, \lambda, 0, .), B_1(0), 0\bigr) = 1 \quad \textit{for every } \lambda \geqslant k.$$

Proof (based on ideas from [5]). For every $t \in [0, 1]$, $U = [u, v] \in \mathbb{H} \times \mathbb{H}$ and $\lambda \geqslant k$ with k large enough let us define homotopies H_1, H_2 by

$$H_1(t, \lambda, U) = \begin{bmatrix} u - \frac{t}{d_1}(b_{11} A u + b_{12} A v) + \frac{\lambda}{d_1} A u \\ v - t P_v\bigl((d_2)^{-1}(b_{21} A u + b_{22} A v - \lambda A v)\bigr) \end{bmatrix},$$

$$H_2(t, \lambda, U) = \begin{bmatrix} u + \frac{t\lambda}{d_1} A u \\ v \end{bmatrix}.$$

Obviously

$$H_1(1,\lambda,U) = T(d,\lambda,0,U),\ H_1(0,\lambda,U) = H_2(1,\lambda,U) \text{ and } H_2(0,\lambda,U) = U.$$

Therefore, by virtue of the homotopy invariance of the Leray-Schauder degree it is sufficient to prove that for k large enough

$$H_i(t,\lambda,U) \neq 0 \quad \forall t \in [0,1],\ \forall U \in \mathbb{H}\times\mathbb{H},\ |U| \neq 0,\ \forall \lambda \geqslant k,\ i = 1,2.$$

First let us assume that it does not hold for $i = 2$. Then obviously $v = 0$ and there are also $t_n \to t$, $\lambda_n \to \infty$ and $u_n \rightharpoonup u$ with $\|u_n\| = 1$ such that

$$u_n + \frac{t_n\lambda_n}{d_1} A u_n = 0. \tag{3}$$

Multiplying (3) by u_n we obtain $\langle u_n, u_n\rangle + \frac{t_n}{d_1}\langle Au_n, u_n\rangle \lambda_n = 0$ and if $u_n \neq 0$ then both terms on the left-hand side are strictly positive and we have a contradiction.

The situation for $i = 1$ is more complicated, again by contradiction: Let $U_n = [u_n, v_n] \in \mathbb{H}\times\mathbb{H}$, $t_n \in [0,1]$ and $\lambda_n \to \infty$ be such that $|U_n| = 1$, $u_n \rightharpoonup u$, $v_n \rightharpoonup v$, $t_n \to t \in [0,1]$ and $H_1(t_n,\lambda_n,U_n) = 0$ again. This means that

$$\lambda_n A u_n = t_n(b_{11}Au_n + b_{12}Av_n) - d_1 u_n, \tag{4}$$

$$v_n = t_n(d_2)^{-1} P_{d_2 v_n}(b_{21}Au_n + b_{22}Av_n - \lambda_n A v_n). \tag{5}$$

Since the right-hand side in (4) is bounded we have necessarily $Au_n \to 0$. Multiplying (5) by v_n we obtain

$$\begin{aligned} \langle v_n, v_n\rangle &= t_n\big\langle P_{v_n}\big((d_2)^{-1}(b_{21}Au_n + (b_{22}-\lambda_n)Av_n)\big), v_n\big\rangle \\ &\leqslant t_n(d_2)^{-1}\big(b_{21}\langle Au_n, v_n\rangle + (b_{22}-\lambda_n)\langle Av_n, v_n\rangle\big) \end{aligned} \tag{6}$$

using $0 \in K_{v_n}$ in the well-known property of the projection P_{v_n}

$$\langle P_{v_n}z_n - z_n,\, y - P_{v_n}z_n\rangle \geqslant 0 \quad \forall y \in K_{v_n}\ \forall z_n \in \mathbb{V}$$

where we take $z_n = (d_2)^{-1}\big(b_{21}Au_n + (b_{22}-\lambda_n)Av_n\big)$ and $v_n = t_n P_{v_n} z_n$ from (5). From the inequality (6) it is easy to show that

