Applications of Mathematics

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Ninth international school on mathematical theory in fluid mechanics

NINTH INTERNATIONAL SCHOOL ON MATHEMATICAL THEORY IN FLUID MECHANICS

FOREWORD*

The Ninth International School on *Mathematical Theory in Fluid Mechanics* was held at the small village of Paseky in the northern part of the Czech Republic in June 2005. The main portion of the program consisted of lecture courses delivered by Diego Córdoba (Madrid, Spain), Volker John (Saarbrücken, Germany), Guiseppe R. Mingione (Parma, Italy) and Ping Zhang (Beijing, China). The texts collected here are based on these lectures.

The problem of existence of singularities in unsteady flows of incompressible fluids is considered to be one of the most interesting problems in contemporary applied mathematics. *Diego Córdoba*, in the first part of his article entitled *On the search for singularities in incompressible flows* provides an overview of certain heuristic physically motivated scenarios of appearance of singularities, and relates them to the regularity properties of the solution. The second part of Córdoba's contribution discusses numerical results that indicate the formation of singularities for the quasi-geostrophic equation and completes them by relevant analytical results.

Next, *Volker John* gives a timely overview of the state of the art in numerical models for turbulent flows. The emphasis is put on the two currently most promising approaches, namely Large Eddy Simulation (LES) and Variational Multiscale Method (VMS). A detailed analysis of several variants of LES approach is presented including the influence of the modeling and computation errors. The VMS approach is derived with a short discussion of numerical aspects of discretization. Relevant references to the current literature are supplied for all topics discussed here.

The lecture notes of *Rosario Mingione* provide a unique survey of regularity/irregularity results concerning minimizers of variational integrals as well as solutions of non-linear elliptic and parabolic systems. The article entitled *Regularity of minima: an invitation to the dark side of the calculus of variations* discusses

in length the up-to-date results concerning the Hölder, Lipschitz, $C^{1,\alpha}$-regularity, higher integrability, partial regularity, constructions of counterexamples, and all this both in the scalar and the vectorial case.

The final paper *Weak solutions to a nonlinear variational wave equation and some related problems*, written by *Ping Zhang*, concerns existence (and possible regularity) of (a suitable class of) solutions to three problems of hyperbolic type. In particular, the first part is devoted to the existence of a dissipative weak solution to the so-called asymptotic wave equation. The second part deals with the global existence of weak solutions to the (full) nonlinear wave equation. The final part is devoted to the so-called vortex density equations, and the global existence of renormalized solution is proved. The proofs employ the modern techniques of compensated compactness, as are Young measures, Div-Curl lemma, the method of estimating the propagation of oscillations and renormalization. These methods are shortly reviewed in the introductory part of Ping Zhang's contribution.

We would like to use the opportunity to thank all participants of the Ninth Fluid Mechanics School at Paseky for their remarkable interest in lecture courses, for presenting their scientific achievements in short communications and in the form of preprints, reprints and book exhibition.

Finally, we would like to announce the forthcoming Tenth School to be held at Paseky between May 26, 2007 and June 1, 2007. The list of lecturers, descriptions of their courses and further information are available on

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`http://www.karlin.mff.cuni.cz/paseky-fluid/`

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