$$\langle v_n, v_n\rangle + t_n(d_2)^{-1}(\lambda_n - b_{22})\langle Av_n, v_n\rangle \leqslant t_n(d_2)^{-1} b_{21}\langle Au_n, v_n\rangle \to 0$$

because $Au_n \to 0$. Therefore $v_n \to v = 0$. Now multiplying (4) by u_n we obtain

$$d_1\langle u_n, u_n\rangle + \lambda_n\langle Au_n, u_n\rangle = t_n\big(b_{11}\langle Au_n, u_n\rangle + b_{12}\langle Av_n, u_n\rangle\big) \to 0$$

and therefore $u_n \to 0$. Hence $U_n \to 0$ and a contradiction again follows. $\square$

Lemma 4. *Let $d^0 = [d_1^0, d_2^0] \in C$ and let the condition* (v) *be satisfied. Then there are $\varepsilon > 0$ and a neighbourhood $\mathcal{U}(d^0)$ of d^0 such that $E_{I_d}(\lambda) = \{[0,0]\}$ (then also $\lambda \notin \sigma_{I_d}$) for any $d \in \mathcal{U}(d^0) \cap D_S$ and $\lambda \in [0, \varepsilon)$.*

P r o o f (we follow the ideas employed in [6] in a more general way). Let $d^n = [d_1^n, d_2^n] \in D_S$, $U_n = [u_n, v_n] \in \mathbb{H} \times \mathbb{H}$ and $\lambda_n \geqslant 0$ be such that $d^n \to d^0$, $|U_n| = 1$, $u_n \rightharpoonup u$, $v_n \rightharpoonup v$ in $\mathbb{H}$, $\lambda_n \to 0$ and

$$d_1^n u_n - b_{11} A u_n - b_{12} A v_n + \lambda_n A u_n = 0, \tag{7}$$

$$d_2^n v_n - P_{d_2^n v_n}(b_{21} A u_n + b_{22} A v_n - \lambda_n A v_n) = 0. \tag{8}$$

According to Lemma 1 we have $u_n \to u$, $v_n \to v$ in $\mathbb{H}$ and $U = [u, v] \in E_{I_{d^0}}(0)$. By Lemma 2 then $U \in E_{d^0}$ and therefore

$$d_1^0 u - b_{11} A u - b_{12} A v = 0, \tag{9}$$

$$d_2^0 v - b_{21} A u - b_{22} A v = 0. \tag{10}$$

First, let d^0 be not a point of intersection of two different hyperbolas: $d^0 \in C_j$, $d^0 \notin C_{j-1}$, $d^0 \notin C_{j+k}$ where k is the multiplicity of the eigenvalue κ_j (see Theorem 0). According to Theorem 0 there are also $\bar{d}^n$, $\overline{U}_n$ such that

$$\bar{d}^n = [\bar{d}_1^n, d_2^n] \in C_j,\ \overline{U}_n = [\bar{u}_n, v] \in E_{\bar{d}^n},\ \bar{d}_1^n < d_1^n,\ \bar{d}^n \to d^0,\ \overline{U}_n \to U \text{ in } \mathbb{H} \tag{11}$$

(see Fig. 1). Indeed, $u = \sum\limits_{i=j}^{j+k-1} c_i e_i$, $v = \sum\limits_{i=j}^{j+k-1} c_i \alpha_i(d^0) e_i$ (see Theorem 0). The general solution $\overline{U} = [\bar{u}, \bar{v}] \in E_d$ is for any $d \in C_j$ also in the form $\bar{u} = \sum\limits_{i=j}^{j+k-1} \bar{c}_i e_i$, $\bar{v} = \sum\limits_{i=j}^{j+k-1} \bar{c}_i \alpha_i(d) e_i$ with some constants $\bar{c}_i$ and we can choose $\bar{c}_i$ so that $\bar{c}_i \alpha_i(d) = c_i \alpha_i(d^0)$ and thus $\bar{v} = v$. Let us note that this fact also implies that if the condition (v) is satisfied for some $d \in C_j$ which is not a point of intersection of two different hyperbolas then it holds on the whole C_j.

Since $\overline{U}_n = [\bar{u}_n, v] \in E_{\bar{d}^n}$ then

$$\bar{d}_1^n \bar{u}_n - b_{11} A \bar{u}_n - b_{12} A v = 0, \tag{12}$$

$$d_2^n v - b_{21} A \bar{u}_n - b_{22} A v = 0. \tag{13}$$

Multiplying (7) and (12) by $\bar{u}_n$ and u_n simultaneously we obtain after subtracting the resulting equations and taking into account the symmetry of the operator A the equation

$$(\bar{d}_1^n - d_1^n)\, \langle u_n, \bar{u}_n \rangle = \lambda_n\, \langle A u_n, \bar{u}_n \rangle - b_{12}\big(\, \langle A v_n, \bar{u}_n \rangle - \langle A v, u_n \rangle \,\big). \tag{14}$$

Multiplying (8) and (13) by v and v_n we similarly obtain using formula (14) that

$$\begin{aligned} (15) \qquad & \left\langle P_{d_2^n v_n}(b_{21}Au_n + b_{22}Av_n - \lambda_n Av_n) - (b_{21}Au_n + b_{22}Av_n - \lambda_n Av_n), v\right\rangle \\ & -\frac{b_{21}}{b_{12}}(d_1^n - \bar{d}_1^n)\langle u_n, \bar{u}_n\rangle - \lambda_n\Big(\frac{b_{21}}{b_{12}}\langle Au_n, \bar{u}_n\rangle + \langle Av_n, v\rangle\Big) = 0. \end{aligned}$$

We can write $v = (v + d_2^n v_n) - d_2^n v_n$ and we have also $v + d_2^n v_n \in K_{d_2^n v_n}$. Using (8) and the basic facts about projections we conclude that the first term on the left-hand side of (15) is not negative. But the second term is even positive for n sufficiently large according to (SIGN) because $|U| = 1$ and $\langle u_n, \bar{u}_n\rangle \to \|u\|^2 > 0$. It remains to prove that the limit of the third term is not negative, either (see also in [5]).

It is sufficient to show that $\frac{b_{21}}{b_{12}}\langle Au, u\rangle + \langle Av, v\rangle$ is negative: For $[u, v] \in E_{d^0}$ the equations (9) and (10) hold. Multiplying (9) by u and (10) by v and using the symmetry of A we obtain

$$(16) \qquad b_{12}\langle Au, v\rangle = d_1^0\|u\|^2 - b_{11}\langle Au, u\rangle,$$

$$(17) \qquad b_{21}\langle Au, v\rangle = d_2^0\|v\|^2 - b_{22}\langle Av, v\rangle.$$

Since the condition (SIGN) holds the equation (17) gives

$$(18) \qquad b_{21}\langle Au, v\rangle > 0.$$

Multiplying (10) by $b_{12}u$ and putting it into (16) we obtain

$$d_2^0 b_{12}\langle u, v\rangle = b_{12}b_{21}\langle Au, u\rangle + b_{12}b_{22}\langle Au, v\rangle = d_1^0 b_{22}\|u\|^2 - (b_{11}b_{22} - b_{12}b_{21})\langle Au, u\rangle.$$

From the assumption (SIGN) for $\mathcal{D}et\left(\{b_{ij}\}_{i,j=1}^2\right)$ and for b_{22} we obviously have $b_{12}\langle u, v\rangle < 0$ and hence

$$(19) \qquad b_{21}\langle u, v\rangle > 0.$$

Finally, multiplying (9) and (10) by $\frac{v}{b_{12}}$ and $\frac{u}{b_{12}}$ respectively and summing the resulting equations we know according to (18), (19) and (SIGN) that

$$\begin{aligned} \frac{b_{21}}{b_{12}}\langle Au, u\rangle + \langle Av, v\rangle &= (b_{12})^{-1}\big((d_1^0 + d_2^0)\langle u, v\rangle - (b_{11} + b_{22})\langle Au, v\rangle\big) \\ &= (b_{12}b_{21})^{-1}\big((d_1^0 + d_2^0)b_{21}\langle u, v\rangle - (b_{11} + b_{22})b_{21}\langle Au, v\rangle\big) < 0. \end{aligned}$$

Summing up, we have proved that the left-hand side in the equation (15) is positive for n large enough and this is a contradiction with zero on the right-hand side and the proof is complete in the case of d^0 being not a point of intersection.

It remains to study the case $d^0 \in C_j \cap C_{j+k}$, $d^0 \notin C_{j-1}$. According to Theorem 0 we have $U = \overline{U} + \tilde{U}$, $\overline{U} = [\bar{u}, \bar{v}]$, $\tilde{U} = [\tilde{u}, \tilde{v}]$, $\bar{u} = \sum_{i=j}^{j+k-1} \bar{c}_i e_i$, $\bar{v} = \sum_{i=j}^{j+k-1} \bar{c}_i \alpha_i(d^0) e_i$, $\tilde{u} = \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i e_i$, $\tilde{v} = \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i \alpha_i(d^0) e_i$ and there are $\tilde{d}^n$, $\tilde{U}_n$ and $\bar{d}^n$, $\overline{U}_n$ such that

$$
\begin{aligned}
(20) \qquad & \bar{d}^n = [\bar{d}_1^n, d_2^n] \in C_j, \quad \tilde{d}^n = [\tilde{d}_1^n, d_2^n] \in C_{j+k}, \\
& \bar{d}_1^n < d_1^n, \quad \tilde{d}_1^n < d_1^n, \quad \bar{d}^n \to d^0, \quad \tilde{d}^n \to d^0, \\
& \overline{U}_n = [\bar{u}_n, \bar{v}] \in E_{\bar{d}^n}, \quad \tilde{U}_n = [\tilde{u}_n, \tilde{v}] \in E_{\tilde{d}^n}, \quad \overline{U}_n + \tilde{U}_n \to U \text{ in } \mathbb{H}
\end{aligned}
$$

again (we also can fix the component v on each hyperbola as above so that $\bar{v} + \tilde{v} = v$). Since A is a linear operator, $\overline{U}_n = [\bar{u}_n, \bar{v}] \in E_{\bar{d}^n}$ and $\tilde{U}_n = [\tilde{u}_n, \tilde{v}] \in E_{\tilde{d}^n}$ we obtain

$$
\bar{d}_1^n \bar{u}_n + \tilde{d}_1^n \tilde{u}_n - b_{11} A(\bar{u}_n + \tilde{u}_n) - b_{12} A v = 0, \tag{21}
$$

$$
d_2^n v - b_{21} A(\bar{u}_n + \tilde{u}_n) - b_{22} A v = 0. \tag{22}
$$

Analogously as above, only replacing equations (12) and (13) by (21) and (22), we obtain

$$
\begin{aligned}
(23) \qquad & \left\langle P_{d_2^n v_n}(b_{21} A u_n + b_{22} A v_n - \lambda_n A v_n) - (b_{21} A u_n + b_{22} A v_n - \lambda_n A v_n), v \right\rangle \\
& \quad - \frac{b_{21}}{b_{12}} (d_1^n - \bar{d}_1^n) \langle u_n, \bar{u}_n \rangle - \frac{b_{21}}{b_{12}} (d_1^n - \tilde{d}_1^n) \langle u_n, \tilde{u}_n \rangle \\
& \quad - \lambda_n \Big(\frac{b_{21}}{b_{12}} \langle A u_n, \bar{u}_n + \tilde{u}_n \rangle + \langle A v_n, v \rangle \Big) = 0.
\end{aligned}
$$

The first term is the same as in (15) so it is not negative. Neither is the fourth term negative because the limit is the same as the limit of the corresponding term in (15). Since (SIGN) is assumed it remains to prove that $\langle u_n, \bar{u}_n \rangle > 0$ and $\langle u_n, \tilde{u}_n \rangle > 0$ for n large enough. Theorem 0 and (20) imply similarly as in [3] that $\langle u_n, \bar{u}_n \rangle \to \langle u, \bar{u} \rangle$ and $\langle u_n, \tilde{u}_n \rangle \to \langle u, \tilde{u} \rangle$ where

$$
\bar{u} = \sum_{i=j}^{j+k-1} \bar{c}_i e_i, \quad \tilde{u} = \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i e_i \qquad \text{and} \qquad u = \sum_{i=j}^{j+k-1} \bar{c}_i e_i + \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i e_i.
$$

From the orthonormality of $\{e_j\}_{j=1,\dots}$ we easily obtain

$$
\langle u, \bar{u} \rangle = \sum_{i=j}^{j+k-1} \bar{c}_i^2 \langle e_i, e_i \rangle = \sum_{i=j}^{j+k-1} \bar{c}_i^2 > 0
$$

and

$$
\langle u, \tilde{u} \rangle = \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i^2 \langle e_i, e_i \rangle = \sum_{i=j+k}^{j+k+m-1} \tilde{c}_i^2 > 0.
$$

In conclusion, the left-hand side of (23) is positive again and the contradiction follows also in the second case. $\square$

Lemma 5 (see [6], Lemma 2.4, for related results also [16]). *Let $d^0 = [d_1^0, d_2^0] \in C$ be such that the condition* (v) *is satisfied. Then there is a neighbourhood $\mathcal{W}(d^0)$ of d^0 such that $\deg\big(T(d,0,0,.\,), B_1(0), 0\big) = 0$ for any $d \in \mathcal{W}(d^0) \cap D_S$.*

Lemma 6. *Let $d^0 = [d_1^0, d_2^0] \in C$ and let the condition* (v) *be satisfied. Then there is $\varepsilon > 0$ such that*

$$\deg\big(T(d,\lambda,0,.\,), B_1(0), 0\big) = 0 \quad \forall d \in \mathcal{W}(d^0) \cap D_S \quad \forall \lambda \in [0,\varepsilon)$$

where $\mathcal{W}(d^0)$ is the neighbourhood from Lemma 5.

P r o o f. The assertion follows immediately from Lemma 5 by the homotopy invariance of the Leray-Schauder degree with the homotopy

$$H(t,U) = \begin{bmatrix} u - (d_1)^{-1}(b_{11}Au + b_{12}Av - t\lambda Au) \\ v - P_v\big((d_2)^{-1}(b_{21}Au + b_{22}Av - t\lambda Av)\big) \end{bmatrix}.$$

We can take ε from Lemma 4. For any $d \in \mathcal{W}(d^0)$, $\lambda \in [0,\varepsilon)$, $r > 0$ and $t \in [0,1]$ we know that there is no $U \in \partial B_1(0)$ such that $H(t,U) = 0$ because of $E_{I_d}(\lambda) = \{[0,0]\}$ (see Lemma 4 again). $\square$

P r o o f of Theorem 1. Let $\mathcal{W}(d^0)$ be from Lemma 5, $d \in \mathcal{W}(d^0) \cap D_S$, $\lambda_1 \in (0,\varepsilon)$, $\lambda_2 \geqslant k$. Let there be no solution U of the problem (EP$_\mathrm{W}$) such that $U \neq [0,0]$ and $\|U\| = 1$ for any $\lambda \in [\lambda_1, \lambda_2]$. Then due to the homotopy invariance of the Leray-Schauder degree, Lemma 3 and Lemma 6 we have

$$0 = \deg\big(T(d,\lambda_1,0,.\,), B_1(0), 0\big) = \deg\big(T(d,\lambda_2,0,.\,), B_1(0), 0\big) = 1$$

because we can take the homotopy

$$H(t,U) = \begin{bmatrix} u - (d_1)^{-1}\big(b_{11}Au + b_{12}Av - (t\lambda_1 + (1-t)\lambda_2)Au\big) \\ v - P_v\big((d_2)^{-1}(b_{21}Au + b_{22}Av - (t\lambda_1 + (1-t)\lambda_2)Av)\big) \end{bmatrix}$$

arriving at a contradiction. In conclusion, for any $d \in \mathcal{W}(d^0) \cap D_S$ there is $\lambda \in [\lambda_1, \lambda_2]$ such that the problem (EP$_\mathrm{W}$) has a nontrivial solution. It means that $\lambda \in \sigma_{I_d}$ is a positive eigenvalue. $\square$

P r o o f of Corollary 1. Let $[u_0, v_0]$ be a nontrivial solution of the problem (EP$_\mathrm{W}$) for some $\lambda > 0$ (see Theorem 1). Then $U(t) = [e^{\lambda t}\tau u_0, e^{\lambda t}\tau v_0]$ is a solution of the evolution problem ((RD$_\mathrm{L}$), (UC)) for any $\tau > 0$. Now, for any $\delta > 0$ there is $\tau > 0$ sufficiently small such that for $t_0 = 0$ we have $\|U(0)\| < \delta$ and simultaneously $\lim\limits_{t\to\infty} \|U(t)\| = +\infty$. Hence, by the standard definition (cf., e.g., [3]) the solution $\overline{U} = [0,0]$ is unstable. $\square$

P r o o f of Corollary 2. Let $\omega > 0$ be fixed small enough. It is clear that there is $\varrho > 0$ small enough such that $E_{I_{d(s)}}(\lambda) = \{[0,0]\}$ also holds for all $\lambda \in [0,\varrho)$ and $s \in [0, 1-\omega]$. The reason is that in the opposite case we find $\lambda_n \to 0$, $s_n \in [0, 1-\omega]$ and $U_n = [u_n, v_n] \in \mathbb{H} \times \mathbb{H}$ such that $\|U_n\| = 1$ and $U_n \in E_{I_{d(s_n)}}(\lambda_n)$. We extract a convergent subsequence $s_n \to s \in [0, 1-\omega]$ and obtain by the limit procedure based on Lemma 1 (as in the proof of Lemma 4) also $U_n \to U$, $\|U\| = 1$ and $U \in E_{I_{d(s)}}(0)$. So, we have a contradiction with the assumption.

Now, by the standard homotopy argument as in the proof of Lemma 6 we obtain

$$\deg\bigl(T(d(s),\lambda,0,.\,),B_1(0),0\bigr) = 0 \quad \forall s \in [0,1-\omega] \quad \forall \lambda \in [0,\varrho).$$

Finally, for fixed $s \in [0,1)$, if there is no positive eigenvalue $\lambda \in \sigma_{I_{d(s)}}$ then as in the proof of Theorem 1 according to Lemma 3 by the homotopy argument again we obtain

$$0 = \deg\bigl(T(d(s),\lambda_1,0,.\,),B_1(0),0\bigr) = \deg\bigl(T(d(s),\lambda_2,0,.\,),B_1(0),0\bigr) = 1$$

for some $\lambda_1 \in (0,\varrho)$ and $\lambda_2 \geqslant k$, which yields a contradiction. Thus, there is $\lambda \in [\lambda_1,\lambda_2]$ a positive eigenvalue of the problem ((RD$_\mathrm{L}$), (UC)) with the parameter $d(s)$. $\square$

Acknowledgement. I would like to thank Dr. Milan Kučera for inspiring and fruitful discussions about the subject.

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